MSEAS: Multiscale two-way embedding schemes for free-surface primitive-equations

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Thanks to W.G. Leslie, A. Agarwal, O. Logutov, T. Sapsis and M. Ueckermann

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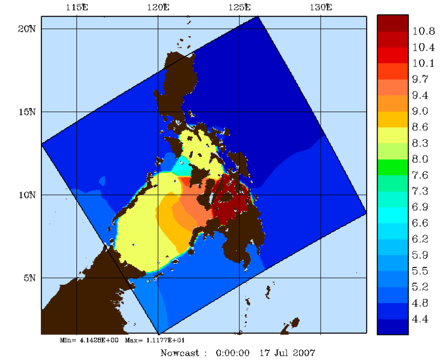
- **Introduction: MSEAS**
- **Motivations**
- **Equations and “Implicit in time and space” 2-way Nesting Schemes**
- **Realistic Simulations: Middle Atlantic Bight , Philippines Archipelago and Taiwan/Kuroshio Region**
- **Conclusions**

Thanks to ONR

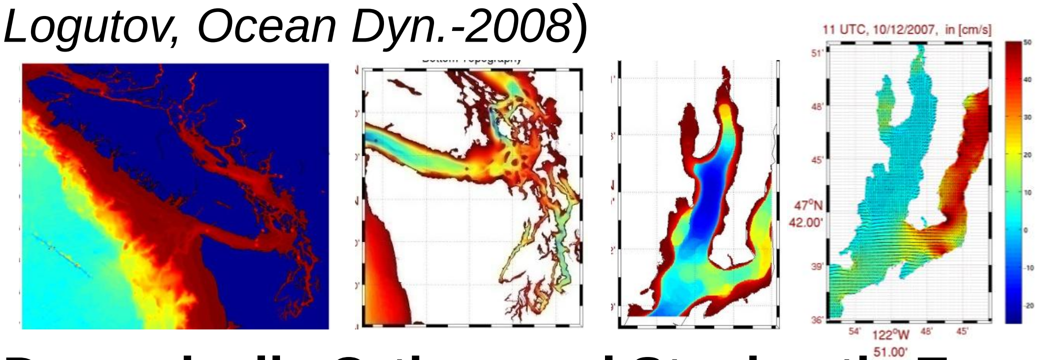
New MSEAS Methods and Codes

- **Two new OA schemes to remove correlations across land but introduce 3D effects** (*Agarwal and Lermusiaux, 2010*) based on:
  1. Fast Marching Method or Level Sets Method
  2. Numerical Diffusion Equation

OA of T at 1000m in Philippines Strait

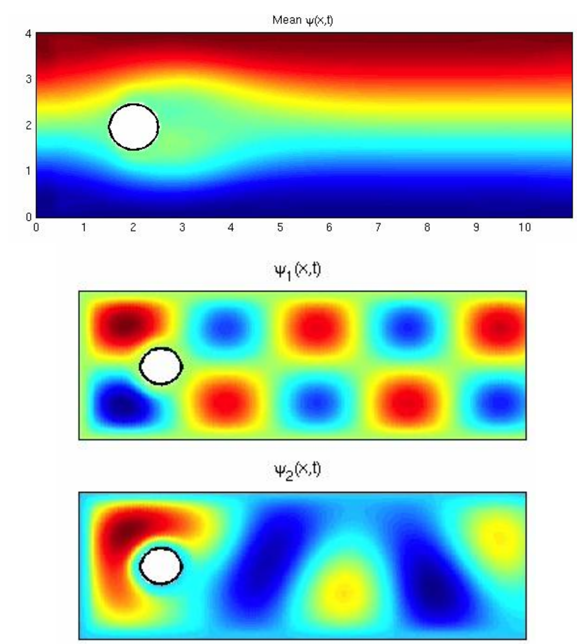


- **New Nested Barotropic Tidal Prediction and Inversion** (*Logutov and Lermusiaux, Ocean Mod.-2008; Logutov, Ocean Dyn.-2008*)



- **Dynamically Orthogonal Stochastic Equations** (*Sapsis and Lermusiaux, Phys-D-2009*): New efficient (S)-PDEs for uncertainty EOFs and coefficients

$$\boldsymbol{u}(\boldsymbol{x},t;\omega)=\bar{\boldsymbol{u}}(\boldsymbol{x},t)+\sum_{i=1}^{N}Y_i(t;\omega)\,\boldsymbol{u}_i(\boldsymbol{x},t)$$



- **Coupled to 2d/3d acoustic models** (*Xu et al, 2008, 2010; Lermusiaux and Xu, 2010*)

Nesting Motivations

- Goal: resolve tidal-to-mesoscale processes/interactions over multi-resolution telescoping domains with complex geometries, focusing on dynamics at steep shelfbreaks, including shallow seas with strong tides and deep ocean interactions
- Why?
  - Shelfbreaks everywhere at edges of coastal and deep regions
  - Found most sensitive to resolution: ultimately, combine structured and unstructured grids (see Ueckermann, Lermusiaux and Haley)
  - Most ocean models developed either for coastal (e.g. POM) or deep oceans (e.g. MOM), then extended to the "other side of the shelfbreak"
- Multiscale Applications:
  - PhilEx: physical-biological processes in complex Philippines Archipelago/Straits
  - Quantifying, Predicting and Exploiting Uncertainty (QPE) in Taiwan-Kuroshio region (Cold Dome, Kuroshio, Eddies, Typhoon, Internal tides/waves, Canyons)
  - Shallow Water-06/AWACS in New Jersey Shelf/Hudson Canyon region (shelfbreak front, shelf-slope, storms, generation/propagation of internal tides).

Our Free-Surface Primitive Equations

(Navier-Stokes equations with thin-film and Boussinesq approximations)

$$\frac{d}{dt}\int_V \vec{u}\,dV + \vec{\Gamma}(\vec{u}) + \int_V f\hat{k}\times\vec{u}\,dV = -\frac{1}{\rho_0}\int_S p\hat{n}_h\cdot dA + \int_V \vec{F}\,dV$$

$$\int_S p\hat{k}\cdot dA = -\int_V \rho g\,dV$$

$$\int_S (\vec{u},w)\cdot dA = 0$$

$$\frac{d}{dt}\int_V T\,dV + \Gamma(T) = \int_V F^T\,dV$$

$$\frac{d}{dt}\int_V S\,dV + \Gamma(S) = \int_V F^S\,dV$$

Vector Notation:

$$\vec{u} = \begin{pmatrix} u \\ v \end{pmatrix}$$

Notation for advection operators:

$$\Gamma(\phi) = \int_s \phi(\vec{u},w)\cdot dA \qquad \vec{\Gamma}(\vec{u}) = \begin{pmatrix} \Gamma(u) \\ \Gamma(v) \end{pmatrix}$$

Haley and Lermusiaux (2010) Multiscale two-way embedding schemes for free-surface primitive-equations in the "Multidisciplinary Simulation Estimation and Assimilation System" (MSEAS) *Ocean Dynamics*. (in review)

Discretized Equations: B-grid in space, leap-frog time differencing

Decompose velocity into depth-averaged and internal modes

Internal eqns

$$\frac{\hat{\delta}\left(\vec{u}\Delta\mathcal{V}\right)}{\tau} + \alpha f \hat{k} \times \hat{\delta}\left(\vec{u}\Delta\mathcal{V}\right) =$$

$$\hat{\mathcal{F}}^{n,n-1} - g\left(\Delta\mathcal{V}\nabla\eta\right)^{\tilde{\alpha}} - f\hat{k} \times \left(\vec{u}\Delta\mathcal{V}\right)^{\tilde{\alpha}}$$

$$\left(\vec{u}'\Delta\mathcal{V}\right)^{n+1} = \left(\widehat{\vec{u}\Delta\mathcal{V}}\right)^{n+1} - \frac{\Delta\mathcal{V}^{MSL}}{H}\sum_{k=1}^{K}\frac{\left(\widehat{\vec{u}\Delta\mathcal{V}}\right)^{n+1}}{\Delta\mathcal{V}^{MSL}} dz^{MSL}$$

$$\frac{\delta(T\Delta V)}{\tau} = F^{T} - \Gamma(T)^{n}; \qquad \frac{\delta(S\Delta V)}{\tau} = F^{S} - \Gamma(S)^{n}$$

Depth-avg eqns:

$$\vec{F}^{n,n-1} = \int_{-H}^{\eta}\left(-\frac{1}{\rho_0}\int_S p_h^n \hat{n}_h \cdot dA - \vec{\Gamma}(\vec{u})^n + \int_V \vec{F}^{n,n-1} dV\right)dz$$

$$\hat{\delta}\vec{U} + \alpha f \tau \hat{k} \times \hat{\delta}\vec{U} = \tau\left\{\vec{\mathcal{F}}^{n,n-1} - g\nabla\eta^{\tilde{\alpha}}\right\} \quad ,$$

$$\alpha\theta g\tau\nabla\cdot\left[\left(H+\eta^n\right)\nabla\delta\eta\right] - \theta\nabla\cdot\left(\left.\vec{u}'^{\,n}\right|_{\eta}\delta\eta\right) - \frac{2\delta\eta}{\tau} =$$

$$\nabla\cdot\left[\left(H+\eta^n\right)\left(\theta\hat{\vec{U}}^{n+1} + \vec{U}^n + (1-\theta)\vec{U}^{n-1}\right)\right] \quad ,$$

$$\vec{U}^{n+1} = \hat{\vec{U}}^{n+1} - \alpha\tau g\nabla\delta\eta + \frac{\left.\vec{u}'^{\,n}\right|_{\eta}}{H+\eta^n}\frac{\delta\eta}{\tau}$$

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where $\vec{u} = \vec{u}' + \vec{U}; \quad \vec{U} = \int_{-H}^{\eta} u\,dz;$

2nd order in time & space. Spherical coordinates.

Decompose pressure into hydrostatic and surface contributions

$$p = p_s + p_h$$

$$p_h(x,y,z) = \int_z^{\eta} g\rho\, d\zeta$$

$$p_s = \rho_0 g\eta$$

$$\delta U = U^{n+1} - U^{n-1}$$

$$\tau = 2\Delta t$$

Coriolis and continuity time discretizations of Dukowicz and Smith (1994)

$$U^{\alpha} = \alpha U^{n+1} + (1-2\alpha)U^{n} + \alpha U^{n-1}$$

$$U^{\theta} = \theta U^{n+1} + (1-\theta)U^{n}$$

Generalized vertical coordinates

Free surface variations spread throughout the water column

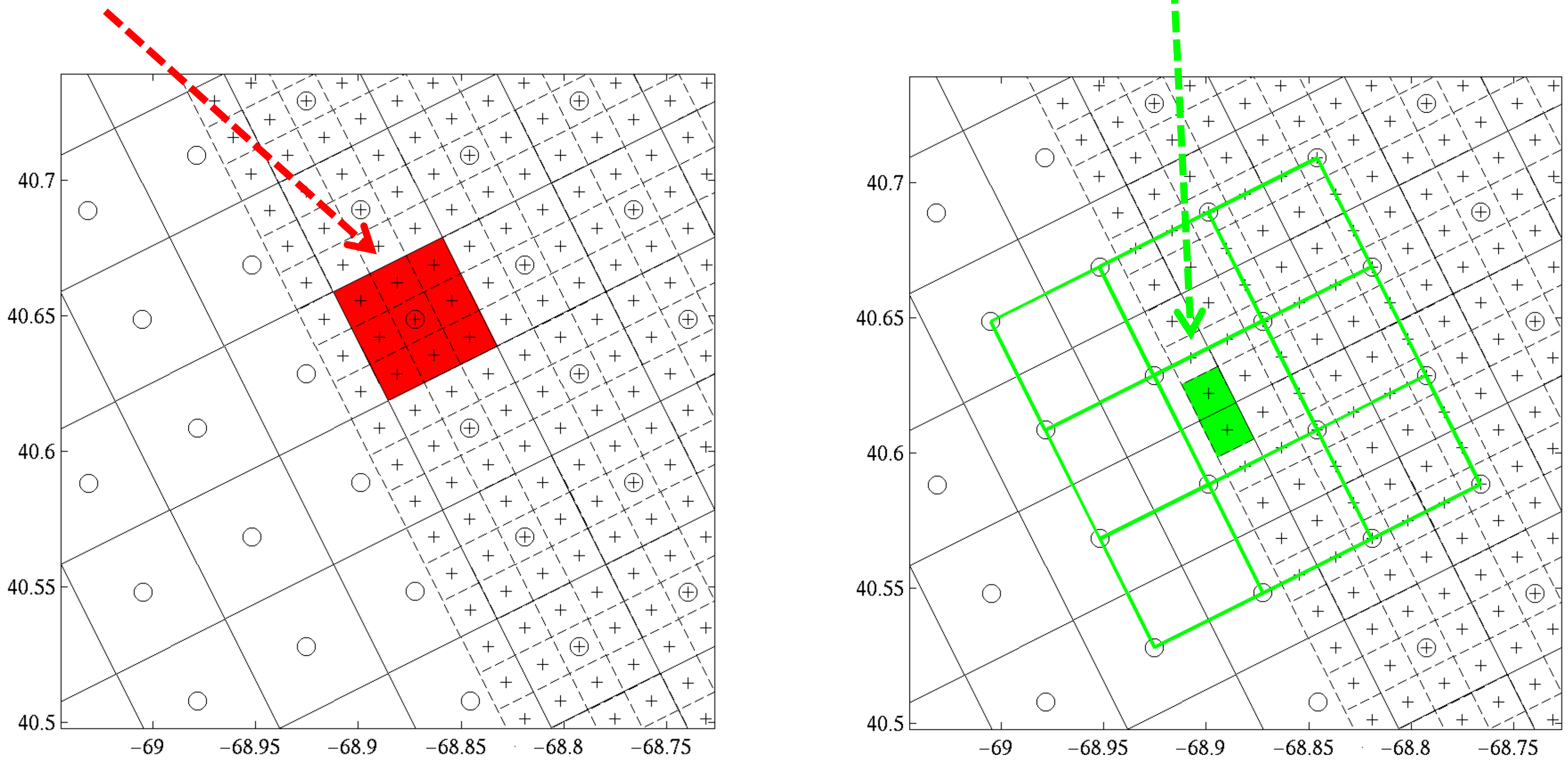
Our Nesting Basics and Grid Overview

Applicable to arbitrary grid refinement ratios.
Examples in this talk illustrated with 3:1 ratio.

- Each coarse grid cell is exactly subdivided into 9 fine grid cells.
- Values in coarse grid cells are replaced by averages of corresponding fine grid cells

- Values in boundary cells of fine grid replaced by bi-cubic interpolation from coarse grid

- Averaging and interpolation operations are applied to topography prior to simulation: cell volumes then satisfy same rules as fields



Implicit vs. Semi-Implicit Two-Way Nesting

Implicit Two-Way Nesting

- goal is to exchange all of the updated fields values as soon as they become available
- analogous to an implicit time stepping algorithm
- values are 2-way exchanged across multiple scales and nested grids within the same time step, for several fields (i.e. implicit in space and time)

Semi-Implicit Two-Way Nesting

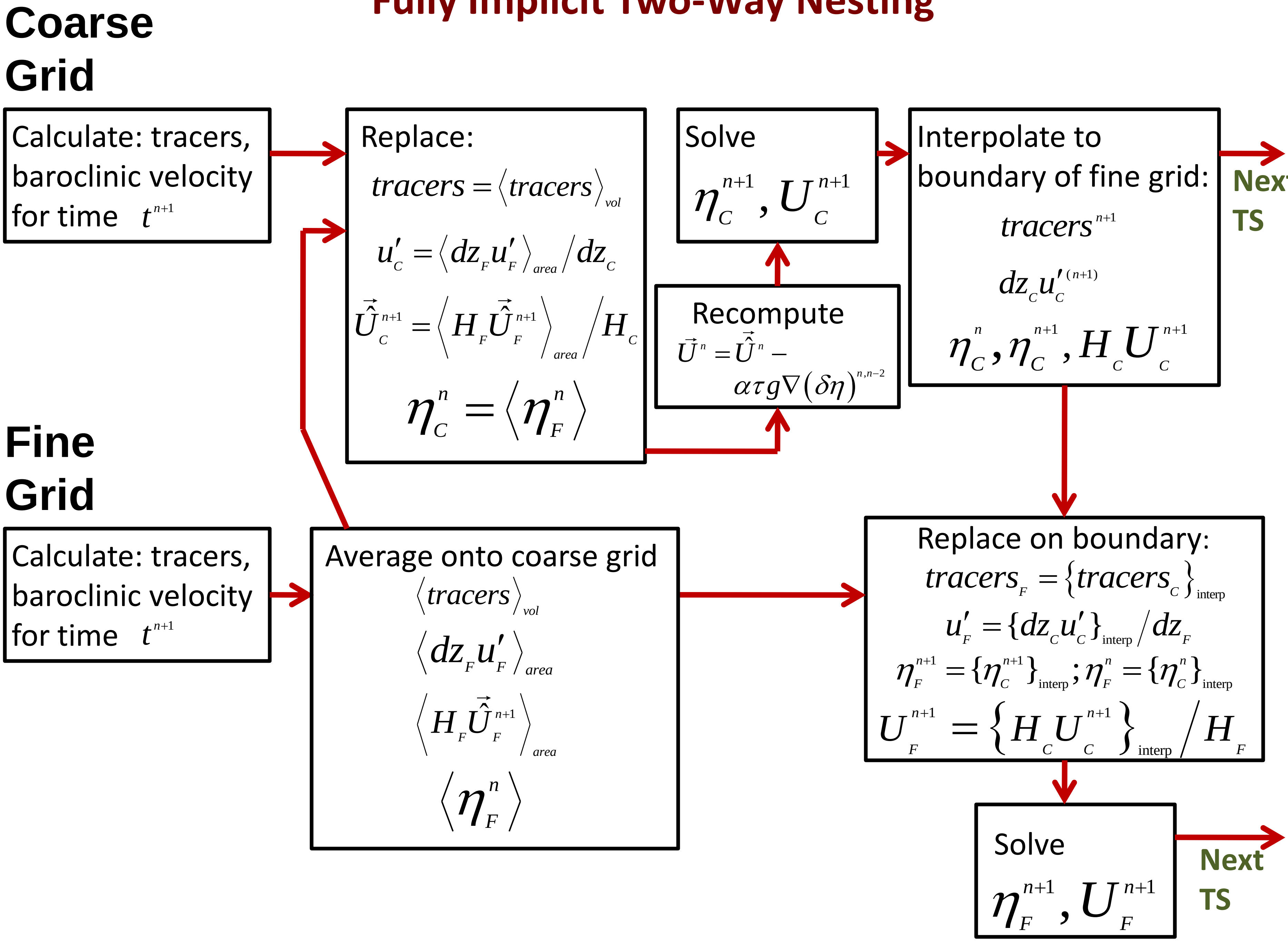
- Not all updated values are used immediately upon availability
- Specifically, coarse-to-fine boundary values do not make use of fine-to-coarse updates.

Implicit two-way nesting can, in some sense, be seen as refining grids in a single domain (e.g. Ginis et al., 1998)

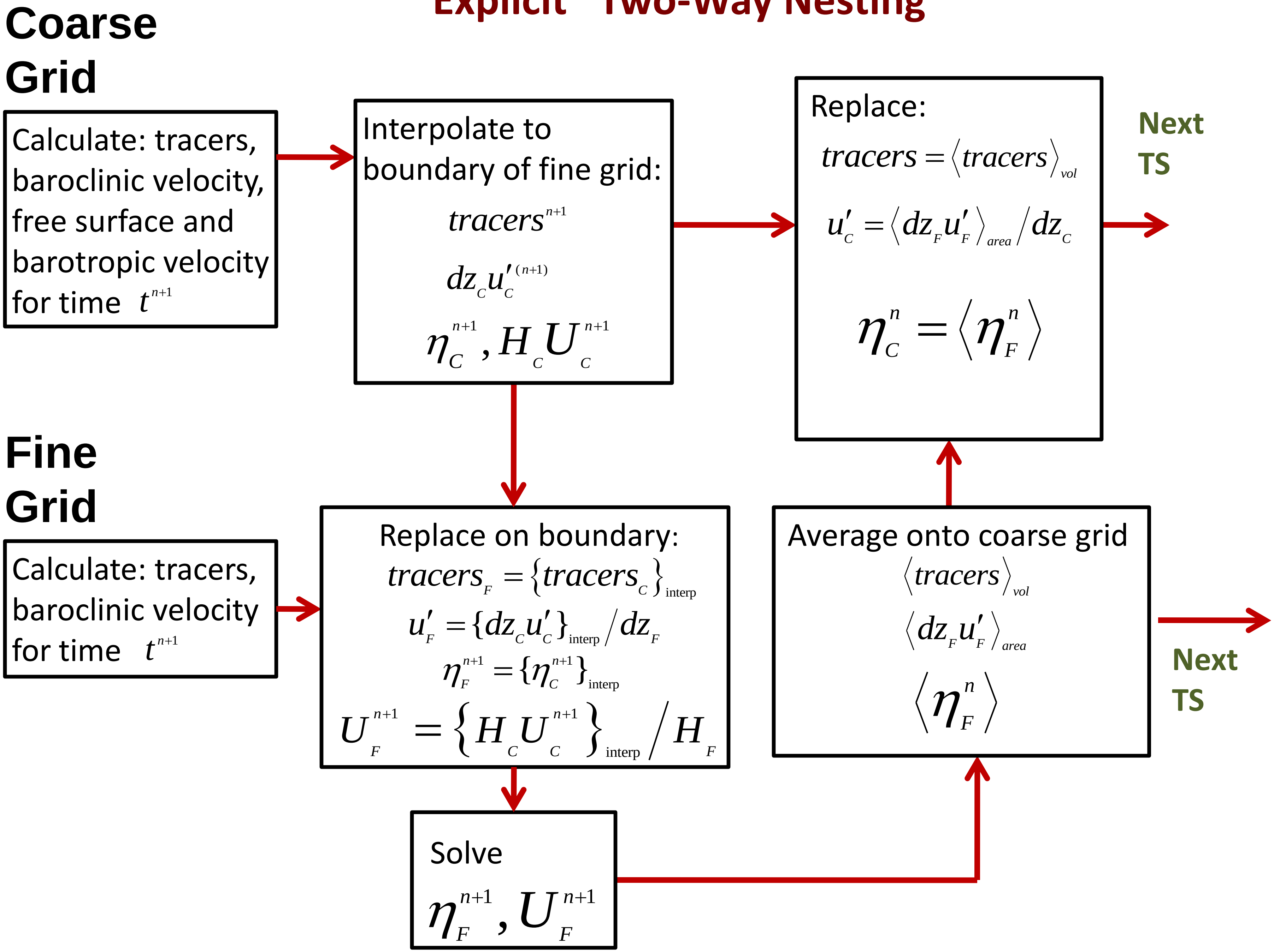
However, there are some advantages to the nesting paradigm.

- Time stepping can be easily refined for the finer domains.
- Model parameterizations (e.g. sub-grid-scales) can be tuned for the different scales in the different domains.
- Fundamentally different dynamics can be employed in the different domains (e.g. Shen and Evans, 2004; Maderich et al., 2008).
- Analogous but extends MIT-gcm 1D super-parameterizations to 3D multi-dynamics 2-way nesting

Fully Implicit Two-Way Nesting

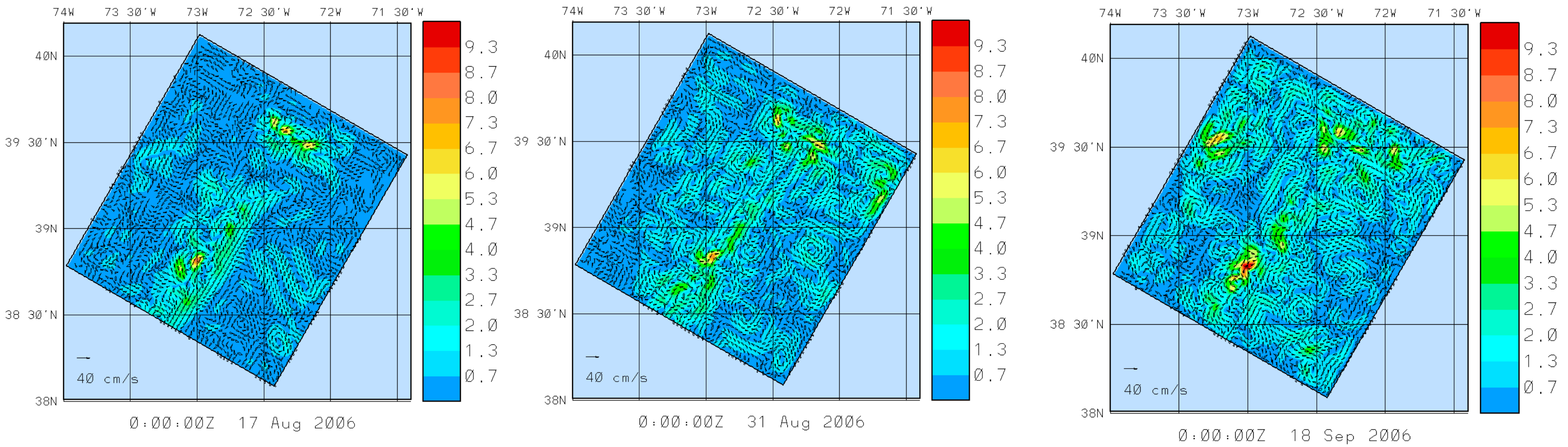


“Explicit” Two-Way Nesting

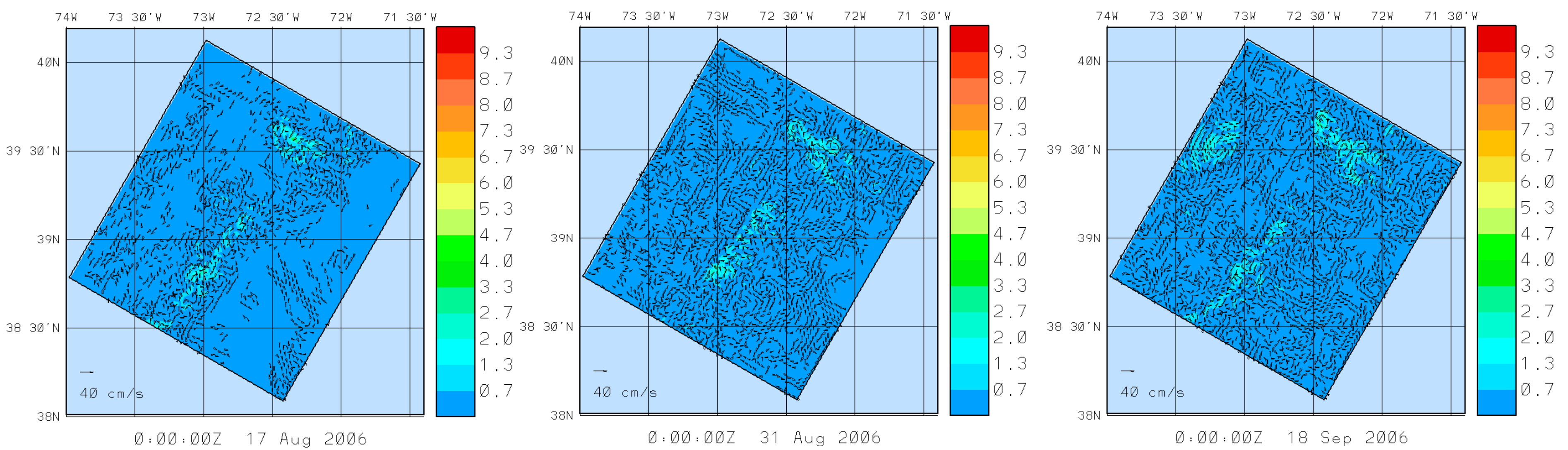


"Explicit" Nesting

(Interpolate from coarse to boundary of fine before averaging fine onto coarse)



Barotropic Velocity Difference (Coarse–Fine on Fine domain)



"Implicit" Nesting: Transfer $H\vec{\hat{U}}^{n+1}$, η^n, make U^n a $f\left(\eta^n\right)$

Error Analysis of Fine-to-Coarse Domain Feedback

Deriving the truncation error of the coarse grid values obtained by averaging fine grid values.

Midpoint approximation, second order accurate

$$\int_{\frac{-\Delta y}{2}}^{\frac{\Delta y}{2}} \int_{\frac{-\Delta x}{2}}^{\frac{\Delta x}{2}} \phi \, dxdy \equiv \left[\phi\big|_{(0,0)} + O\left(\Delta x^2\right) + O\left(\Delta y^2\right) \right] \Delta x \Delta y$$

Truncation error of Fine-to-Coarse averaging operator

$$\phi_{i_c, j_c} = \frac{1}{\Delta x_c \Delta y_c} \sum_{j_f = j_{fc}-1}^{j_{fc}+1} \sum_{i_f = i_{fc}-1}^{i_{fc}+1} \int_{\frac{-\Delta y_f}{2}}^{\frac{\Delta y_f}{2}} \int_{\frac{-\Delta x_f}{2}}^{\frac{\Delta x_f}{2}} \phi \, dxdy$$

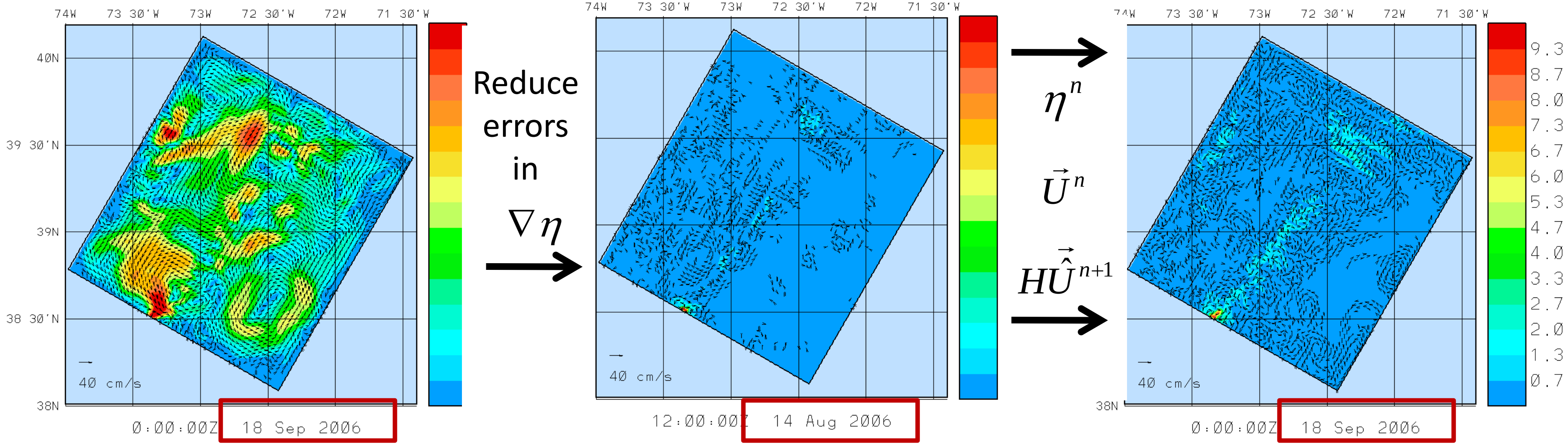
$$= \frac{1}{\Delta x_c \Delta y_c} \sum_{j_f = j_{fc}-1}^{j_{fc}+1} \sum_{i_f = i_{fc}-1}^{i_{fc}+1} \left[\phi_{i_f, j_f} + O\left(\Delta x_f^2\right) + O\left(\Delta y_f^2\right) \right] \Delta x_f \Delta y_f$$

$$= \frac{1}{\Delta x_c \Delta y_c} \sum_{j_f = j_{fc}-1}^{j_{fc}+1} \sum_{i_f = i_{fc}-1}^{i_{fc}+1} \left[\phi_{i_f, j_f} + O\left(\Delta x_f^2\right) + O\left(\Delta y_f^2\right) \right] \frac{1}{9} \Delta x_c \Delta y_c$$

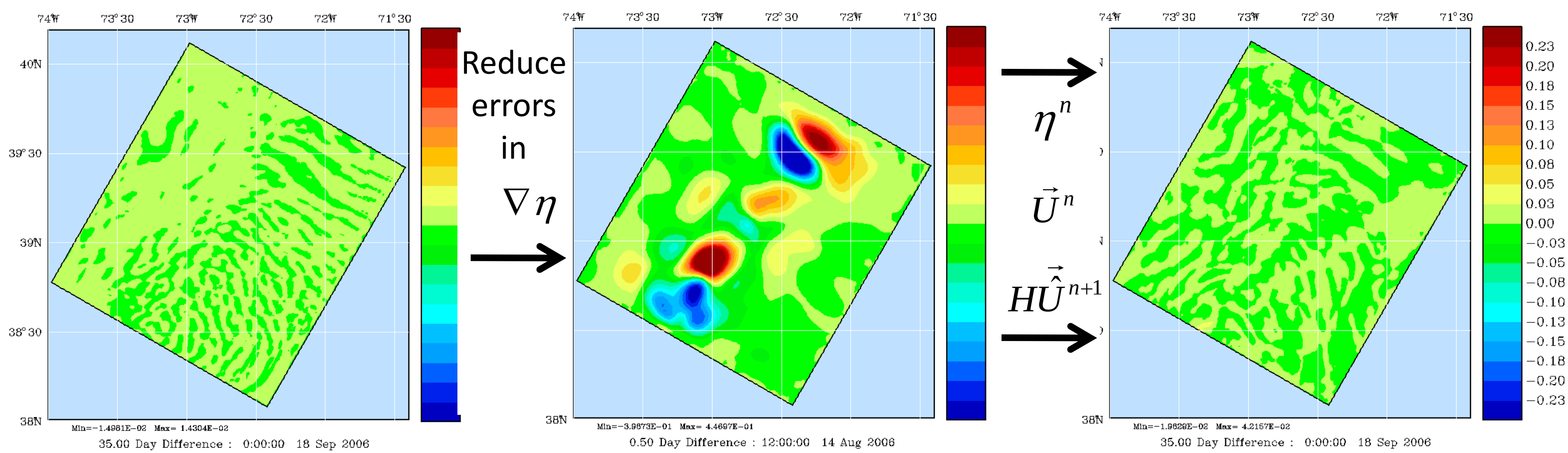
$$= \langle \phi \rangle_{i_{fc} \pm 1, j_{fc} \pm 1} + \langle O\left(\Delta x_f^2\right) \rangle_{i_{fc} \pm 1, j_{fc} \pm 1} + \langle O\left(\Delta y_f^2\right) \rangle_{i_{fc} \pm 1, j_{fc} \pm 1}$$

- Coarse domain value averaged from fine domain is second order in fine grid spacing
  - Even for only a 3:1 refinement, this is an order of magnitude reduction in coarse domain error (second order in coarse grid spacing)
- Averaging operator should be at least as accurate as overall numerical scheme

Original Scheme | **Transfer $\vec{\hat{U}}^{n+1}$ instead of $\vec{F}^{n,n-1}$** | **Transfer $(H\vec{\hat{U}}^{n+1}, \eta^n)$, make U^n a $f(\eta^n)$**



Barotropic Velocity Difference

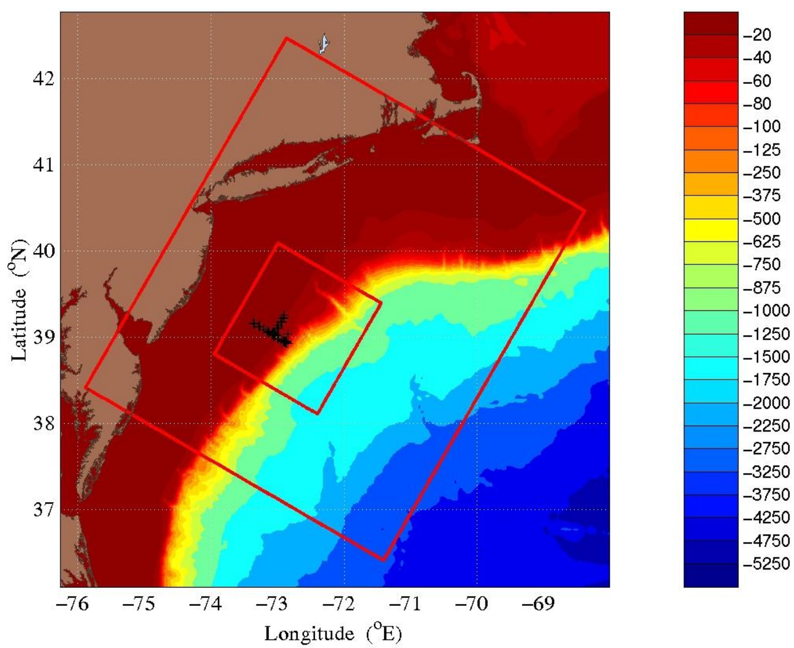


Surface Elevation Difference

Autonomous Wide Aperture Cluster for Surveillance (AWACS)

Goal

Develop and evaluate new environmental-acoustical adaptive sampling and search methodologies, and *improve the modeling of ocean dynamics*, for the environments in which the main AWACS-06, -07 and -09 experiments will occur, using the re-configurable REMUS cluster and coupled data

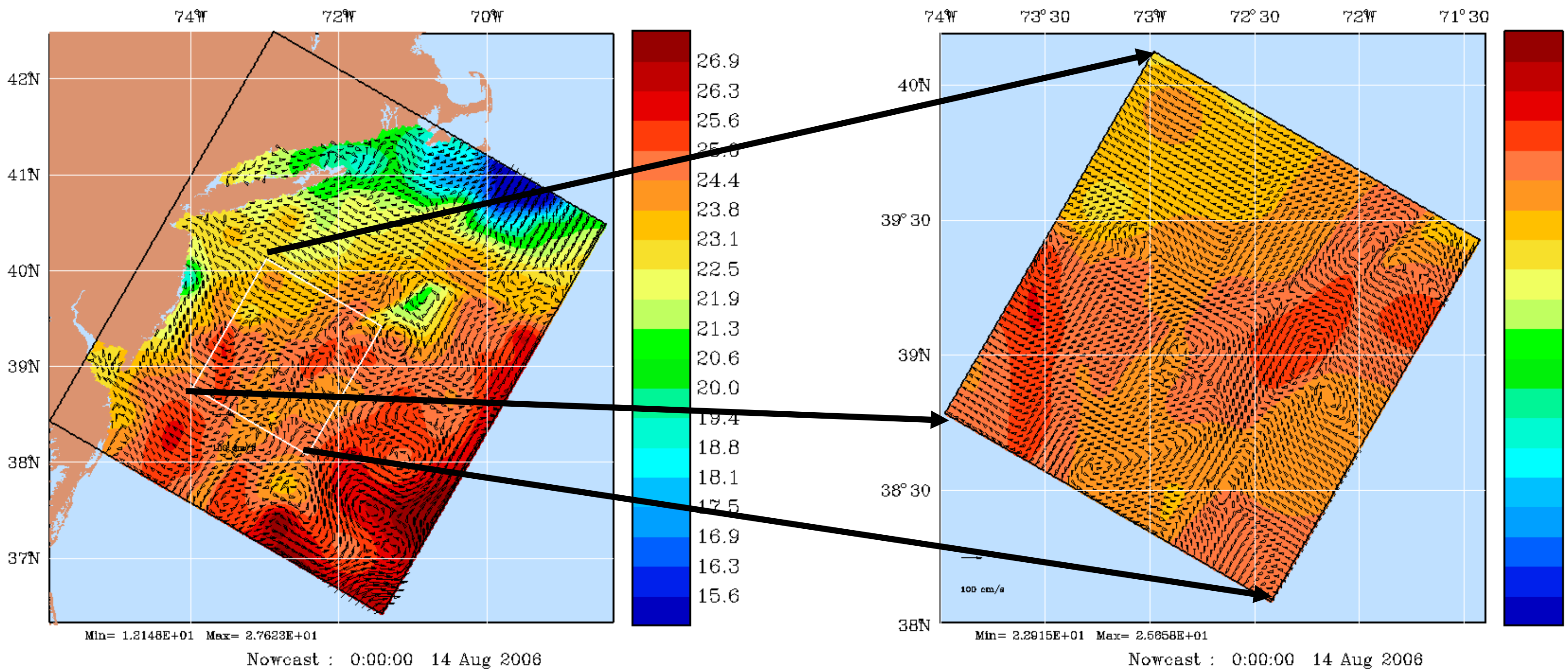


Specific Objectives

1. Evaluate current methods and develop new algorithms for adaptive environmental-acoustical sampling, search and coupled data assimilation techniques
2. Research optimal REMUS configurations for the sampling of interactions of the oceanic mesoscale with inertial oscillations, internal tides and boundary layers
3. *Improve models of (sub)-mesoscale ocean physics* and develop new adaptive ocean model parameterizations for specific regional AWACS processes.
4. Provide near real-time fields and uncertainties in AWACS experiments and *develop algorithms for fully-coupled physical-acoustical data assimilation among relocatable nested 3D domains*
5. Provide adaptive sampling guidance for array performance and surveillance , and link our MIT research with vehicle models and command and control.

Surface Temperature overlaid with surface velocity vectors in Middle-Atlantic Bight – Shelfbreak region: 14 Aug - 26 Sep 2006

2-way nested free-surface simulations, with data assimilation

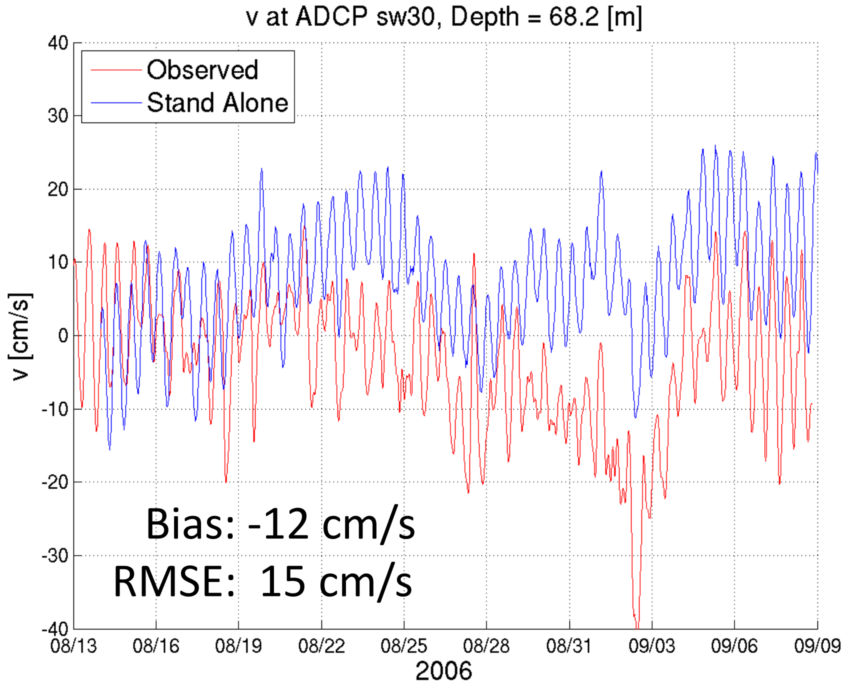


Coarse Grid – 3km **Fine Grid – 1km**

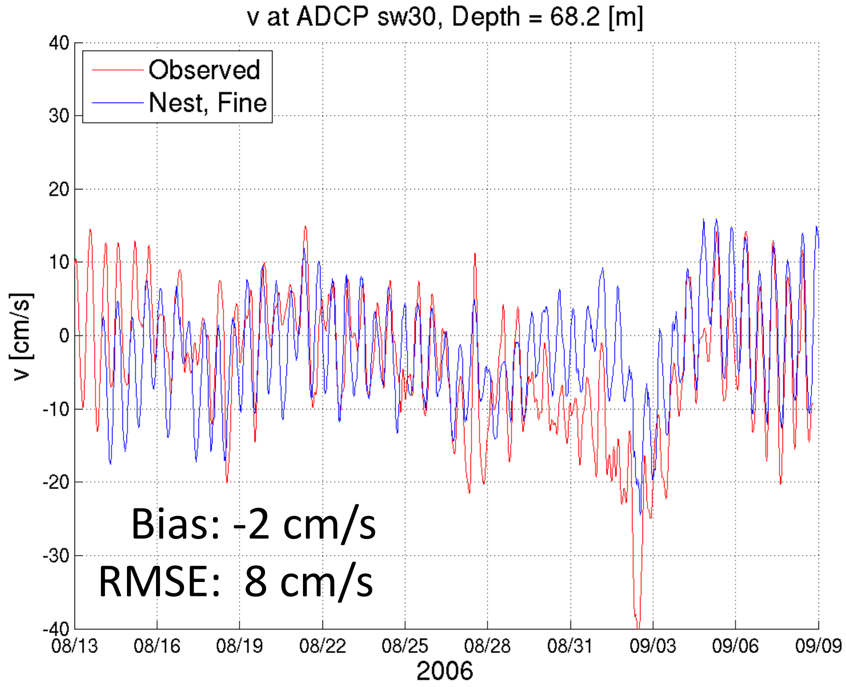
- Tropical storm Ernesto (1 Sep) cooled the surface and advected the shelfbreak front several kilometers offshore.
- In following relaxation, filaments are spun off of the shelfbreak front.

Modeling and Scientific studies of Tides/internal tides and their interactions with mesoscales

3-km grid resolution re-analysis



1-km nested into a 3-km grid resolution re-analysis



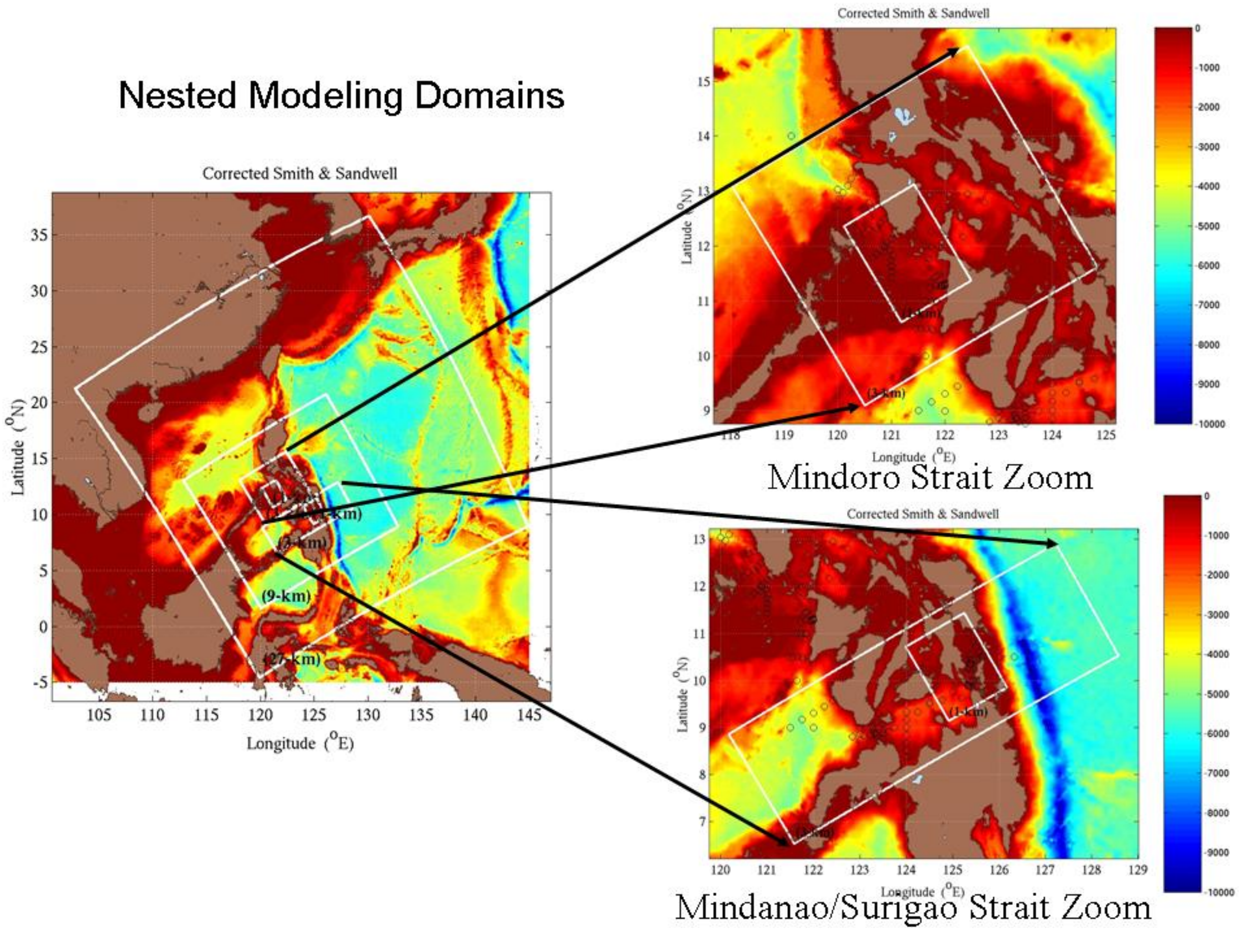
Hourly meridional velocities (v) at 68m depth at the location of mooring SW30, as measured by the moored ADCP (red curves) and as estimated by the re-analysis simulations (blue curves) with atmospheric and barotropic tidal forcing. No mooring data are assimilated.

Parameter sensitivity study shows importance of bottom friction.

PhilEx-IOP09 – February- March 2009

Long-Term Goal: *<u>Explore and better understand</u> interactive <u>dynamics and variability</u> of sub-mesoscale and mesoscale <u>features and processes</u> in the Philippine Straits region and their <u>impacts on local ecosystems</u> through:*

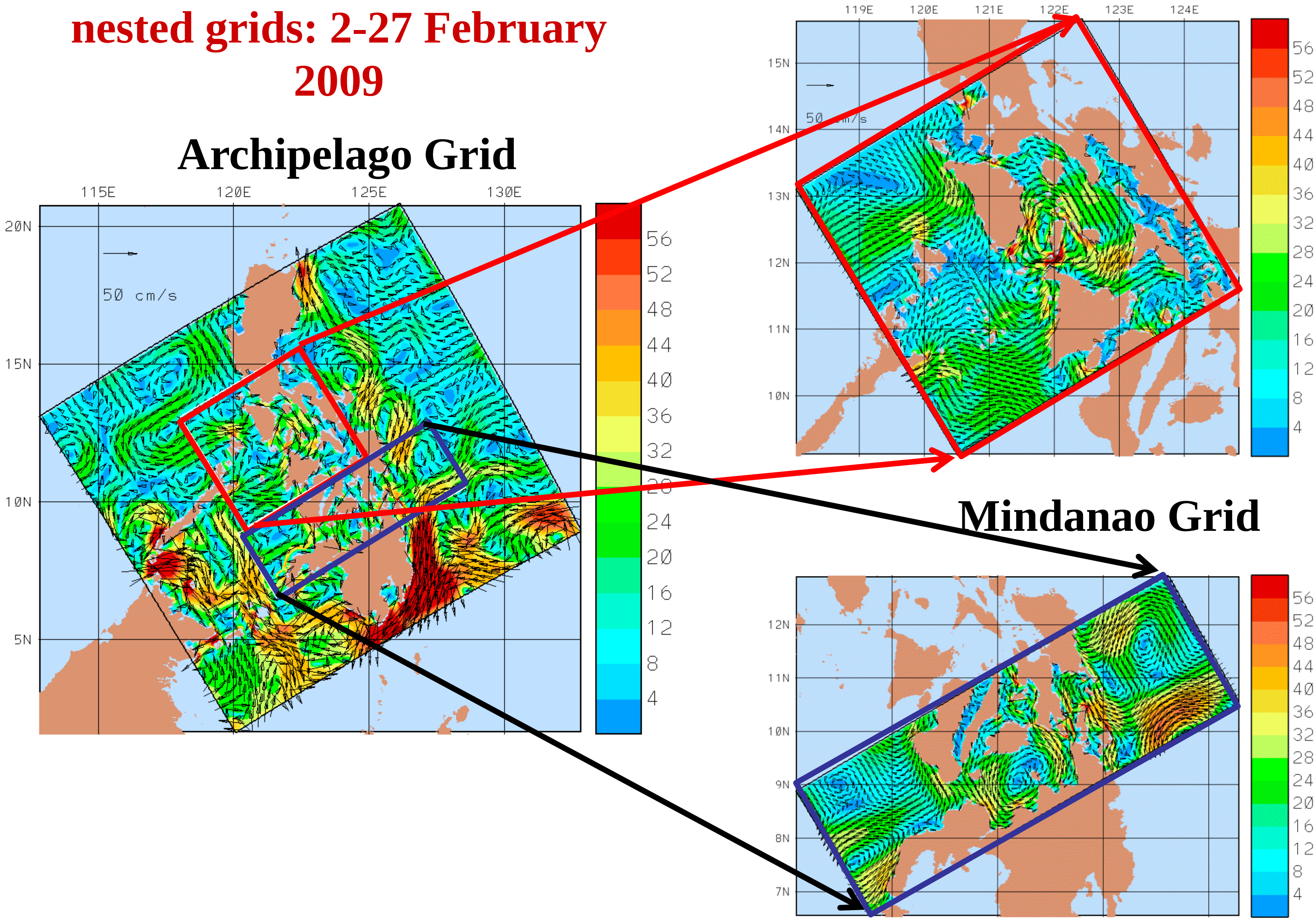
i. *<u>Interdisciplinary</u> physical-biogeochemical-acoustical <u>data assimilation</u> of novel multidisciplinary observations*
ii. *<u>Adaptive</u>, multi-scale physical and biogeochemical <u>modeling</u>*
iii. *<u>Process and sensitivity studies</u> based on a hierarchy of simplified simulations and focused modeling.*



http://mseas.mit.edu/Research/Straits/index.html

Surface Velocity in two-way nested grids: 2-27 February 2009

Mindoro Grid

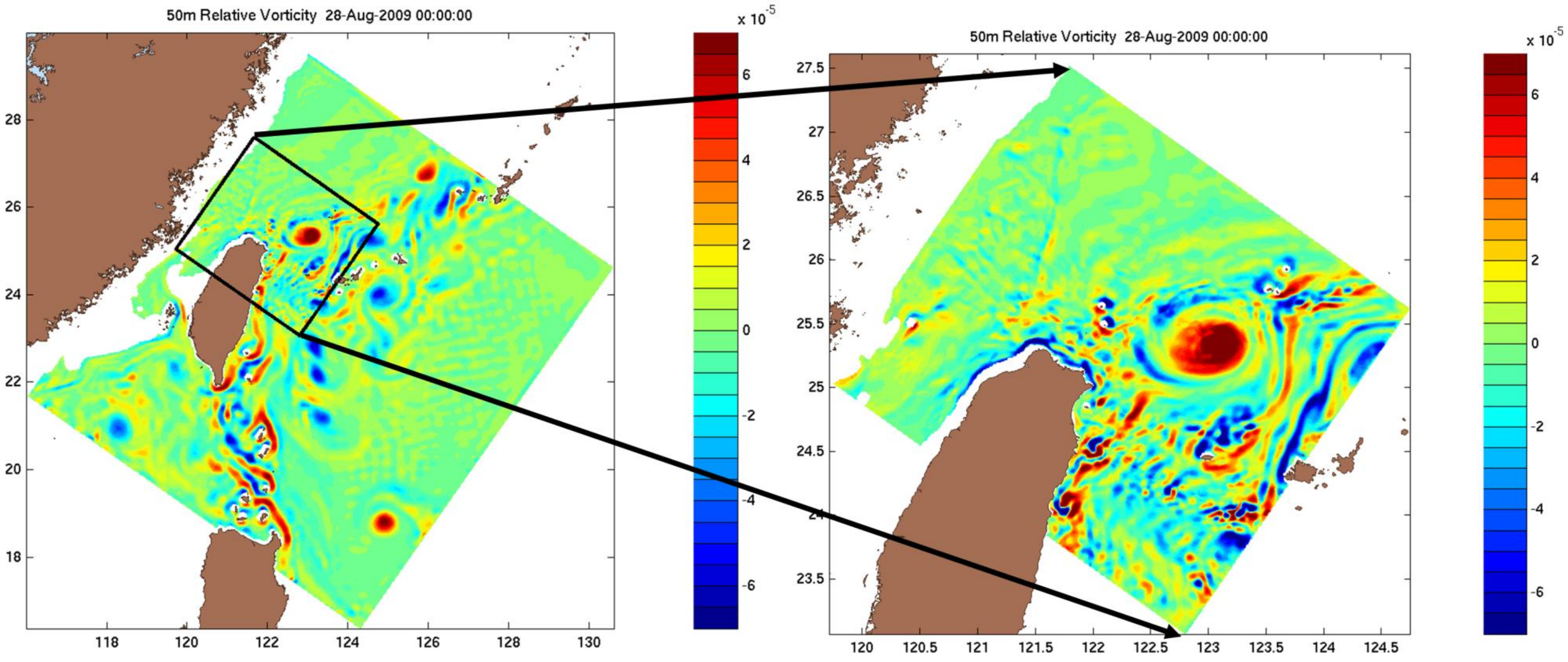


Quantifying, Predicting and Exploiting Uncertainties (QPE)

- **Regional ocean dynamics and modeling focus**
  - Continental shelf and slope northeast of Taiwan, especially
    - Cold Dome, its dynamics, variabilities and uncertainties,
    - Impacts on low-frequency (100 to 1000Hz) acoustic propagation
- **This dynamics is influenced by various processes that can occur simultaneously, very energetically and on multiple scales**
  - Kuroshio: western boundary current interacting with complex topography and influenced by larger-scale Pacific variability;
  - Ocean responses to atmospheric forcing including Typhoons;
  - Mesoscale and sub-mesoscale variability
    - Kuroshio's meanders and eddies, semi-permanent features (Cold Dome) and sub-mesoscale eddies, filaments and thin layers;
  - Taiwan Strait shelf jets/currents and their effects on Kuroshio intrusions;
  - surface and internal tides, internal waves and solitons.

=> Uncertainties!

Kuroshio Grid – 4.5km | Taiwan Grid – 1.5km

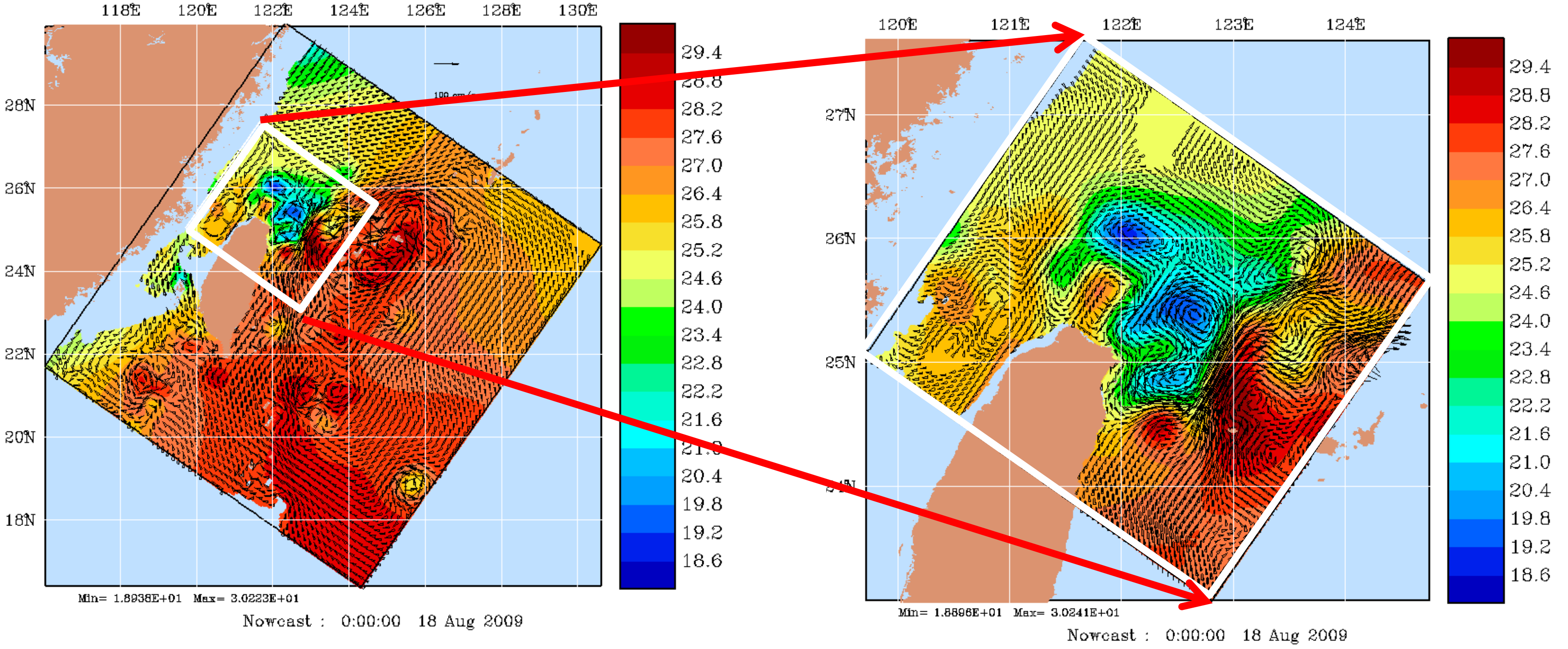


- Representative simulation from an ensemble of simulations
- Vortex generation of the Kuroshio passing over the I-Lan ridge
- well developed vortex wake trailing to the northeast off of Yonaguni island
- Downstream of the I-Lan ridge, an eddy trapped between the Kuroshio and the shelf.
- Interaction of tidal currents with topography produces a tight vorticity signal along the 50m isobath just north of Taiwan.
- Across the mouth of the Taiwan strait, another (weaker) interaction of tidal currents and bathymetry, along the 80m isobath.

50m Temperature with velocity vectors
QPE: 18 August – 5 September 2009

Coarse Grid

Fine Grid



CONCLUSIONS - FINDINGS

- **Derived new two-way embedded (nested) schemes for nonlinear free-surface primitive-equation models**
  - with strong tidal forcing
  - over shallow-to-deep seas
  - realistic computations, including data assimilation
- **Nesting with free-surface, multiscale dynamics requires strong consistency within time-steps**
  - Stronger consistency from "Implicit" nesting than "explicit" nesting
- **Future work:**
  - Multiscale dynamics studies underway
  - Generalize our nesting approach for refined time stepping
  - Compare/Merge our nesting approach with HDG approach

http://mseas.mit.edu