Using Adaptive Mesh Refinement to Model Ocean Flows

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Outline

Goals

- Solve depth averaged equations

$$h_t + (hu)_x + (hv)_y = 0$$

$$(hu)_t + \left(hu^2 + \frac{1}{2}gh^2\right)_x + (huv)_y = -ghb_x - \frac{h}{\rho}(P_A)_x + \frac{1}{\rho}\left[\tau_{sx} + \tau_{bx}\right]$$

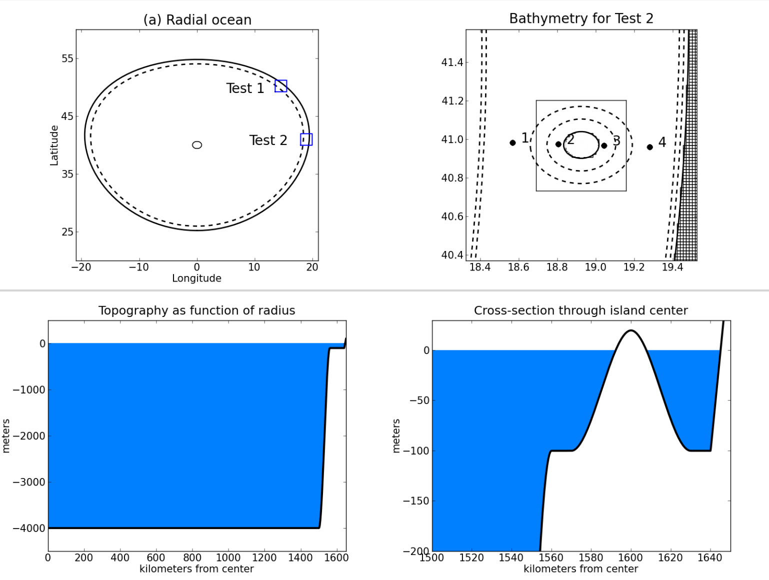
$$(hu)_t + (huv)_x + \left(hv^2 + \frac{1}{2}gh^2\right)_y = -ghb_y - \frac{h}{\rho}(P_A)_y + \frac{1}{\rho}[\tau_{sy} + \tau_{by}]$$

- Resolve ocean to inundation spatial and temporal scales
- Handle arbitrary bathymetry and inundation
- Determine grids based on both physical and numerical error estimation

Numerical Method

- Wave-propagation algorithm, class of high-resolution finite volume methods using a Godunov type scheme
    - Evolve grid cell averages by determining waves at edges of grid cells
    - Riemann solution preserves non-trivial steady states and handles dry-states and inundation
- Adaptive mesh refinement:
    - Structured, patch based scheme with properly nested grids
    - Time steps chosen to satisfy CFL condition on current grid
    - Conservative fix-ups required along boundaries using ghost cells
- GeoClaw
    - Download at: `http://www.clawpack.org/`
    - Open source research code incorporating:
        - Solvers for shallow water systems using finite volume methods
        - Adaptive mesh refinement for complex topography
        - Dry-states and inundation handling
        - Tools for handling multiple topography specifications

Circular Ocean



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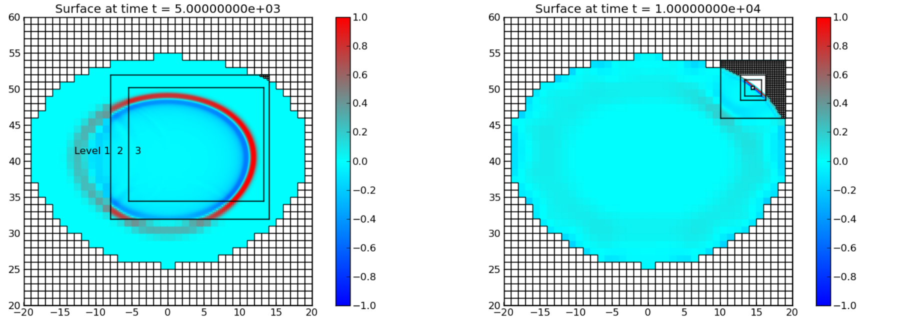

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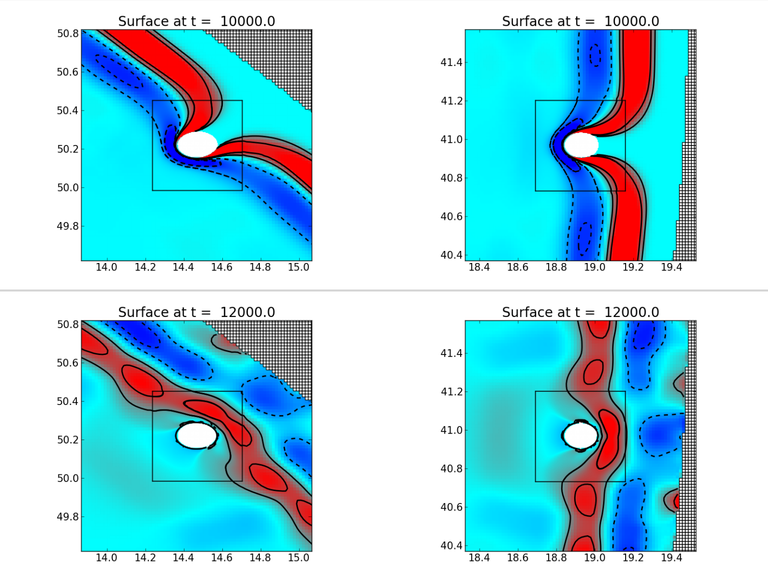

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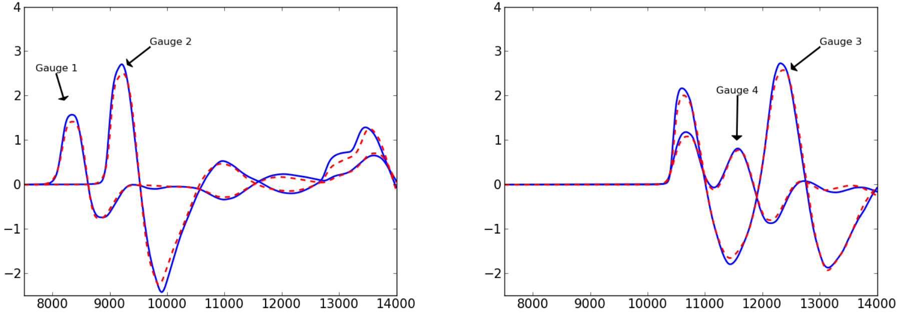
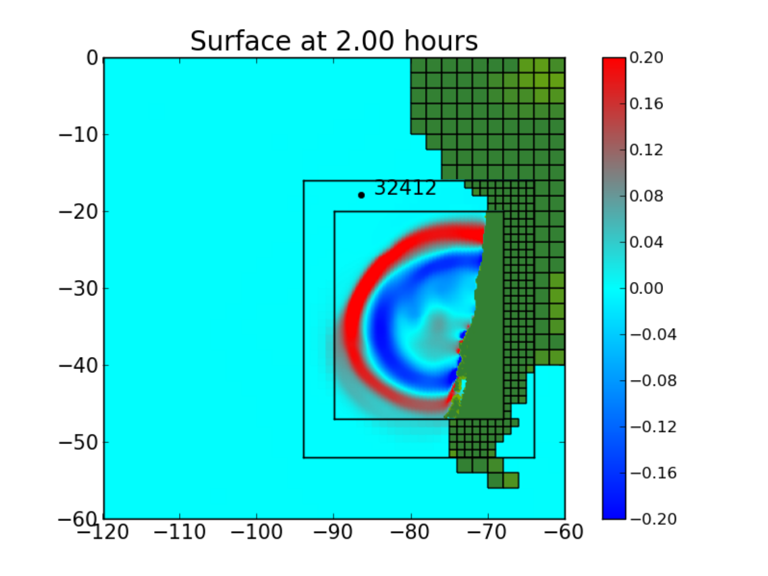


Chile Tsunami



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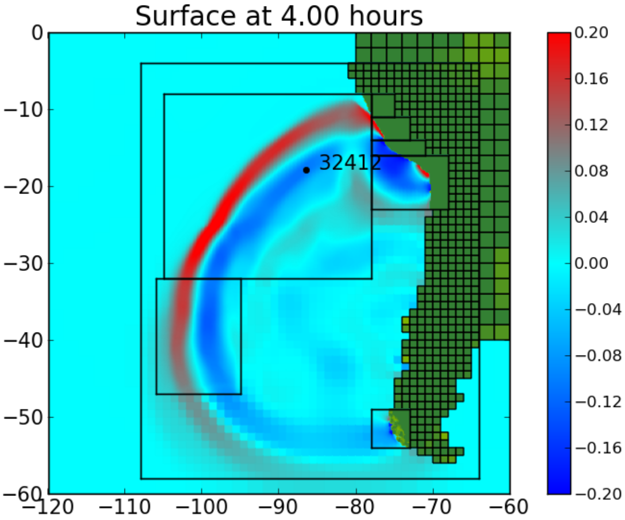

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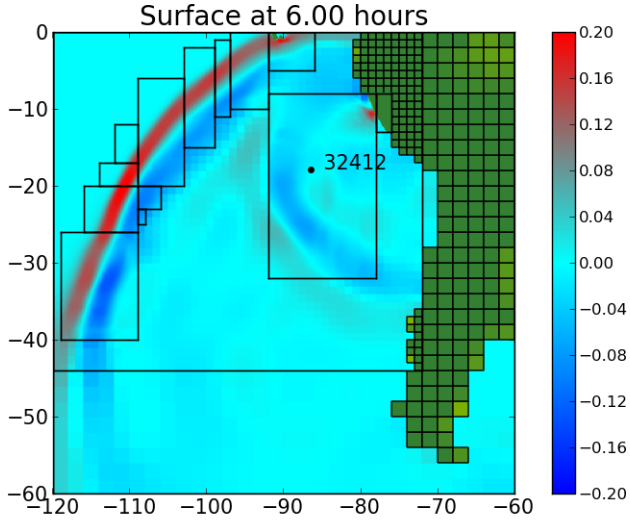

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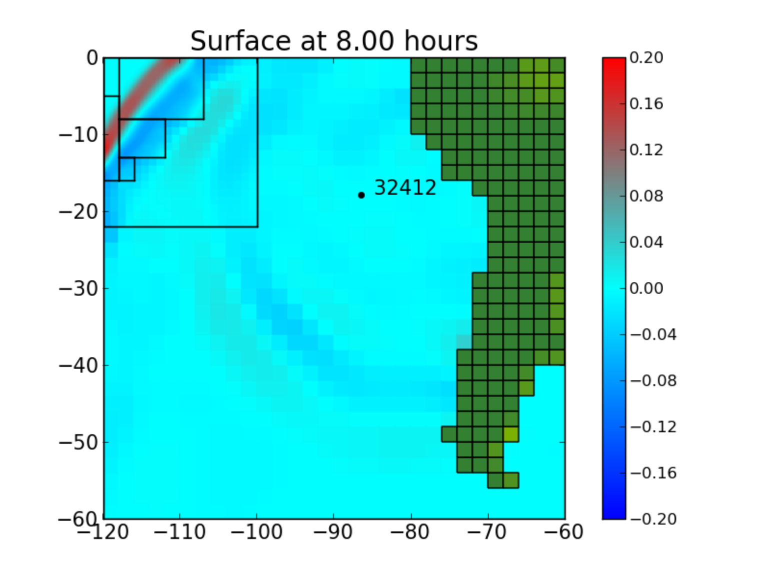


Storm Surge Preliminary Results

Storm Surge Preliminary Results

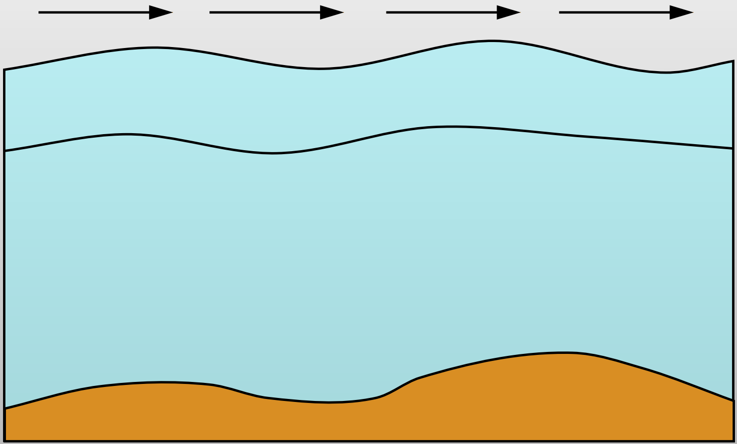
Future Work

Multilayer shallow water equations:

$$
\begin{aligned}
&\frac{\partial}{\partial t}(\rho_1 h_1) + \frac{\partial}{\partial x}(\rho_1 h_1 u_1) = 0, \\
&\frac{\partial}{\partial t}(\rho_1 h_1 u_1) + \frac{\partial}{\partial x}\left(\rho_1 h_1 u_1^2 + g\rho_1 \frac{h_1^2}{2} + g\rho_2 h_1 h_2\right) = \rho_2 g h_2 \frac{\partial h_1}{\partial x} - \rho_1 g h_1 b'(x) \\
&\frac{\partial}{\partial t}(\rho_2 h_2) + \frac{\partial}{\partial x}(\rho_2 h_2 u_2) = 0, \\
&\frac{\partial}{\partial t}(\rho_2 h_2 u_2) + \frac{\partial}{\partial x}\left(\rho_2 h_2 u_2^2 + g\rho_2 \frac{h_2^2}{2}\right) = -\rho_2 g h_2 \frac{\partial h_1}{\partial x} - \rho_2 g h_2 b'(x)
\end{aligned}
$$

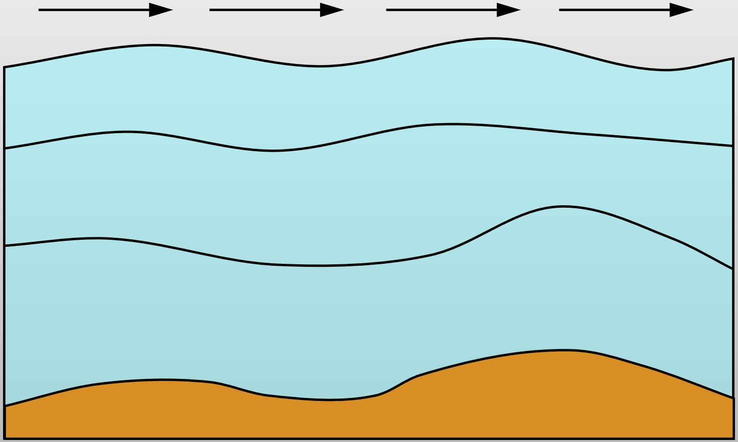
Future Work

Multilayer shallow water equations:



Future Work

Multilayer shallow water equations:



Special Thanks To:

OFFICE OF NAVAL RESEARCH