Extension of Brinkman Penalization Method for Ocean Circulation Modeling using Adaptive Wavelet Collocation Method

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Motivation and Objectives

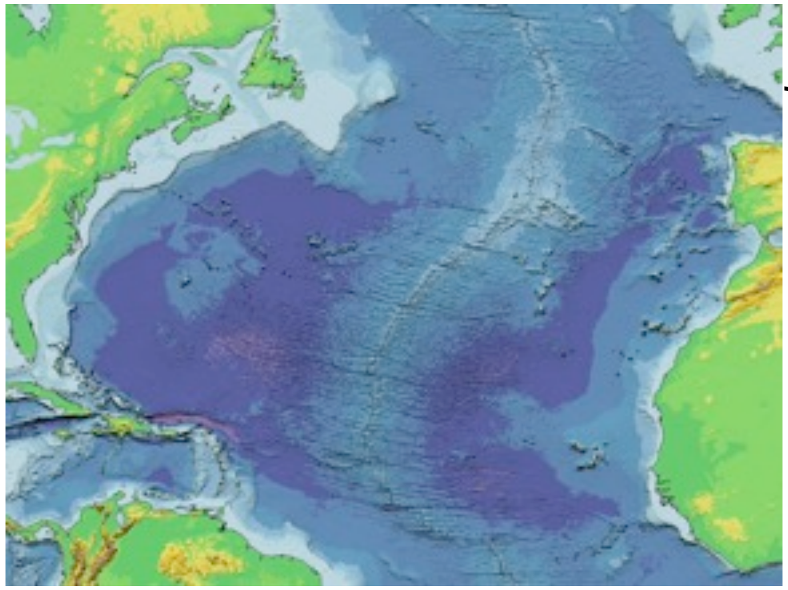
1. Improve accuracy and efficiency of basin-scale, wind-driven ocean circulation models

http://www.eng.warwick.ac.uk

boundary currents

2. Develop better representation for boundaries with complex geometry

continental topology



http://www.windows.ucar.edu

ocean bathymetry

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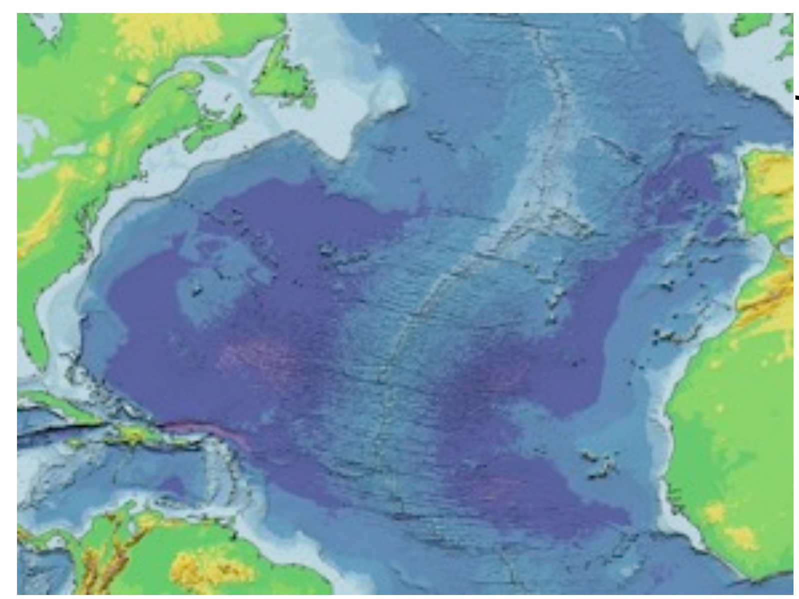

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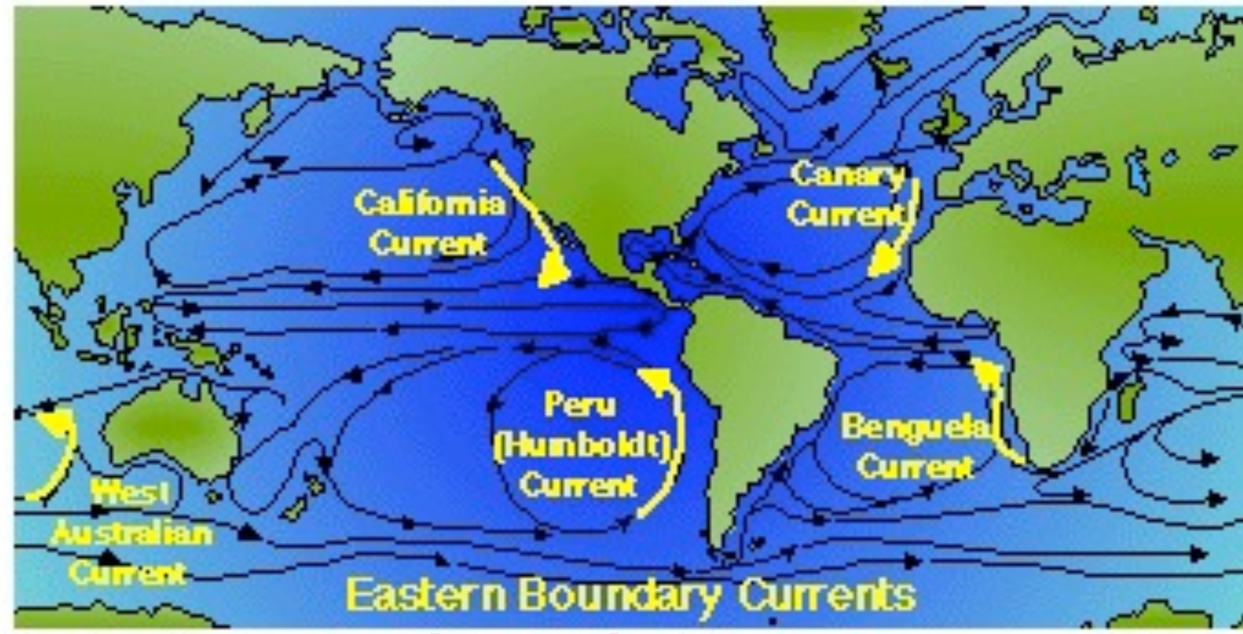

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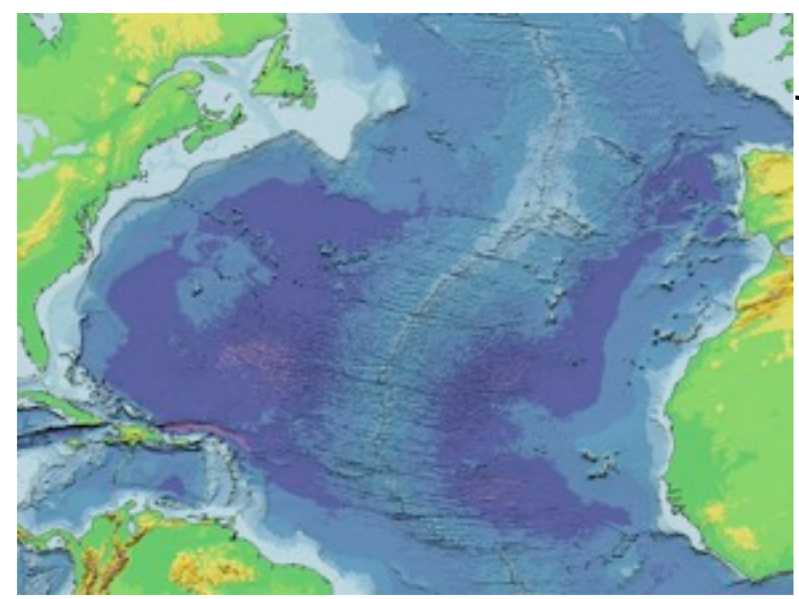

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ocean bathymetry

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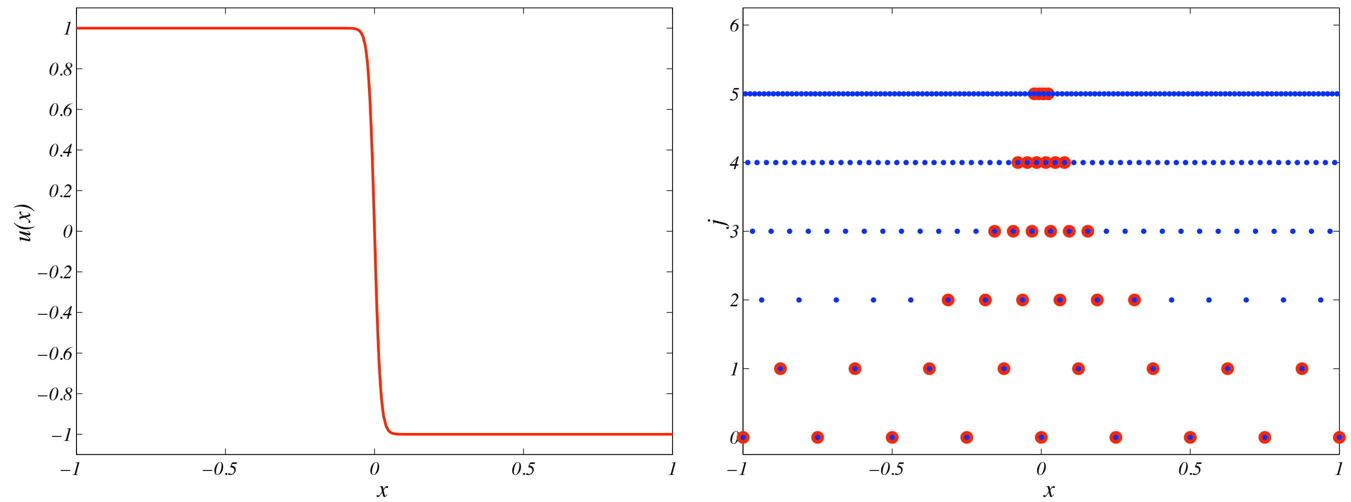
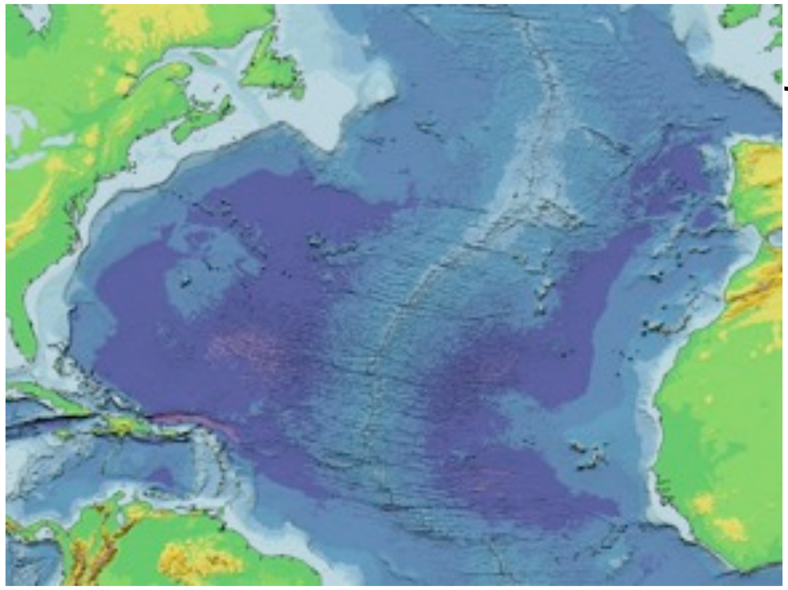
Adaptive Wavelet Collocation Method



2. Develop better representation for boundaries with complex geometry



http://www.windows.ucar.edu

continental topology **ocean bathymetry**

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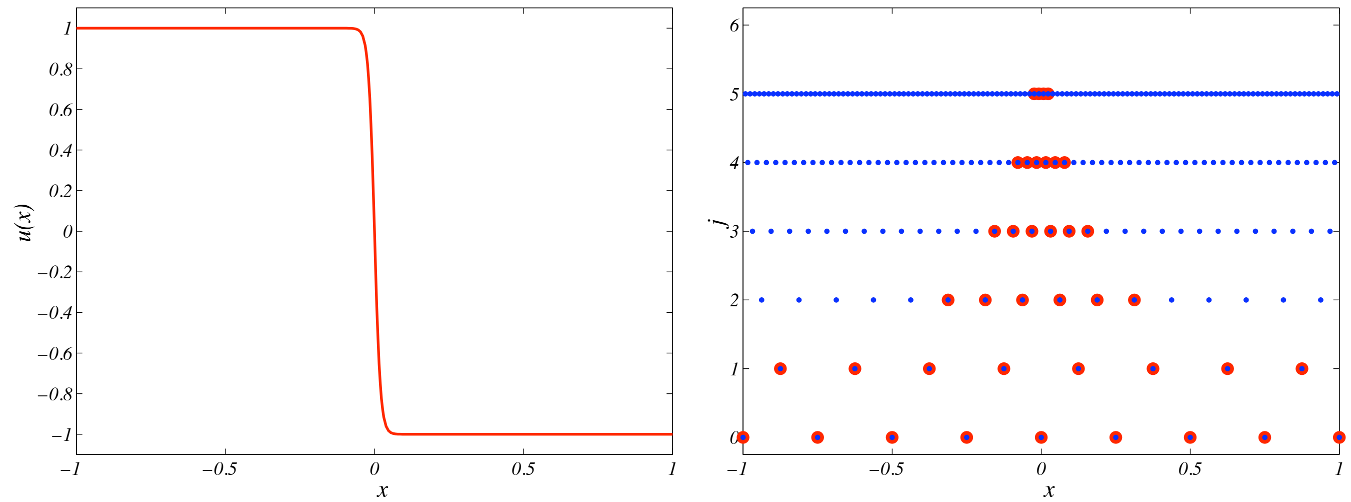
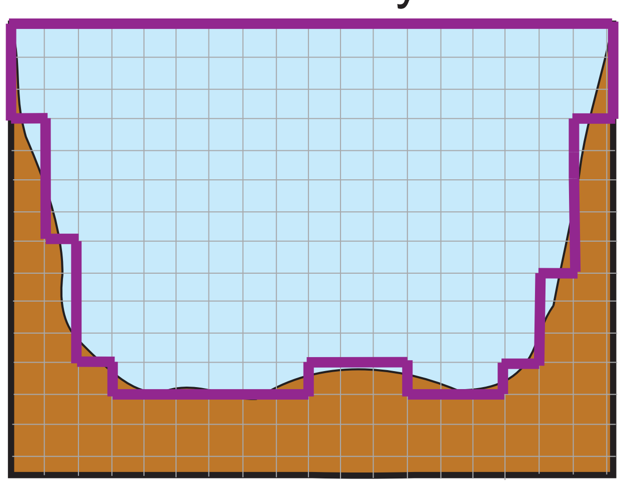
Adaptive Wavelet Collocation Method



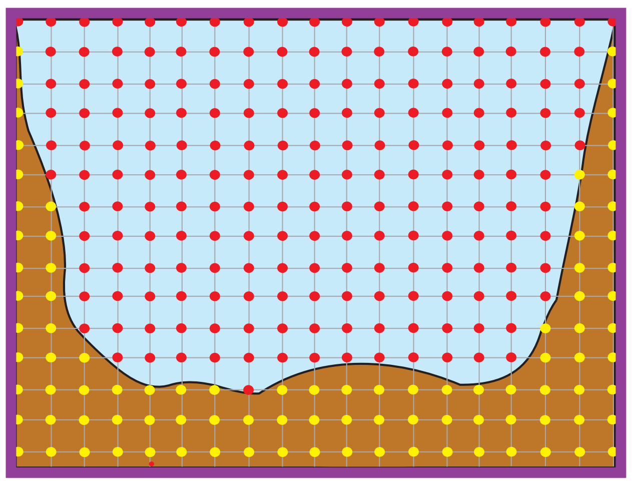
2. Develop better representation for boundaries with complex geometry

Brinkman Penalization Technique

Traditional Boundary Conditions

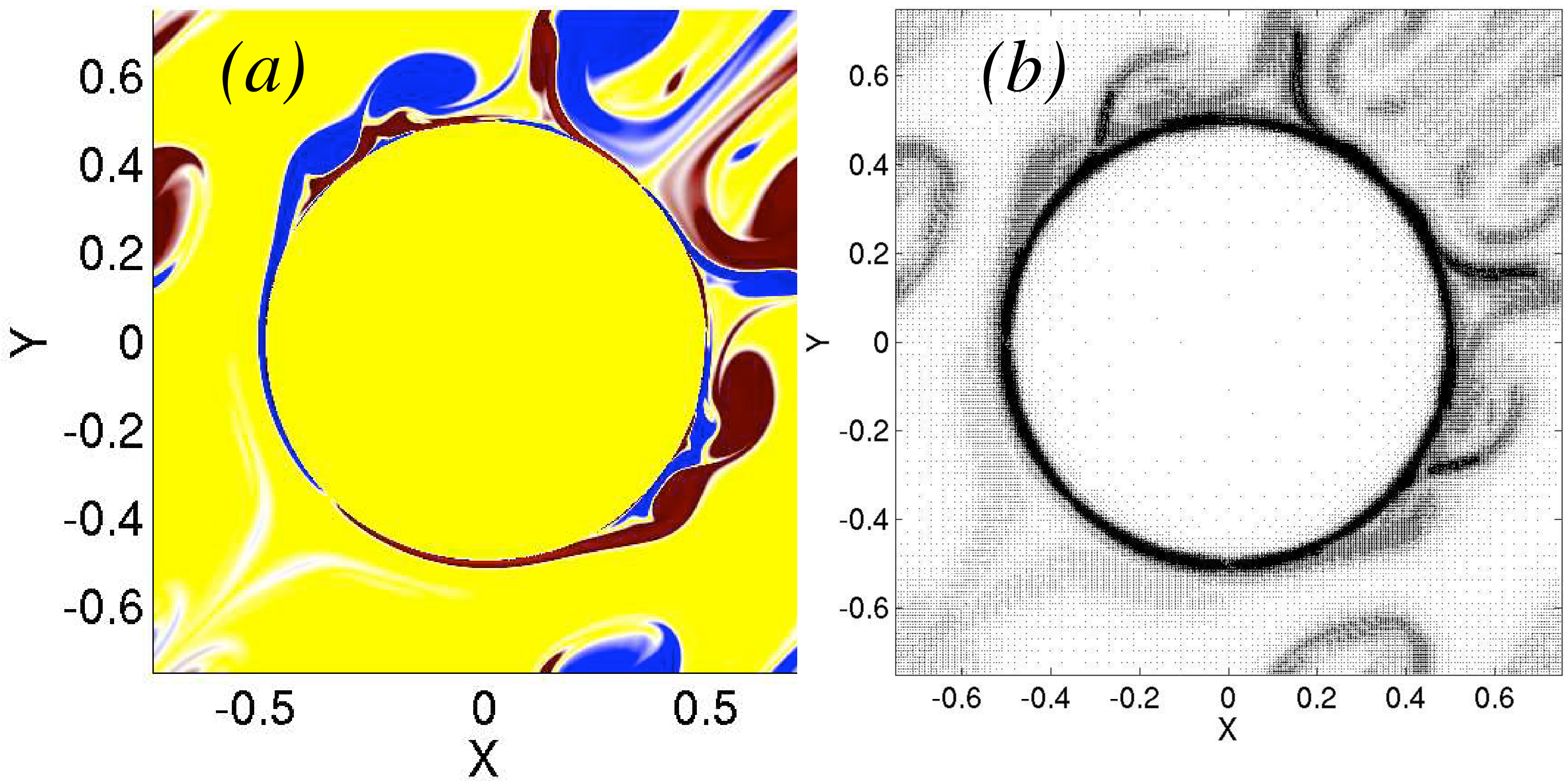


Brinkman Penalization



Why dynamic adaptivity?

- dynamically resolves and "tracks" dominant flow structures
- more computationally efficient
- direct error control of accuracy



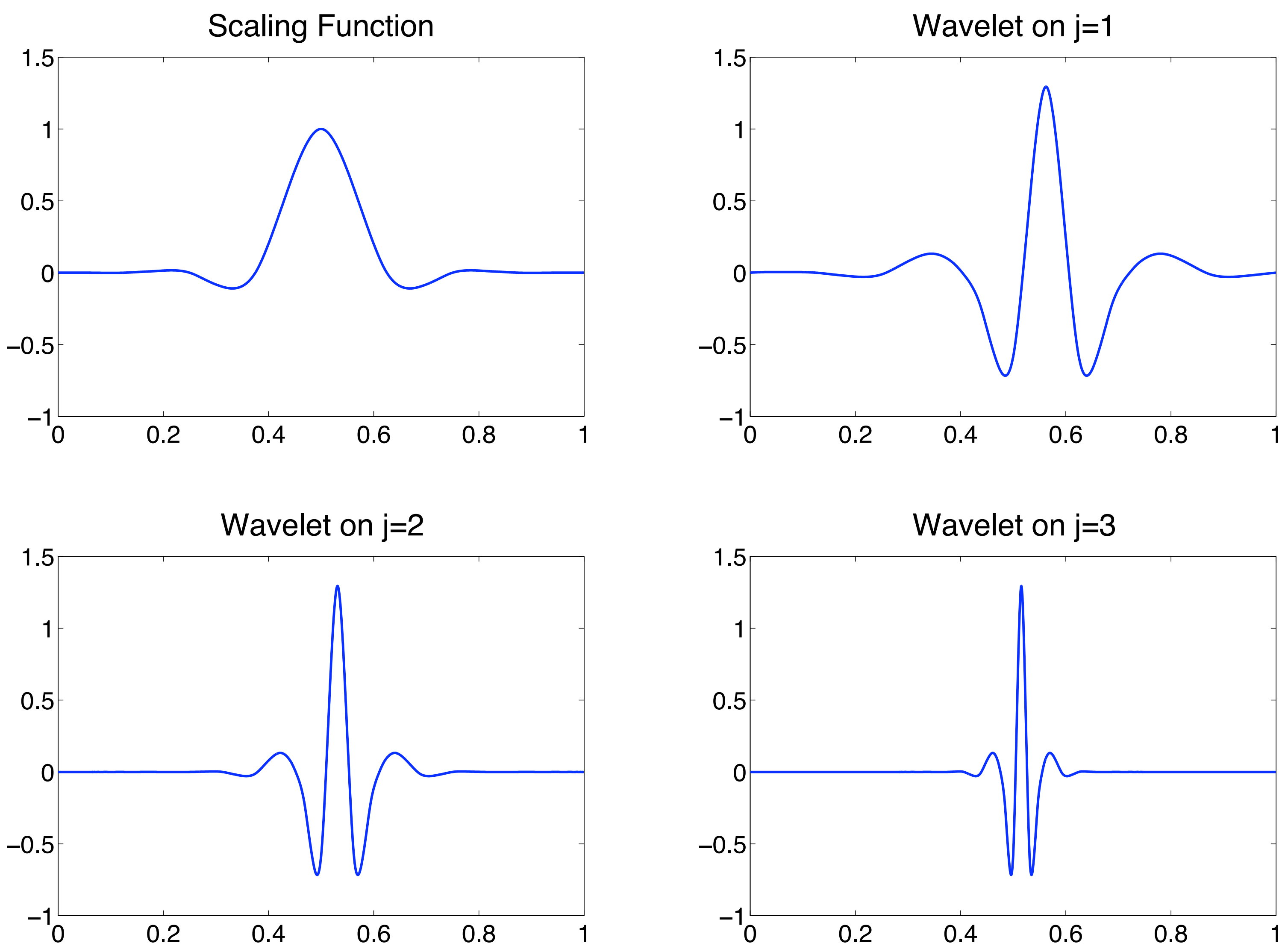
**Kevlahan NKR, Vasilyev OV. 2005. SIAM J. Sci. Comput. 26:1894–915*

Wavelet Transform

$$u(\mathbf{x}) = \sum_{j=0}^{+\infty} \sum_{\mathbf{k} \in \mathcal{K}^j} d_{\mathbf{k}}^{j} \psi_{\mathbf{k}}^{j}(\mathbf{x})$$

Level — Location

- Wavelets used for bases functions
- Localized in wave number and physical space
- Provides both frequency and position information



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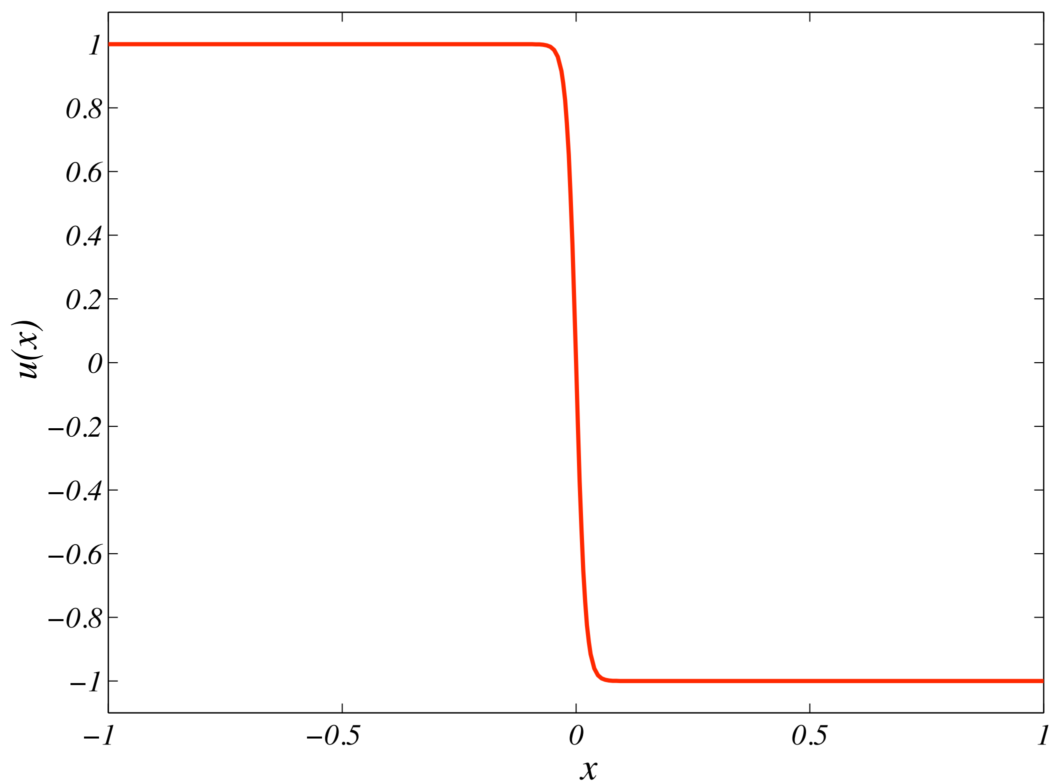
Wavelet Compression

Wavelet Transform:

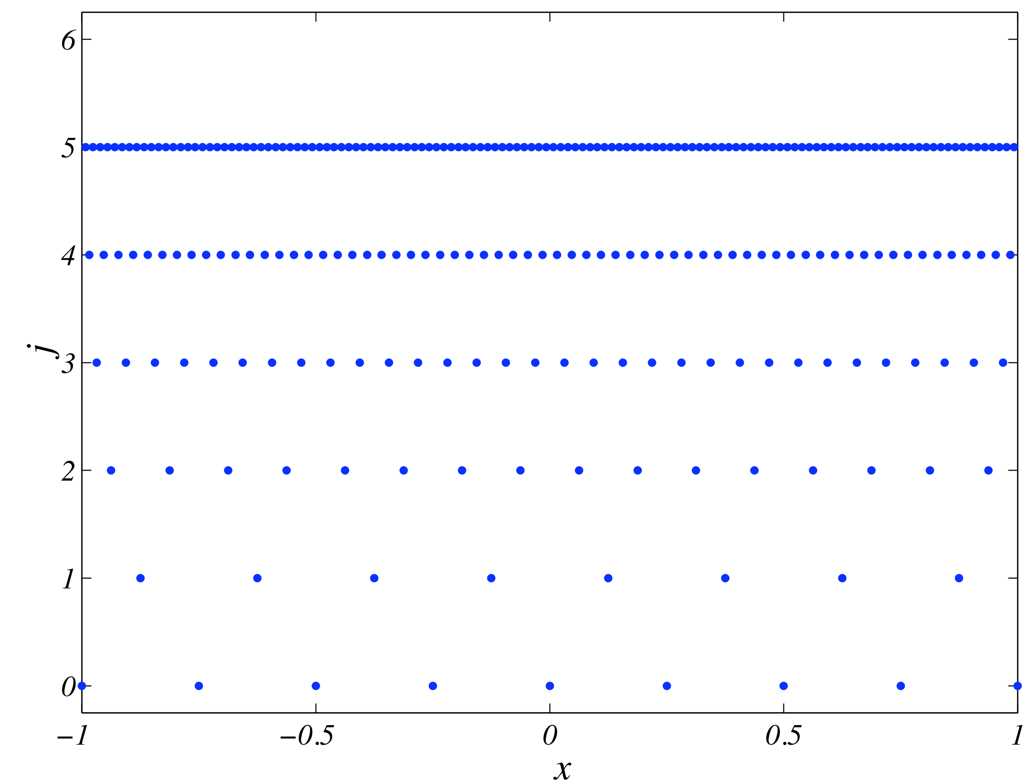
$$u_{\geq}(\mathbf{x}) = \sum_{j=0}^{+\infty} \sum_{\mathbf{k} \in \mathcal{K}^j} d^j_{\mathbf{k}} \psi^j_{\mathbf{k}}(\mathbf{x})$$

level (j)

location ($\mathbf{k}$)



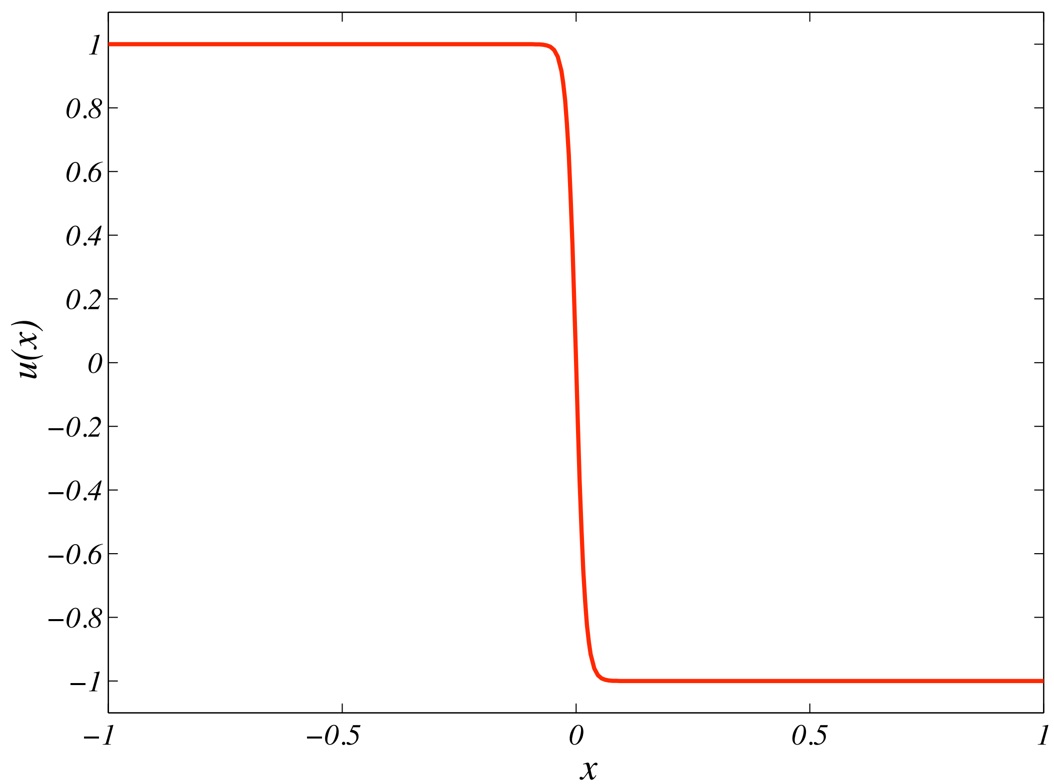
Function $u(x)$



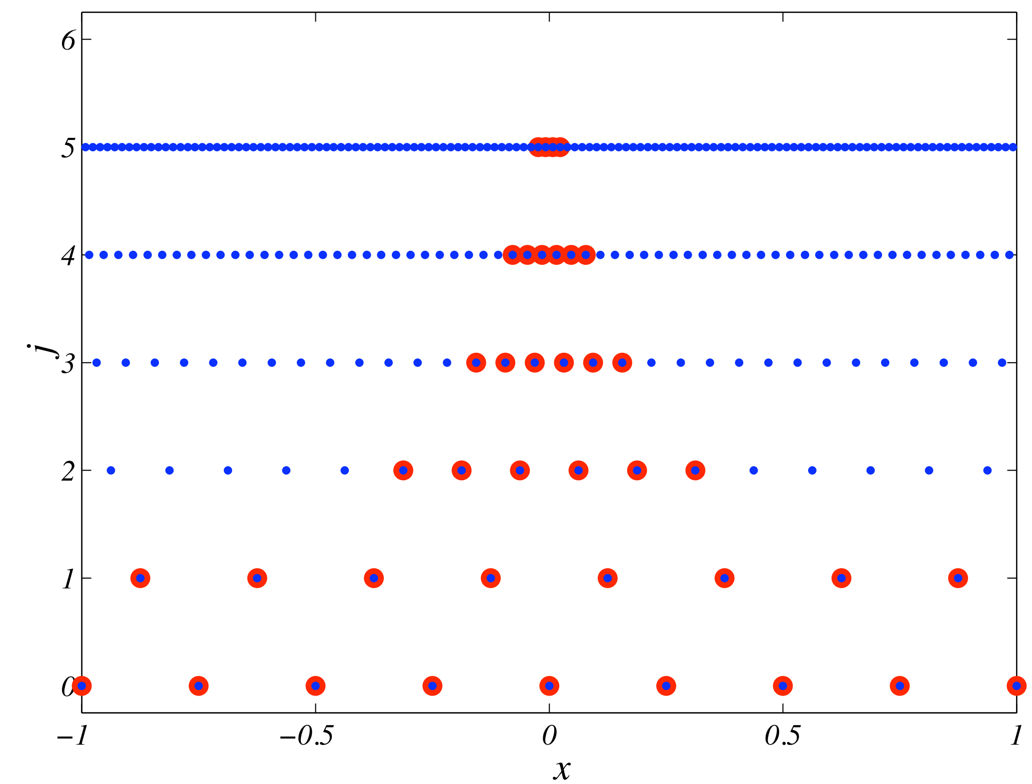
Wavelet coefficient locations $d^j_{\mathbf{k}}$

Wavelet
Thresholding Filter :

$$u_{\geq}(\mathbf{x}) = \sum_{j=0}^{+\infty} \sum_{\substack{\mathbf{k} \in \mathcal{K}^j \\ |d^j_{\mathbf{k}}| \geq \epsilon \|u\| = 10^{-3}}} d^j_{\mathbf{k}} \psi^j_{\mathbf{k}}(\mathbf{x})$$



Function $u(x)$



Wavelet coefficient locations $d^j_{\mathbf{k}}$

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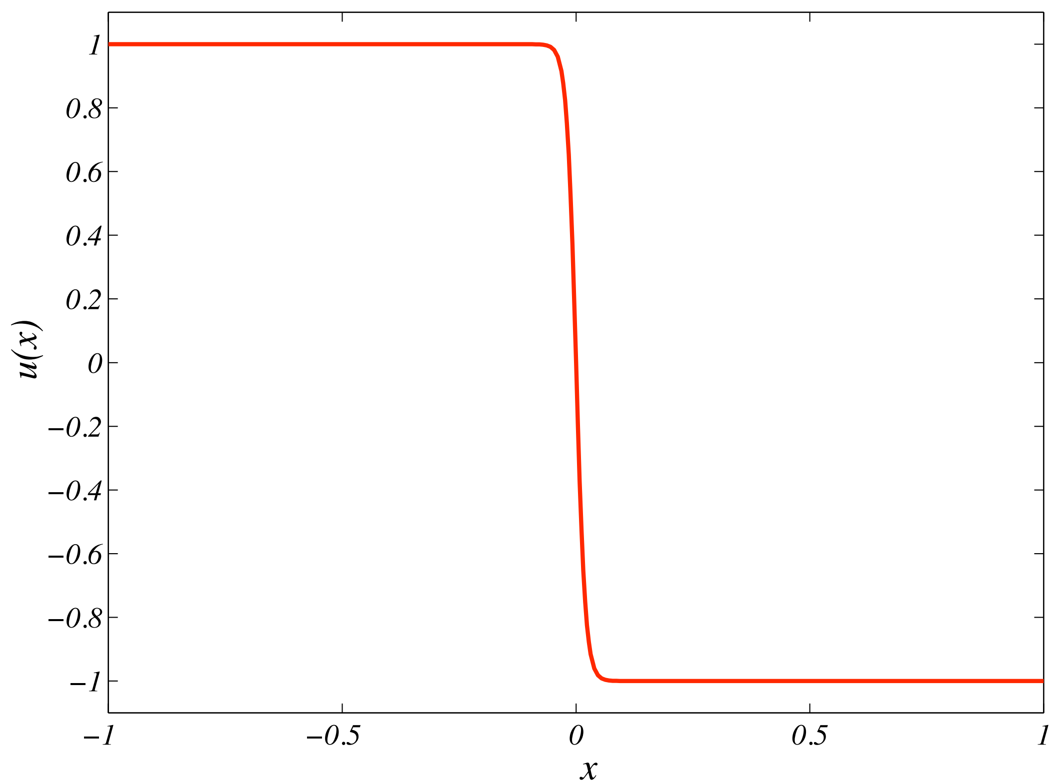
Wavelet
Thresholding Filter :

Error Due
to Thresholding:

$$\|u(\mathbf{x}) - u_{\geq}(\mathbf{x})\| \leq C_1 \epsilon \, \|u\|$$



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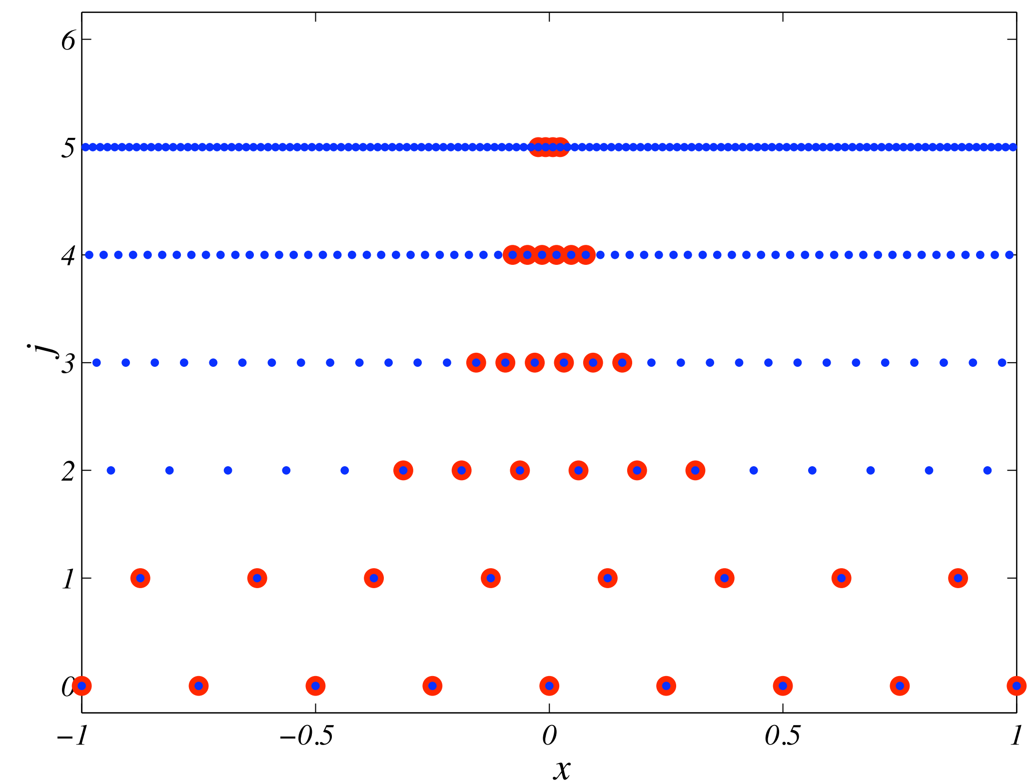
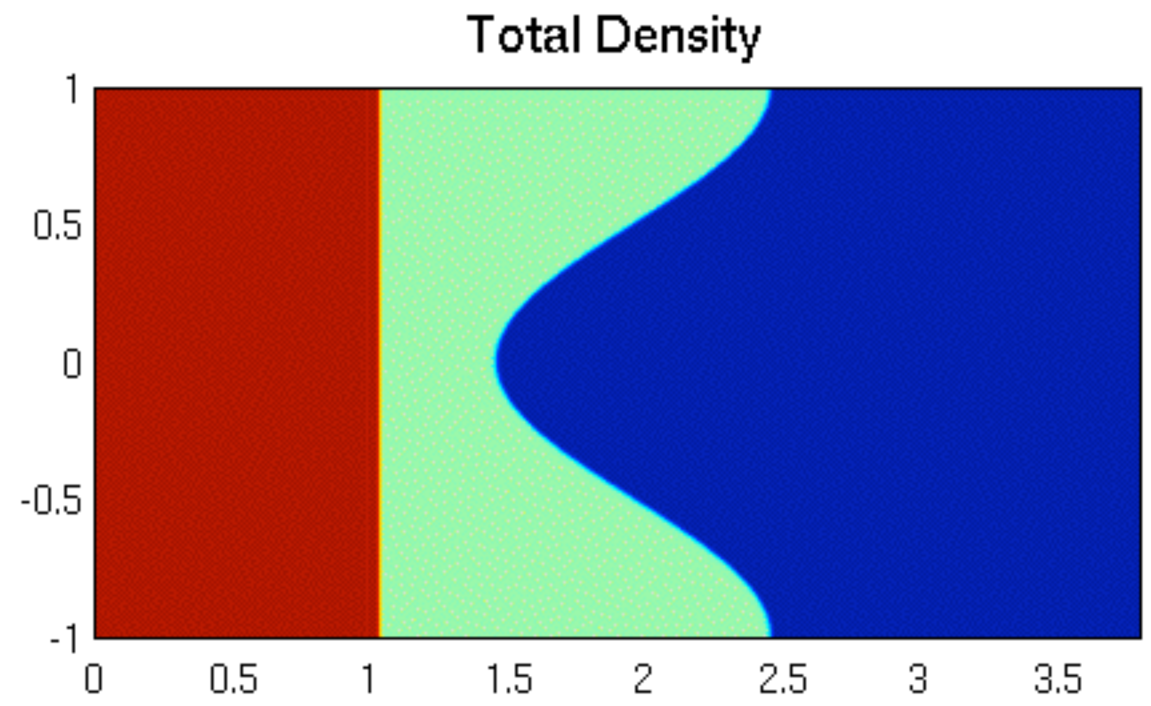


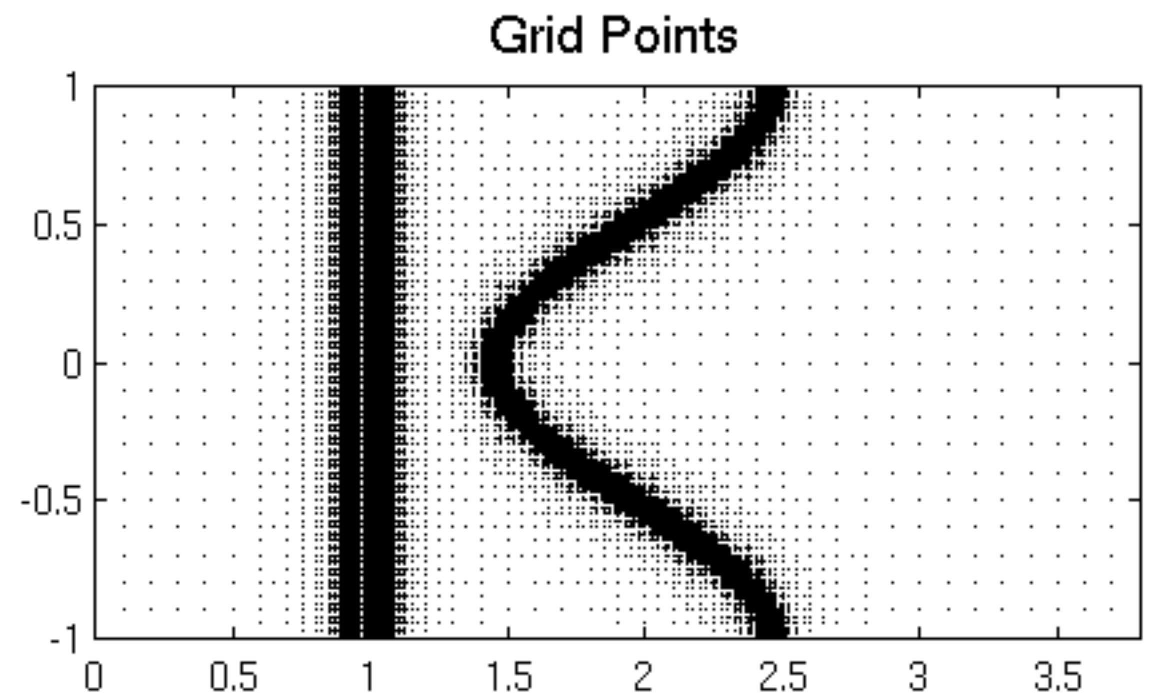
Wavelet coefficient locations $d^{j}_{\mathbf{k}}$

AWCD Procedure

- Forward Wavelet Transform
- Significant Points - above threshold parameter
- Adjacent Points - buffer zone
- Reconstruction Points - needed for wavelet transform
- Ghost Points - needed for derivative calculations
- Inverse Wavelet Transform
    - Derivatives are taken at this step
- Time advancement

EX: Richtmyer-Meshkov Instability





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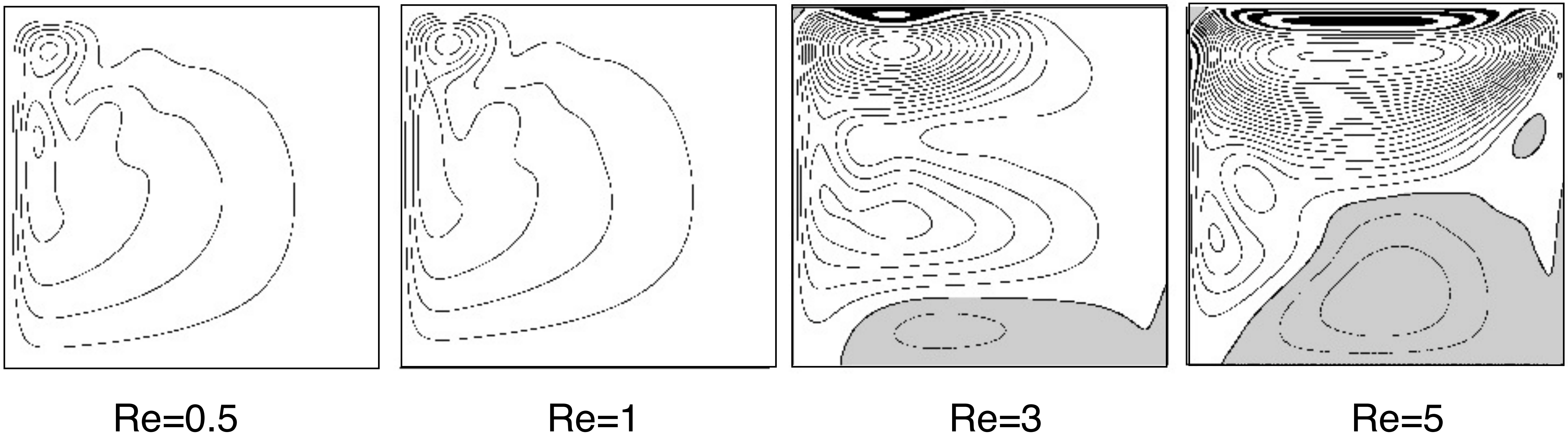
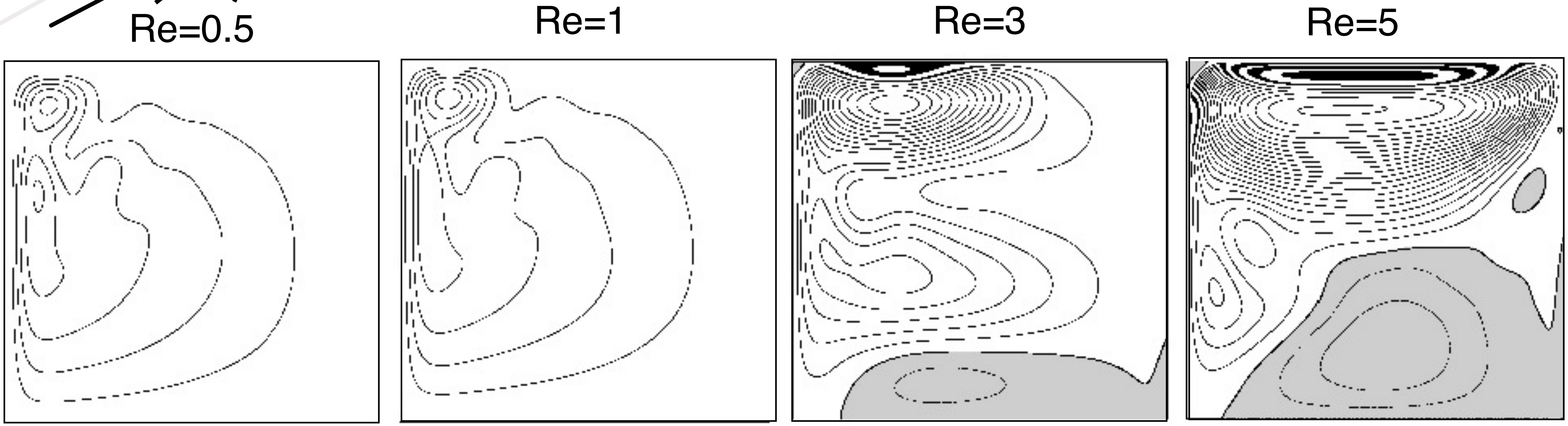
Validation: Test Case



-Contours of time-mean streamfunction: averaged over second half of 4000 days.
-Regions of negative streamfunction are shaded

*Fox-Kemper B.; Pedlosky J., Journal of Marine Research, Volume 62, Number 2, 1 March 2004 , pp. 169-193(25)

Validation: Test Case



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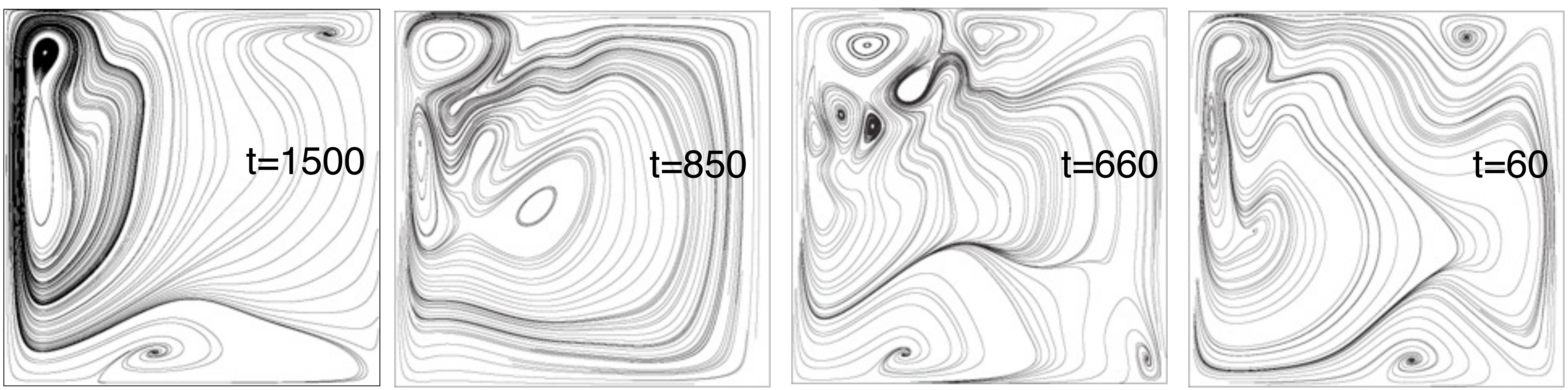

Most recent snapshot in time for current runs (not averaged as with Fox-Kemper results)

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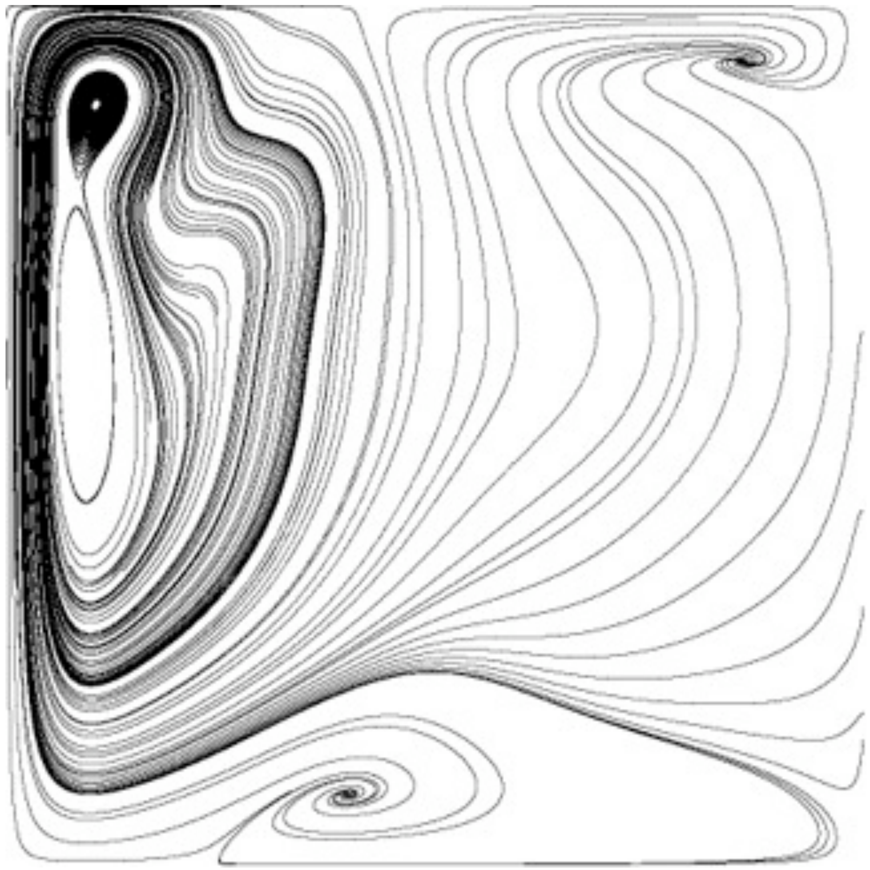
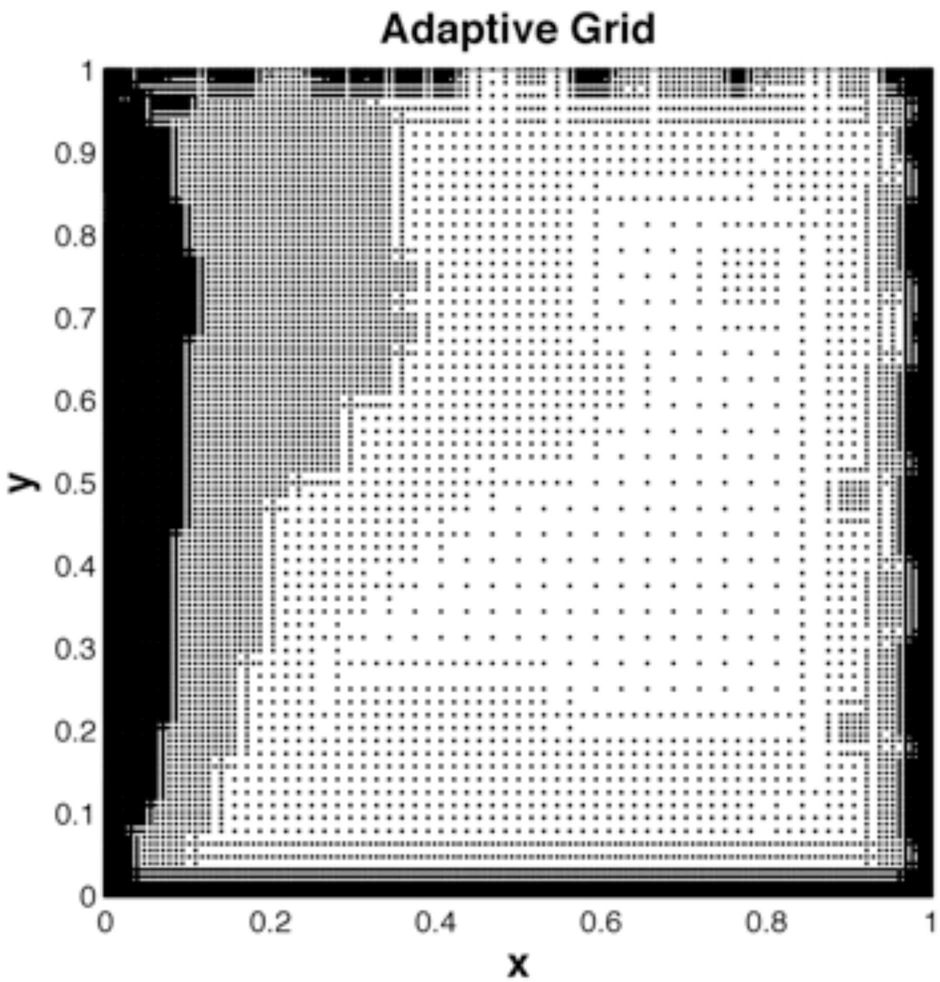
Example of Effective Resolution





- Number of points used: 53,740
- Effective Resolution: 2048 x 2048 = 4,194,304
- 1.3% of grid points used (98.7% compression)

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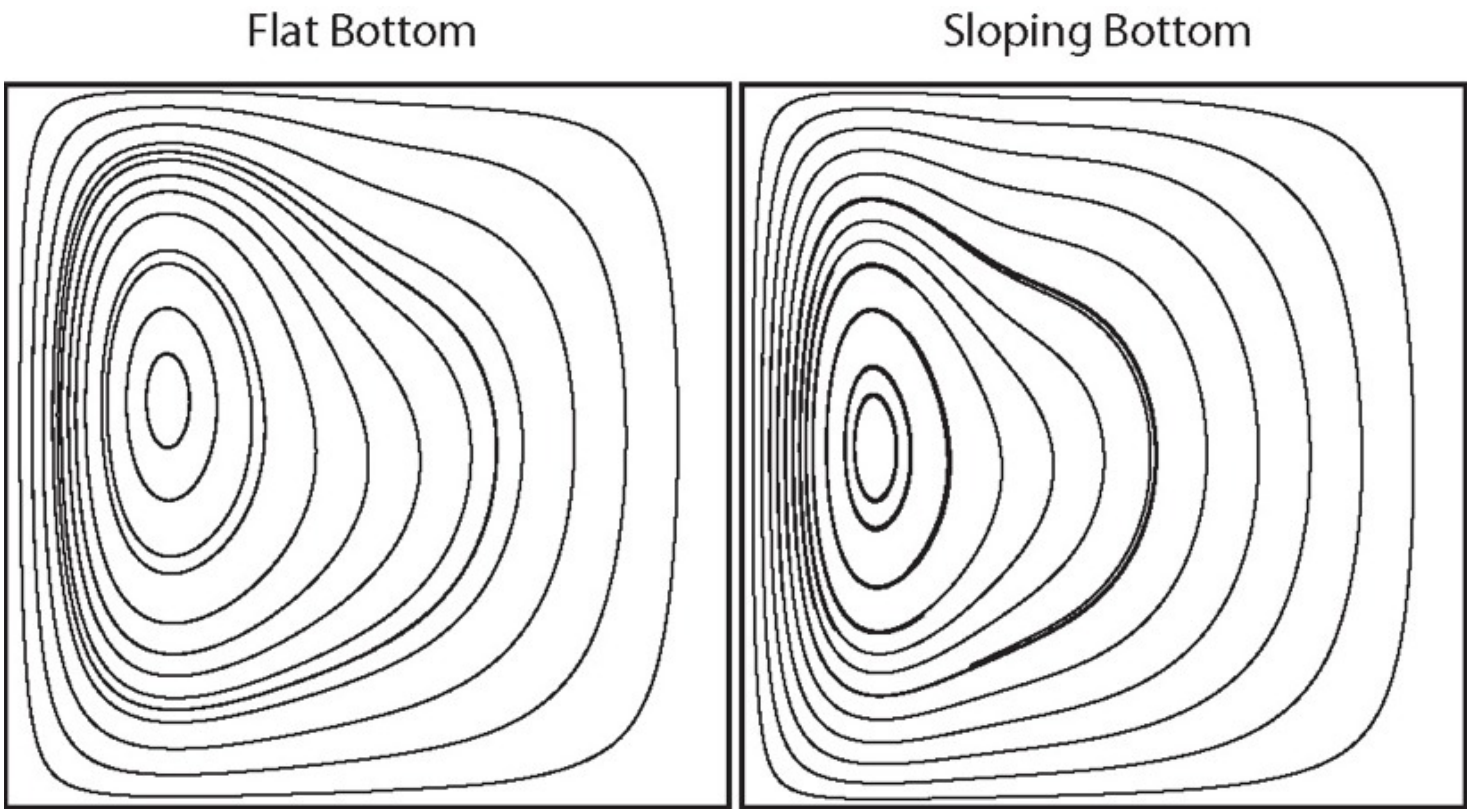
Sloping Bottom

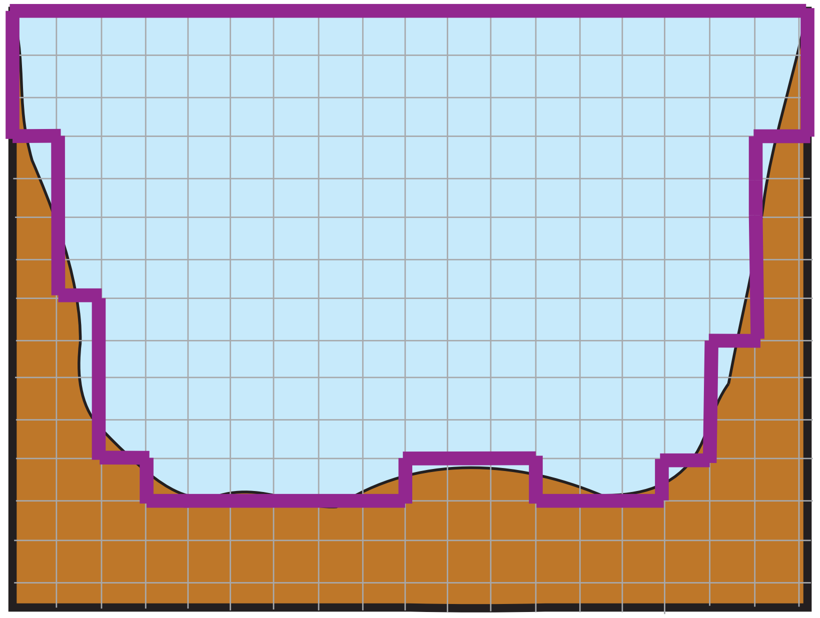


Re=0.2

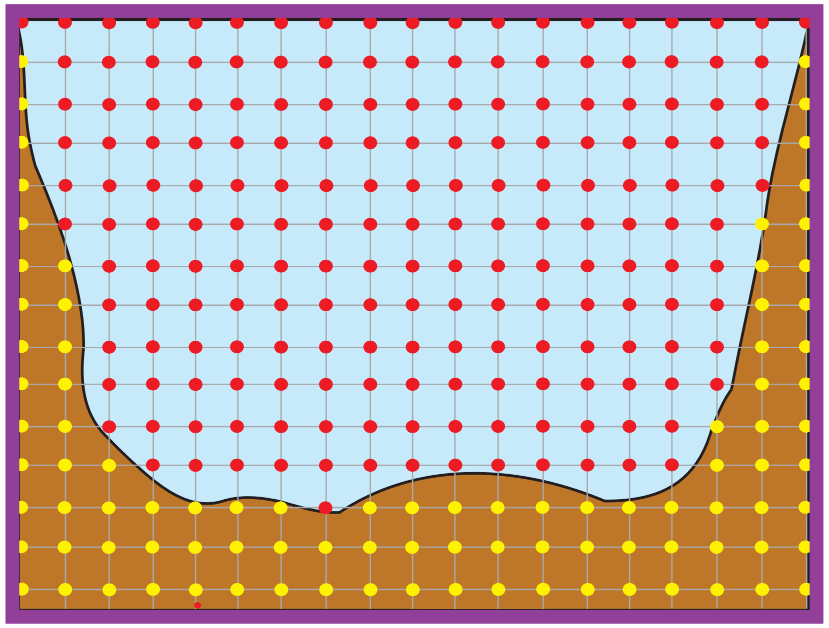
Brinkman Penalization

- an immersed boundary method
  - good for representation of complex geometry
  - altering governing equations in such a way that boundary conditions are automatically satisfied
- models solid as porous media
- no slip boundary conditions
- conducive to adaptive grids

Traditional Boundary Conditions



Brinkman Penalization



Why Brinkman Penalization?

- Not specific to any numerical method or grid
- Error can be estimated rigorously in terms of penalization parameter
- Solutions of the penalized incompressible Navier-Stokes equations strongly converges to the exact solution as penalization parameter tends to zero.

Compressible Brinkman Penalization Formulation

$$\frac{\partial \rho}{\partial t} = -\left[1 + \left(\frac{1}{\phi} - 1\right)\chi\right]\frac{\partial}{\partial x_j}(\rho u_j),$$

$$\frac{\partial \rho u_i}{\partial t} = -\frac{\partial}{\partial x_j}(\rho u_i u_j) - \frac{\partial p}{\partial x_i} + \frac{\partial \tau_{ij}}{\partial x_j} - \frac{\mu\chi}{\eta}u_i,$$

$$\frac{\partial e}{\partial t} = -\frac{\partial}{\partial x_j}\left[(e+p)u_j\right] + \frac{\partial}{\partial x_j}(u_i\tau_{ij}) + \frac{\partial}{\partial x_j}\left(k\frac{\partial T}{\partial x_j}\right) - \frac{h\chi}{\phi}(T - T_o),$$

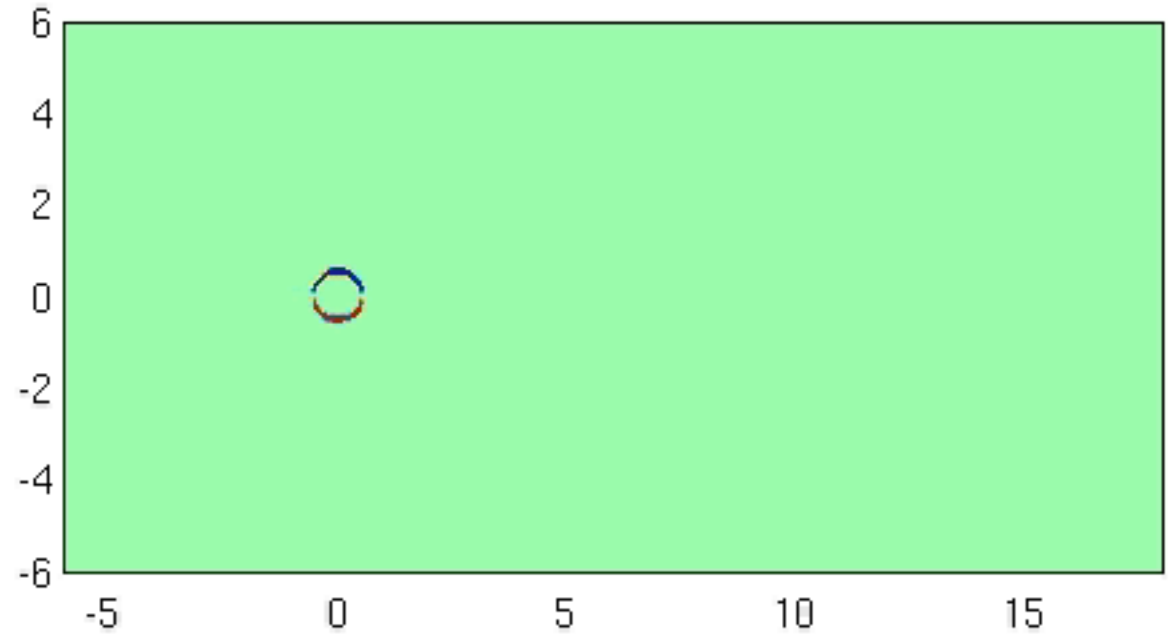
$$\chi(\mathbf{x}, t) = \begin{cases} 1 & \text{if } \mathbf{x} \in O_i \\ 0 & \text{otherwise} \end{cases}$$

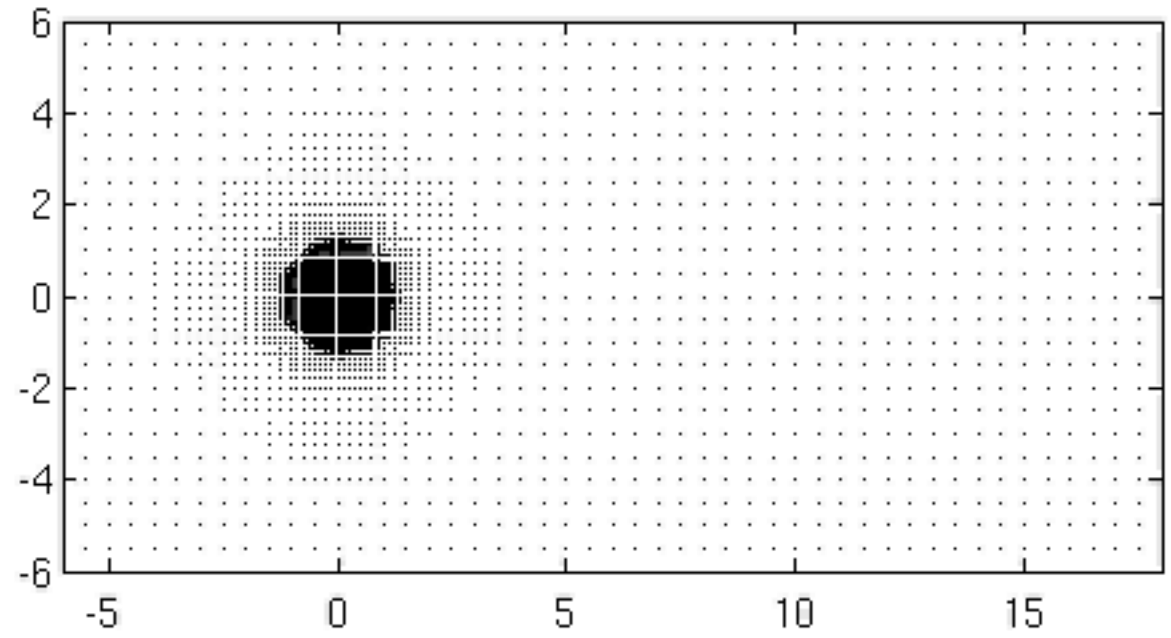
- Asymptotic analysis → amplitude and phase errors are $O(\phi, \eta)$

*Liu Q, Vasilyev OV. 2007. J. Comp. Phys. 227:946–66.

Brinkman Penalization - Compressible Case*

- *Re* = 1000, M_{in} = 0.2
- Effective grid resolution: *769 x 385*





*Liu Q, Vasilyev OV. 2007. J. Comp. Phys. 227:946–66.

Shallow Water Model

Continuity Equation

$$\frac{\partial \eta}{\partial t} = -\left[1 + \left(\frac{1}{\phi} - 1\right)\chi\right] \nabla \cdot (\eta \mathbf{u})$$

Momentum Equations

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} + \frac{1}{Ro} f \hat{\mathbf{k}} \times \mathbf{u} = -\frac{1}{Fr^2} \nabla \eta - \frac{\chi}{\eta_{pen}} \mathbf{u}$$

$\eta_{pen} \ll 1$, Brinkman penalization parameter

$\phi \ll 1$, porosity parameter

$$\chi(\mathbf{x}, t) = \left\{ \begin{array}{ll} 1 & \text{if } \mathbf{x} \in O(\mathbf{x}), \\ 0 & \text{otherwise} \end{array} \right\}$$

Comparison

	Compressible Gas Dynamic Equations	Shallow Water Equations
Wave speeds	$c = \sqrt{\frac{\gamma p}{\rho}}$	$c = \left\{ \begin{array}{ll} \sqrt{\frac{gH}{\phi}} & \text{if } x_i \in O_i, \\ \sqrt{gH} & \text{otherwise} \end{array} \right\}$
Impedance	$Z \sim \frac{1}{\phi}$	$Z \sim \frac{1}{\phi^{\frac{3}{2}}}$
Asymptotic Analysis	boundary layer	no boundary layer

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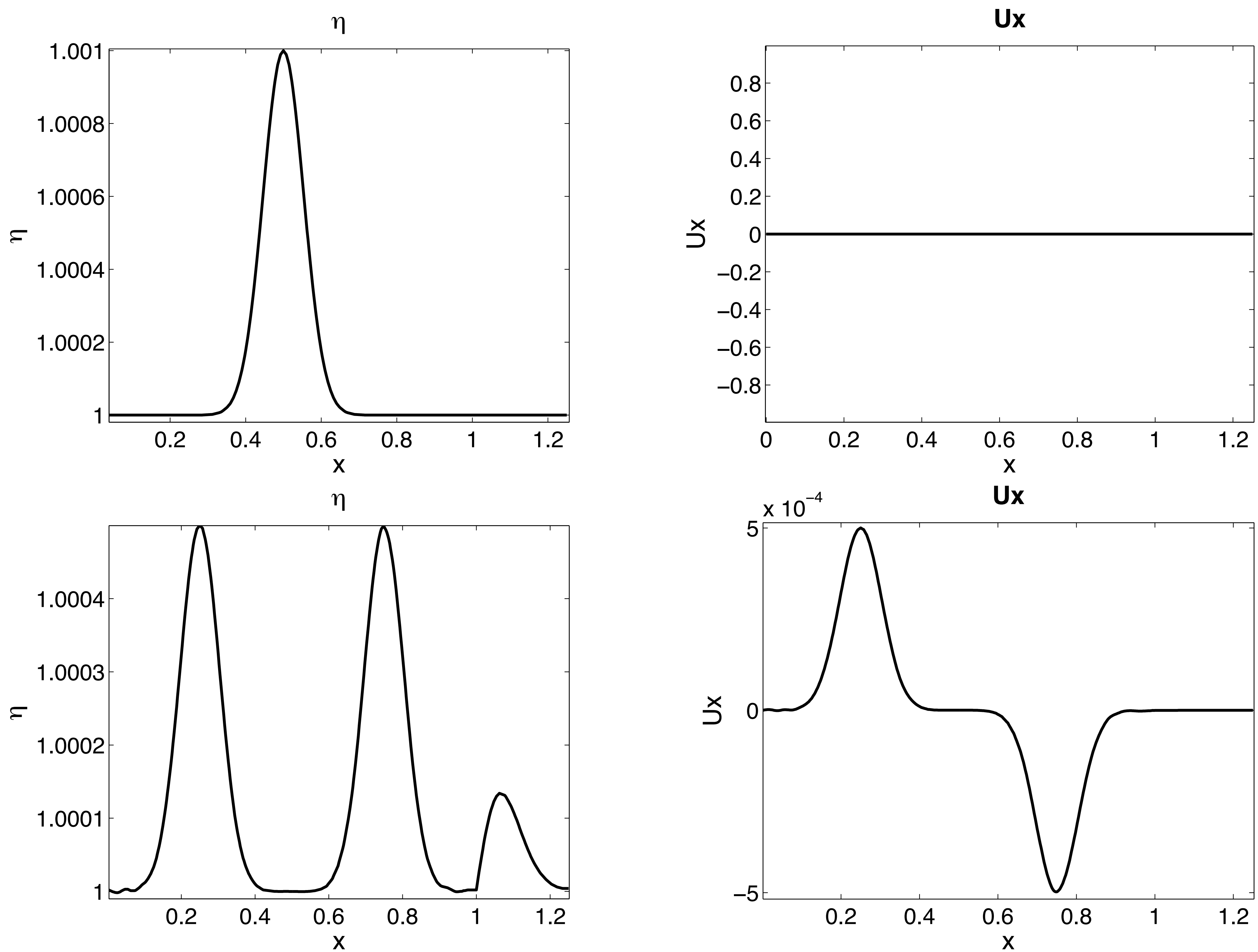
1D Convergence Test

initial conditions

after reflection



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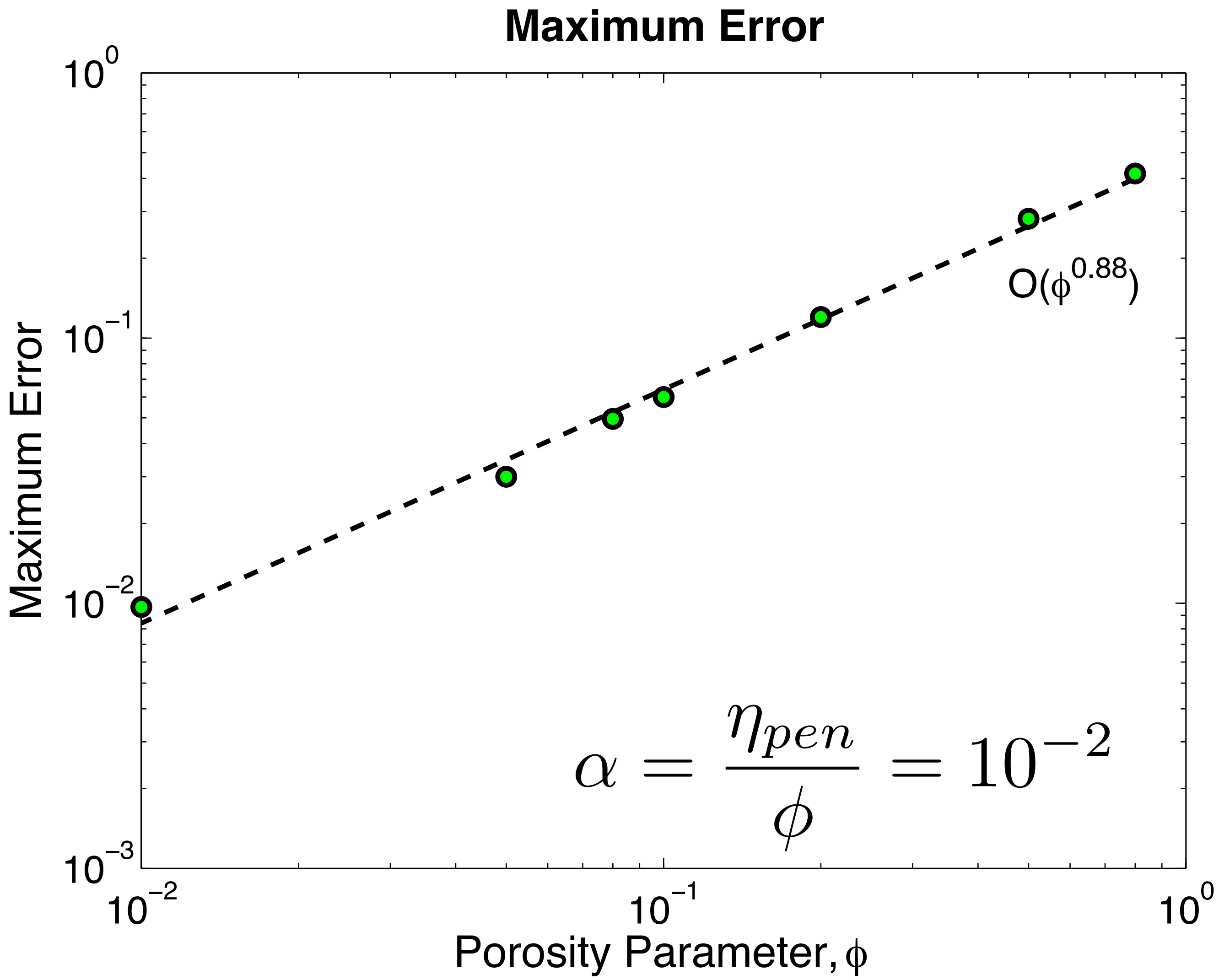
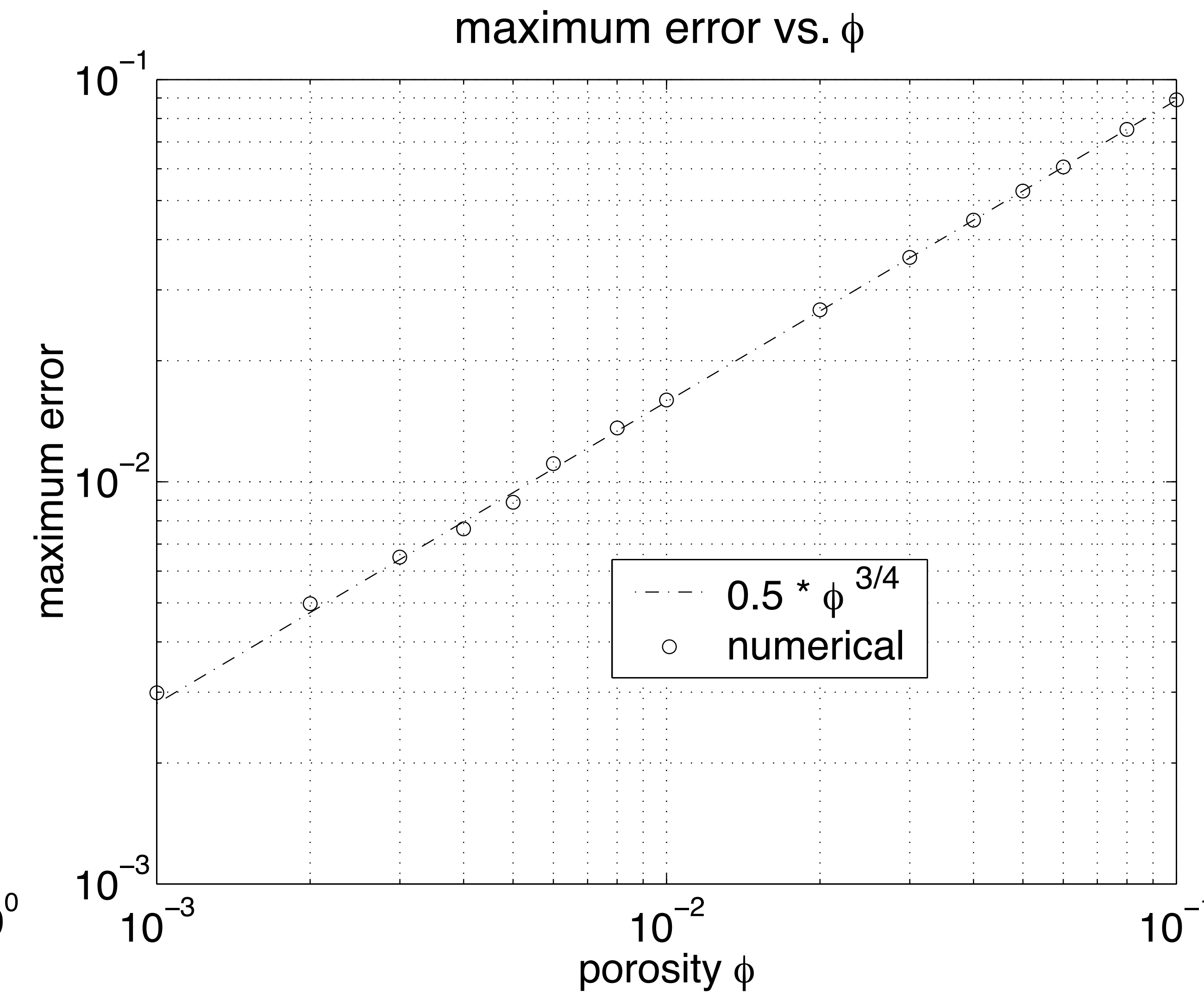
1D Results

SW Case

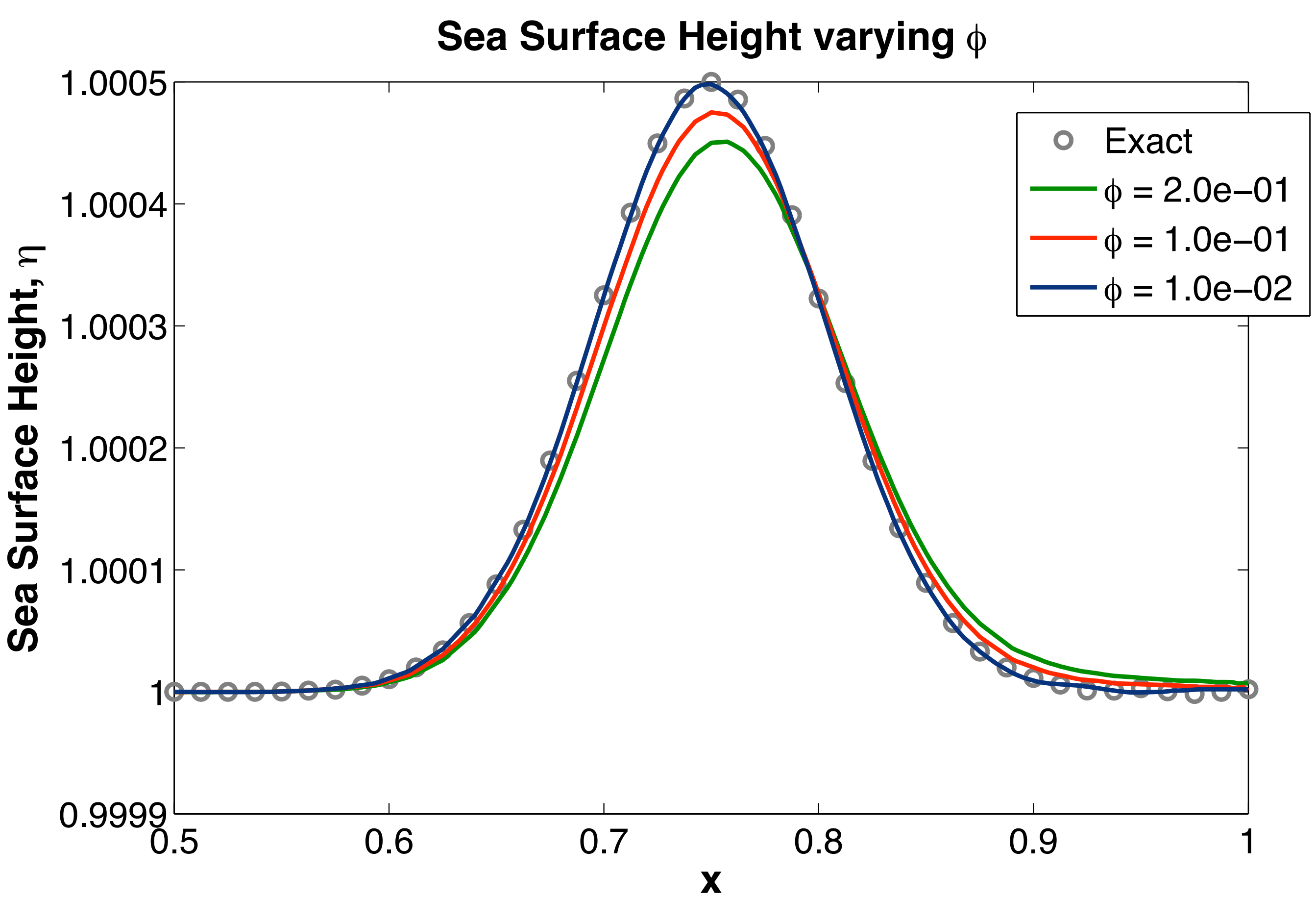


Compressible Case*

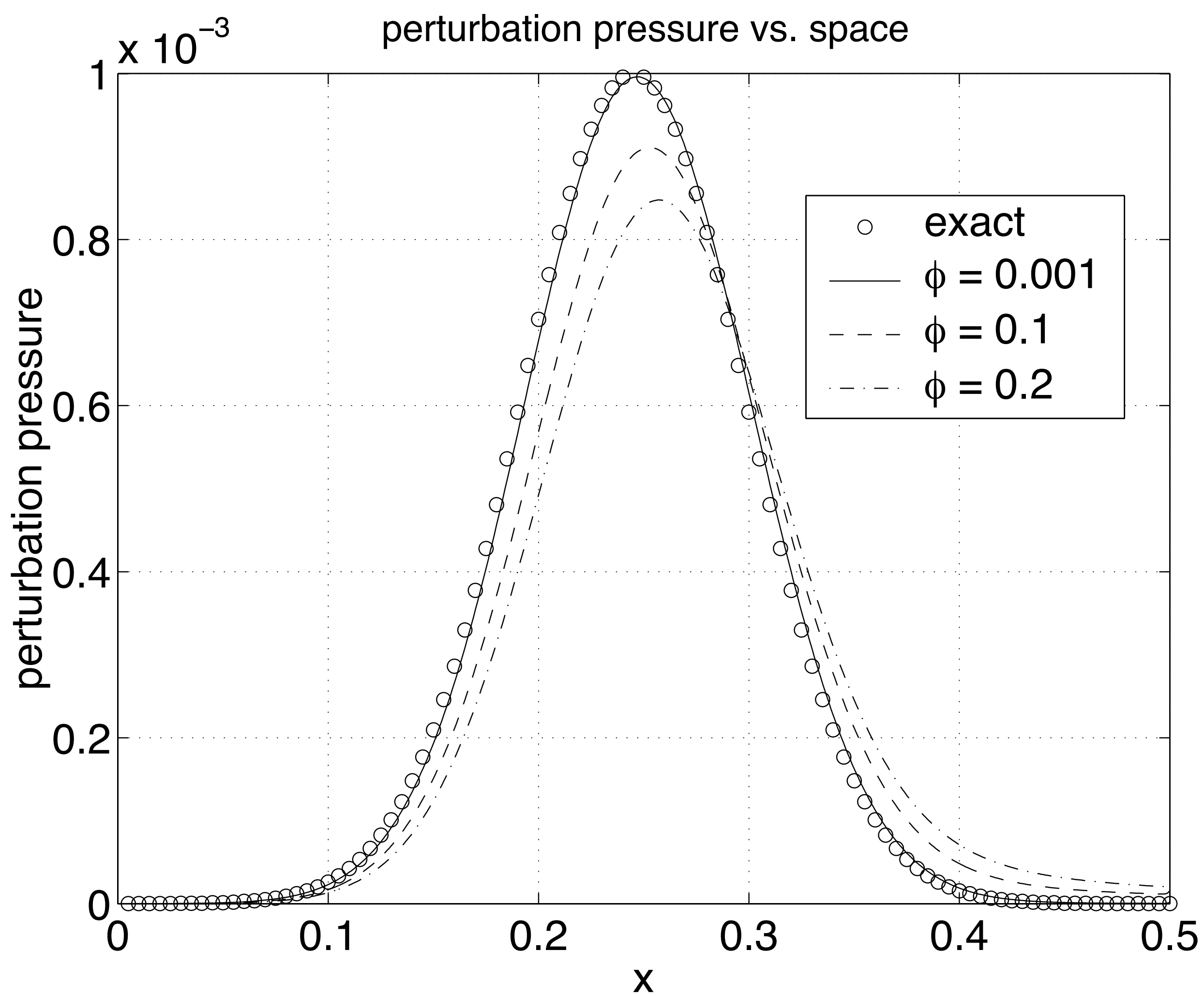


*Liu, Q. and Vasilyev, O.V.,Brinkman Penalization Method for Compressible Flows in Complex Geometries, Journal of Computational Physics, 227(2), pp. 946-966, 2007

SW Case

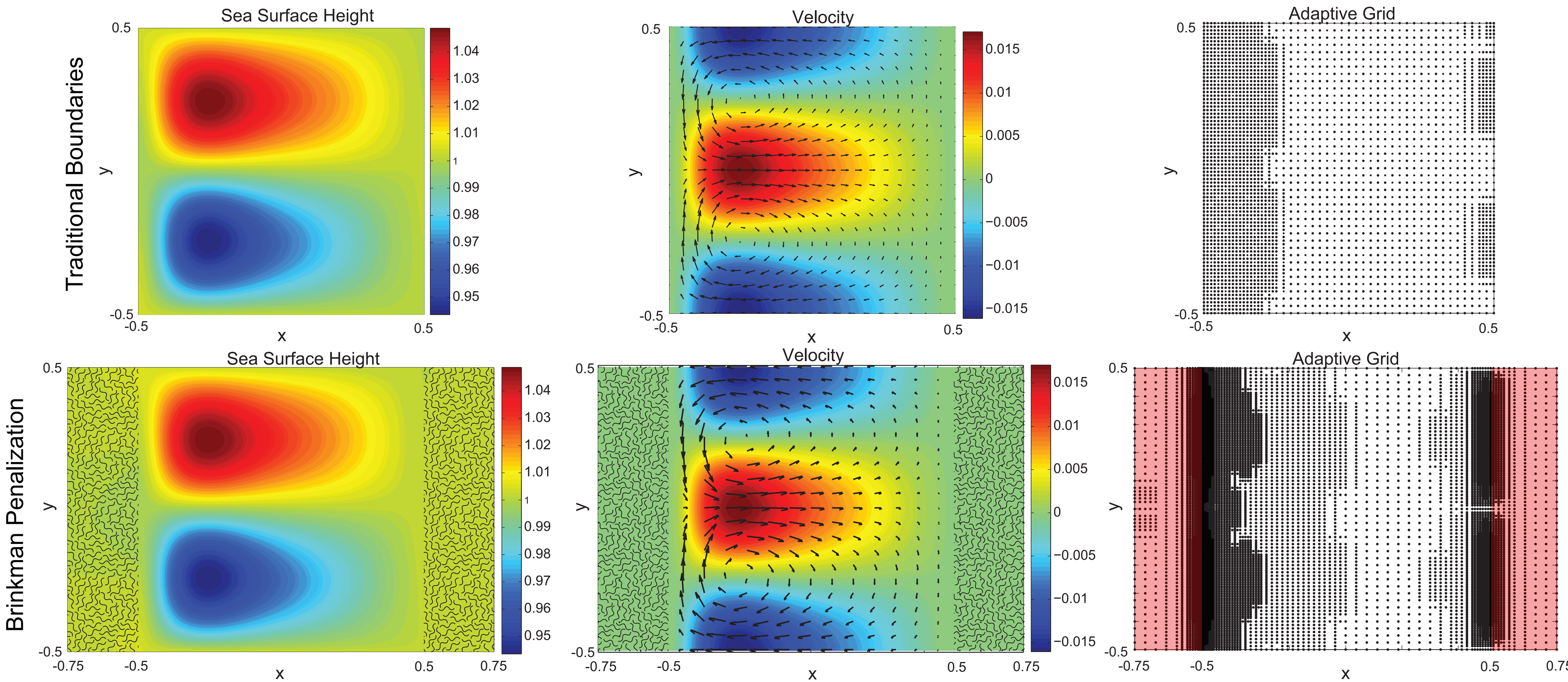


Compressible Case*

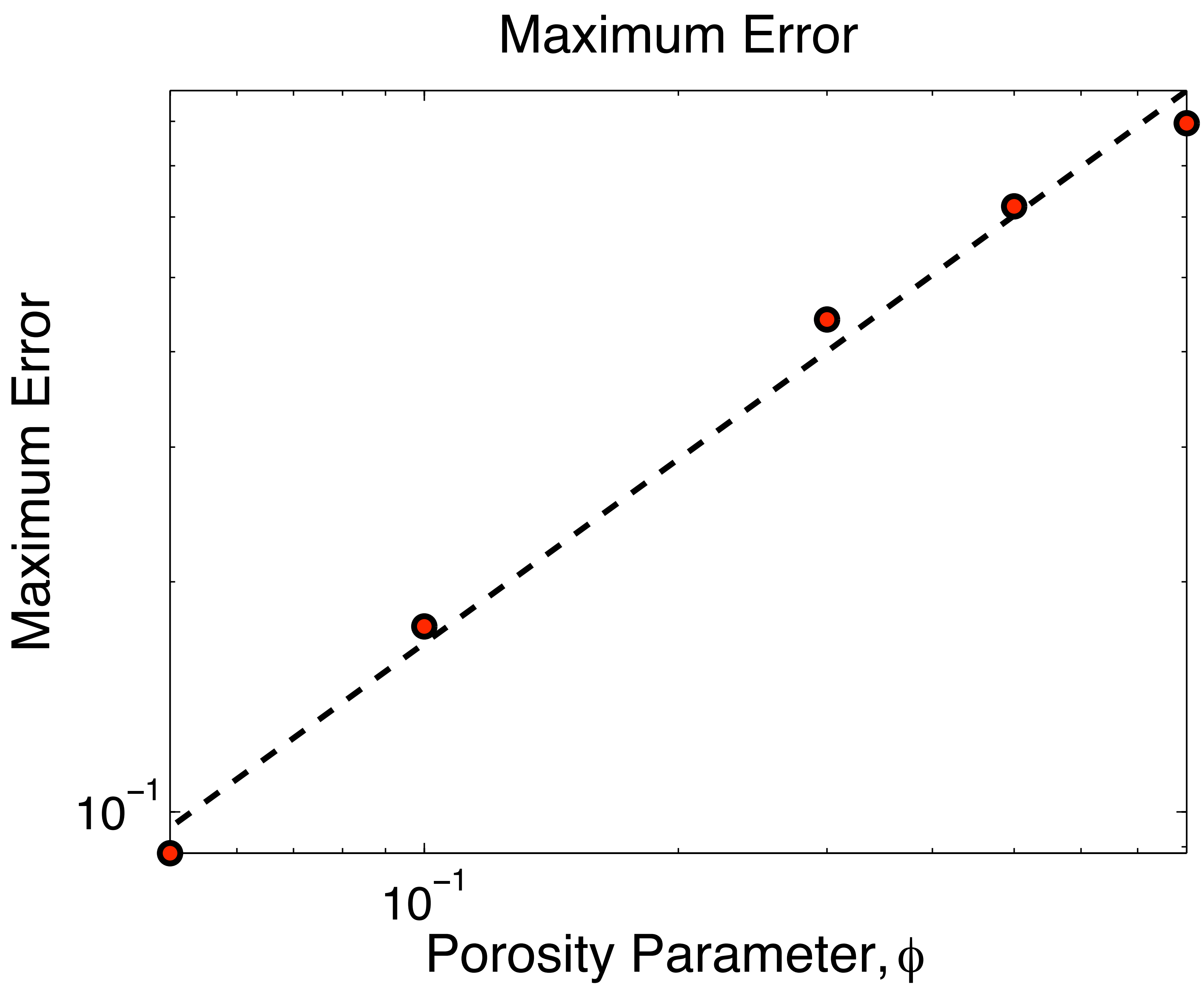


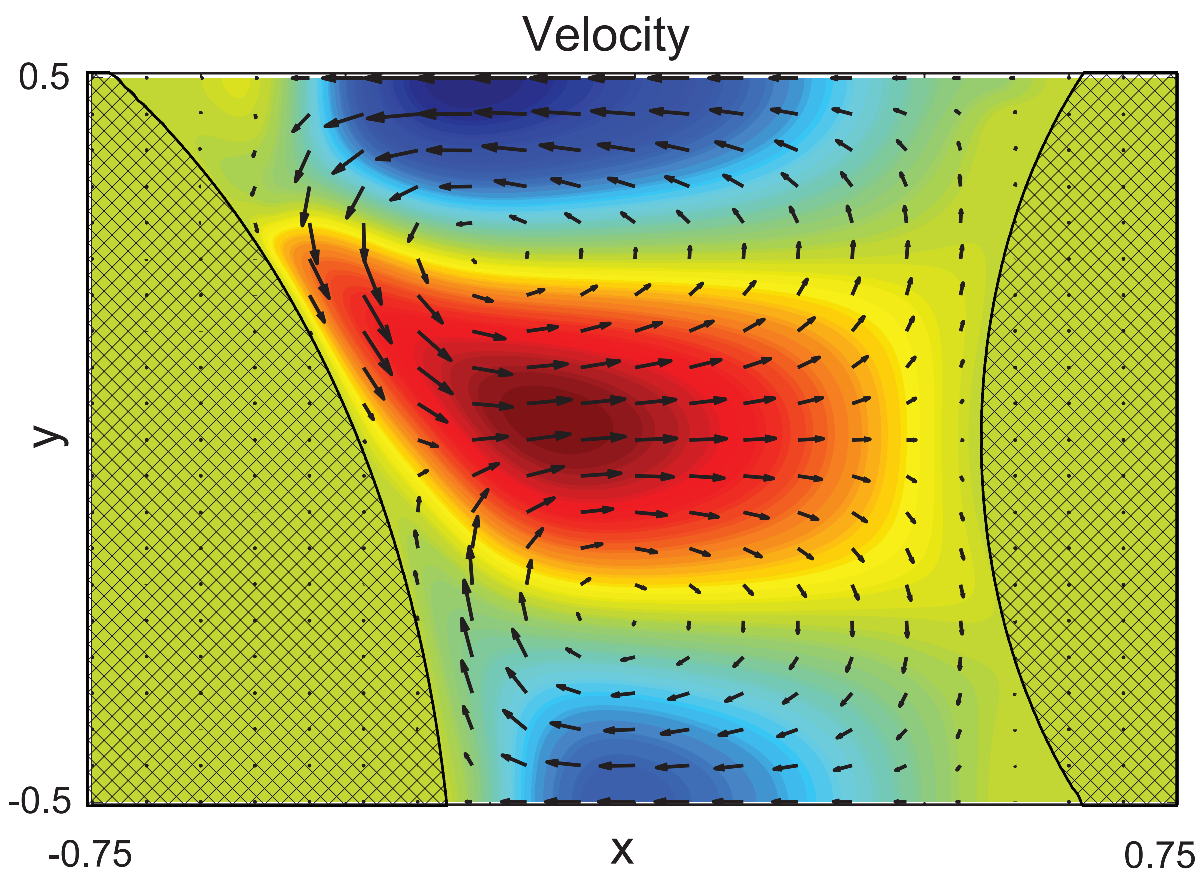
*Liu, Q. and Vasilyev, O.V.,Brinkman Penalization Method for Compressible Flows in Complex Geometries, Journal of Computational Physics, 227(2), pp. 946-966, 2007

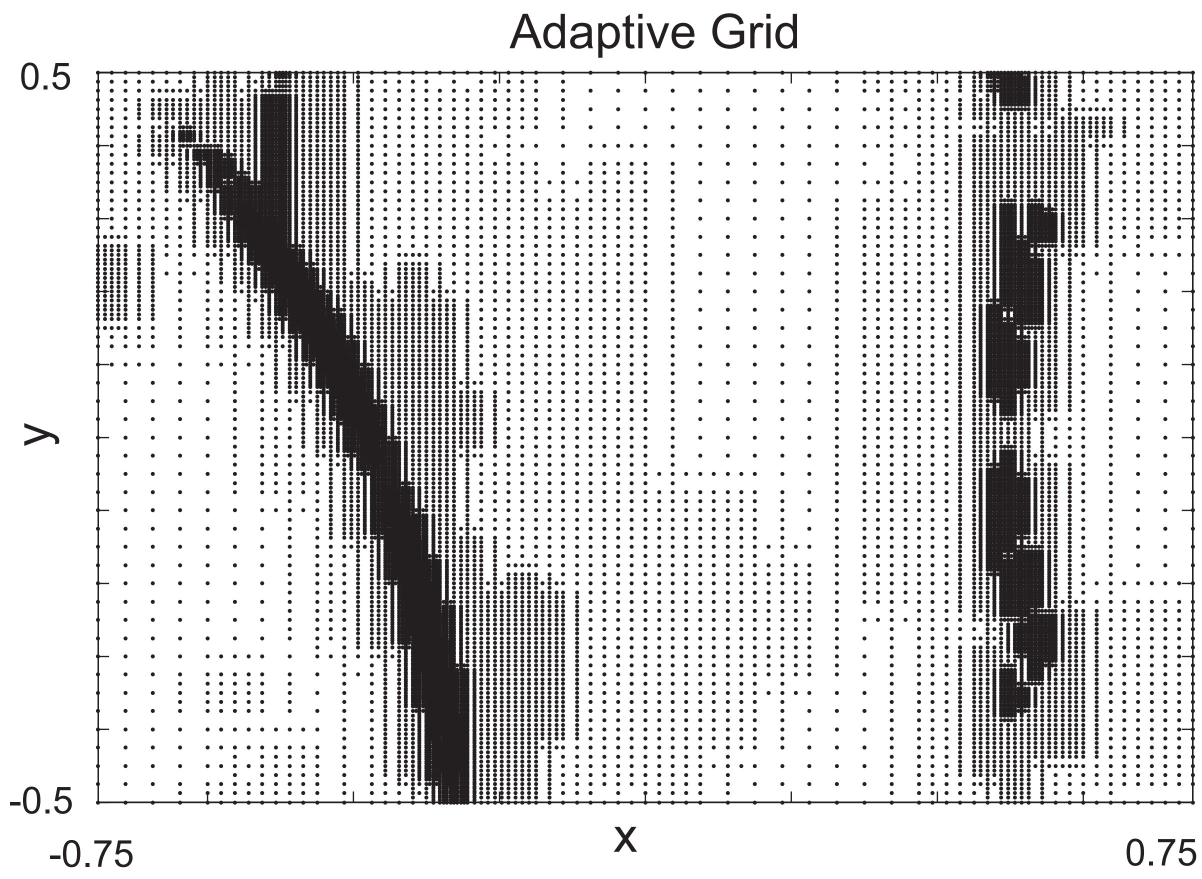
2D Results


Traditional Boundaries
Brinkman Penalization
Sea Surface Height
Velocity
Adaptive Grid
x
y
0.5
-0.5
-0.75
0.75
1.04
1.03
1.02
1.01
1
0.99
0.98
0.97
0.96
0.95
0.015
0.01
0.005
0
−0.005
−0.01
−0.015

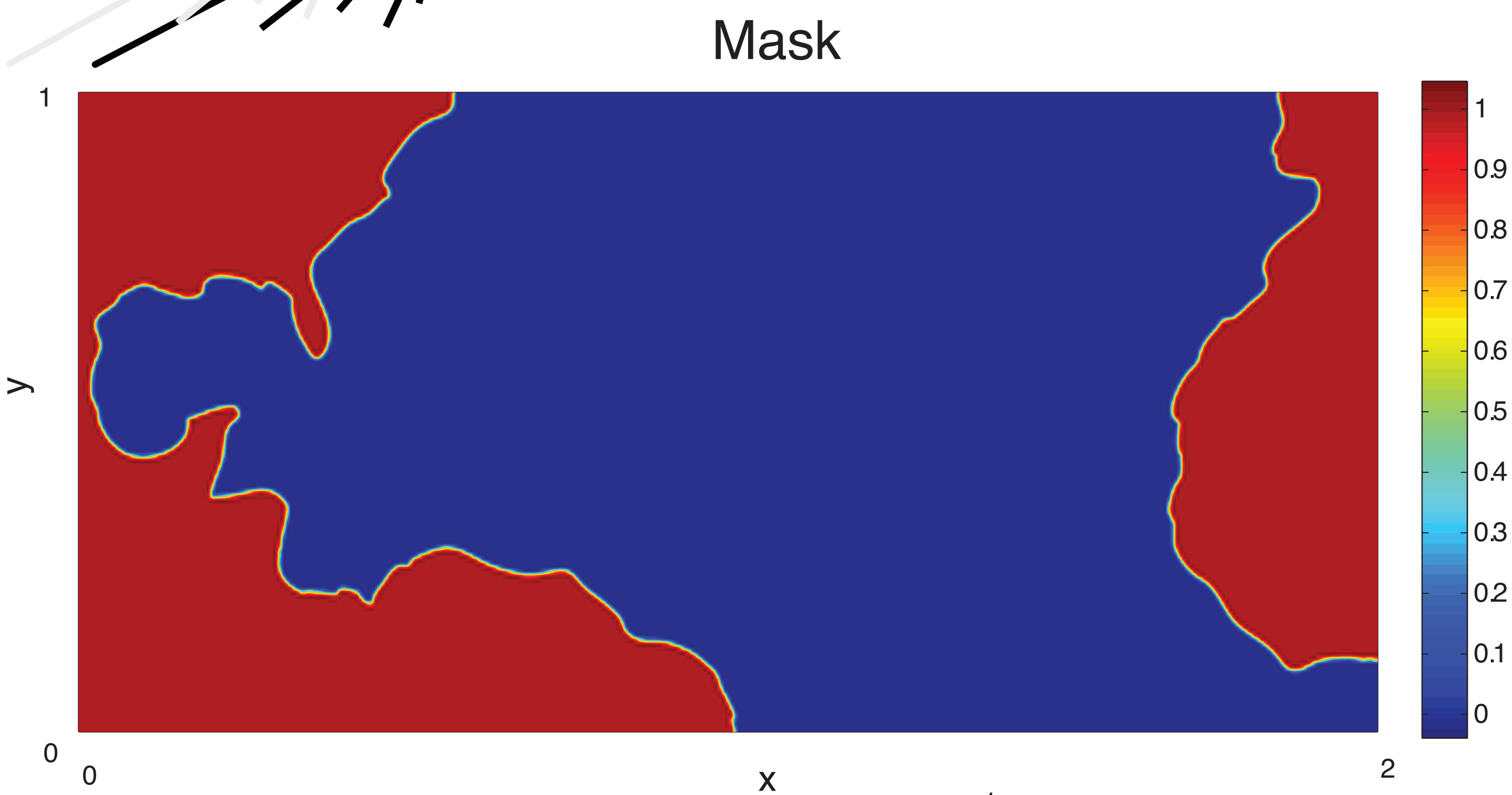
2D Convergence Results

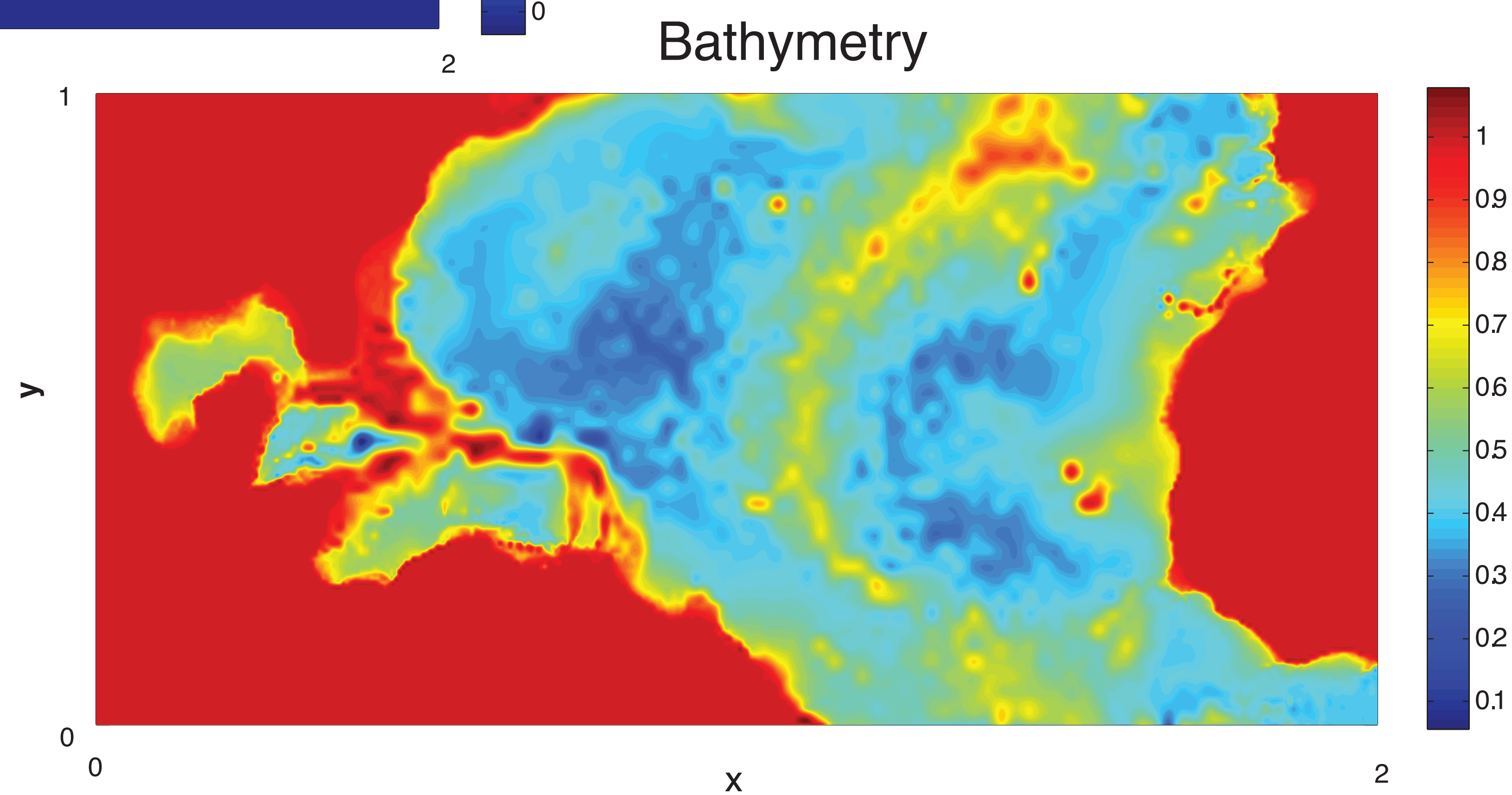



Velocity
0.5
-0.5
y
-0.75
x
0.75


Adaptive Grid
0.5
-0.5
y
-0.75
x
0.75

North Atlantic Case

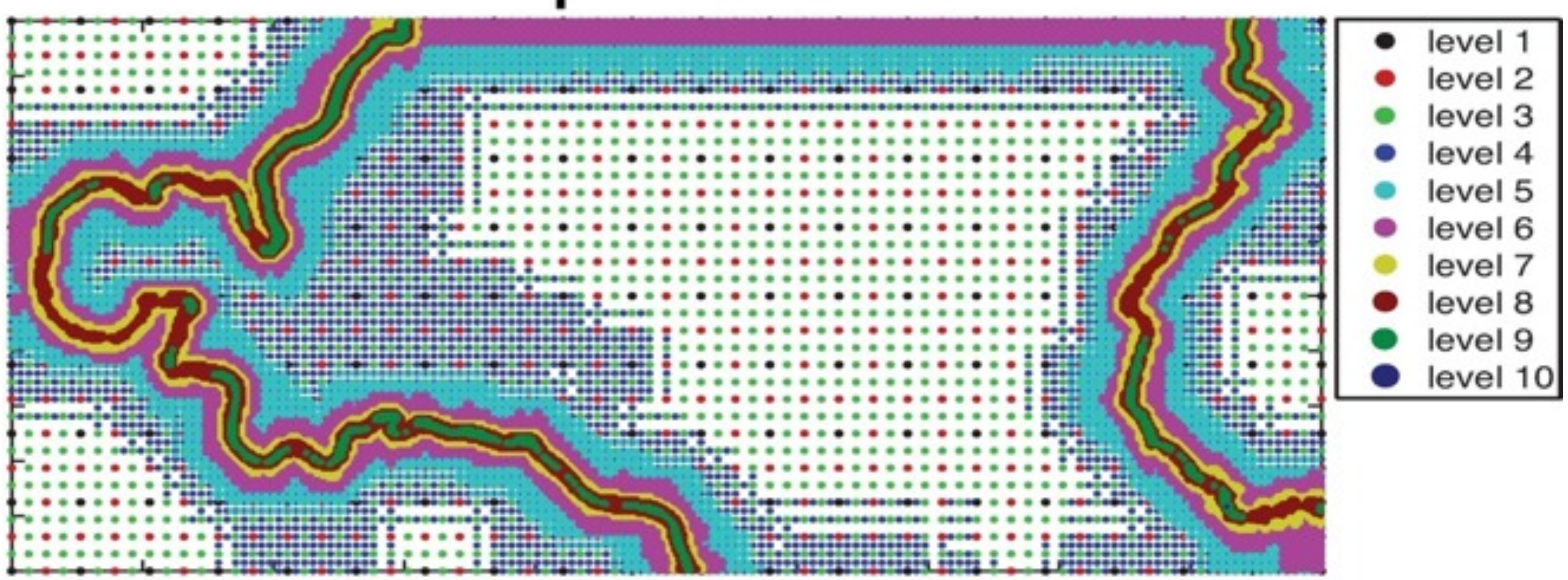




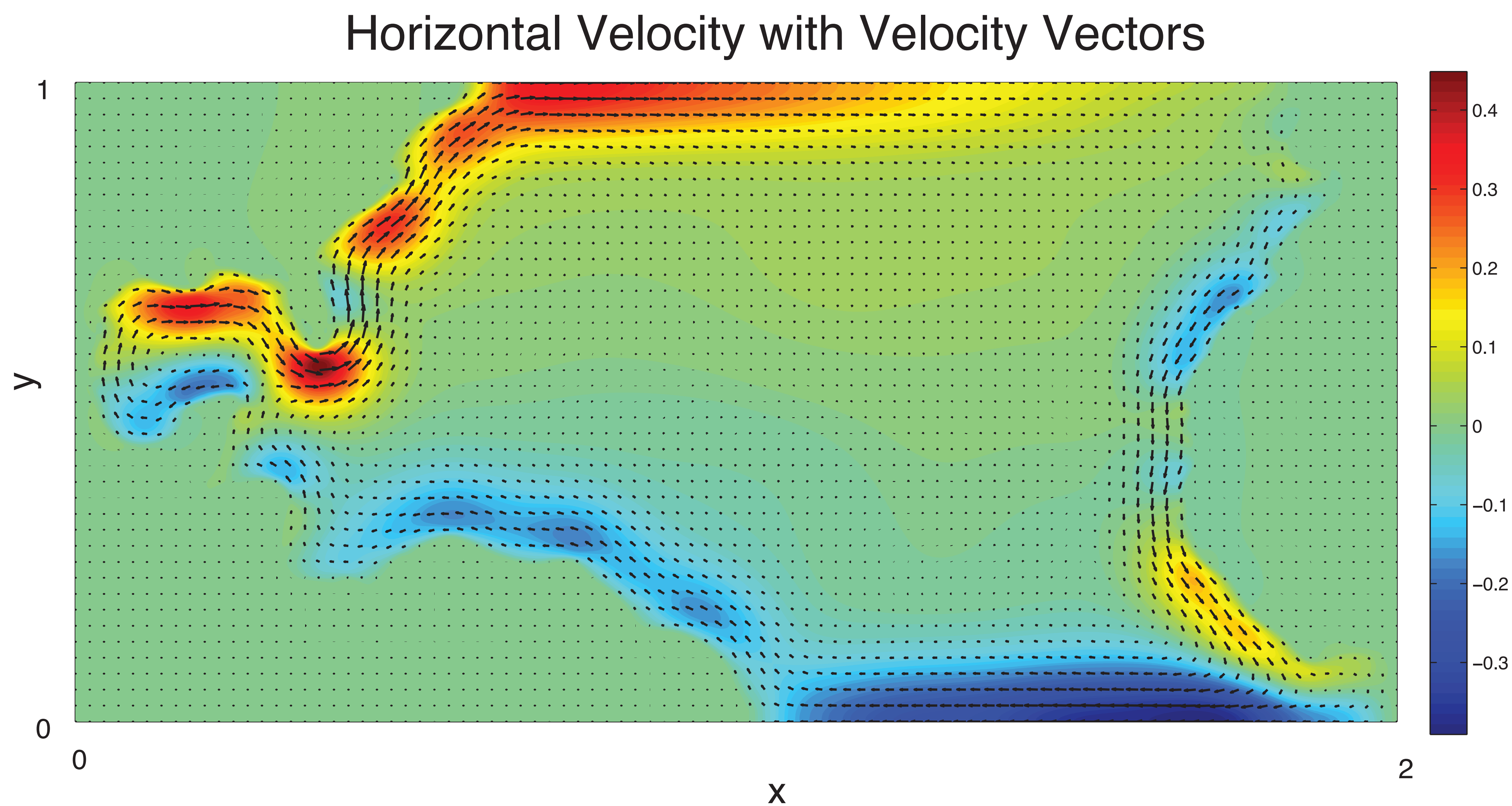
North Atlantic Case

Vorticity

Adaptive Grid



North Atlantic Case



- Number of points used: 193,539
- Effective Resolution: 11,776 x 5120 = 60,293,120
- 0.3% of grid points used (99.7% compression)

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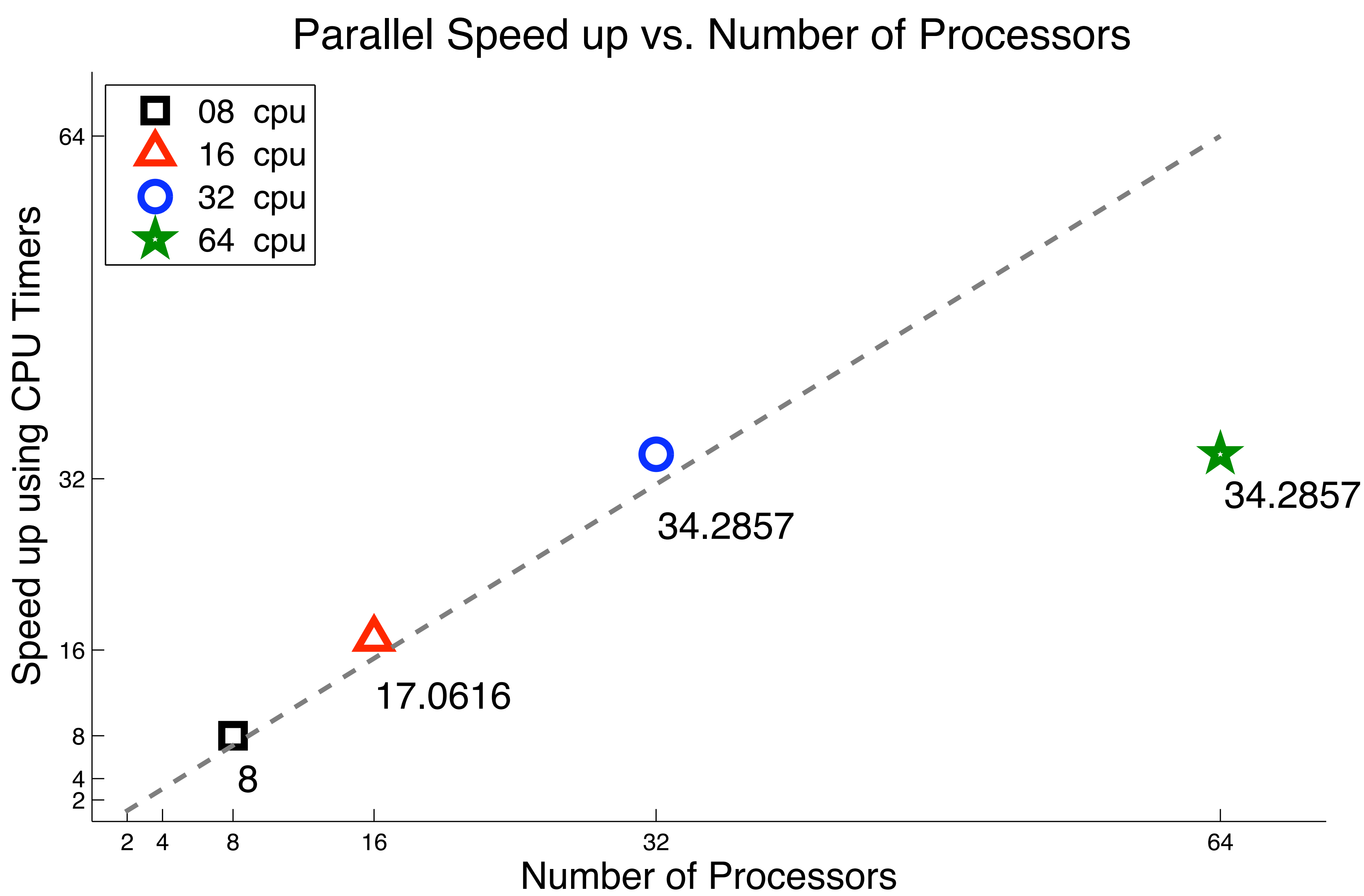
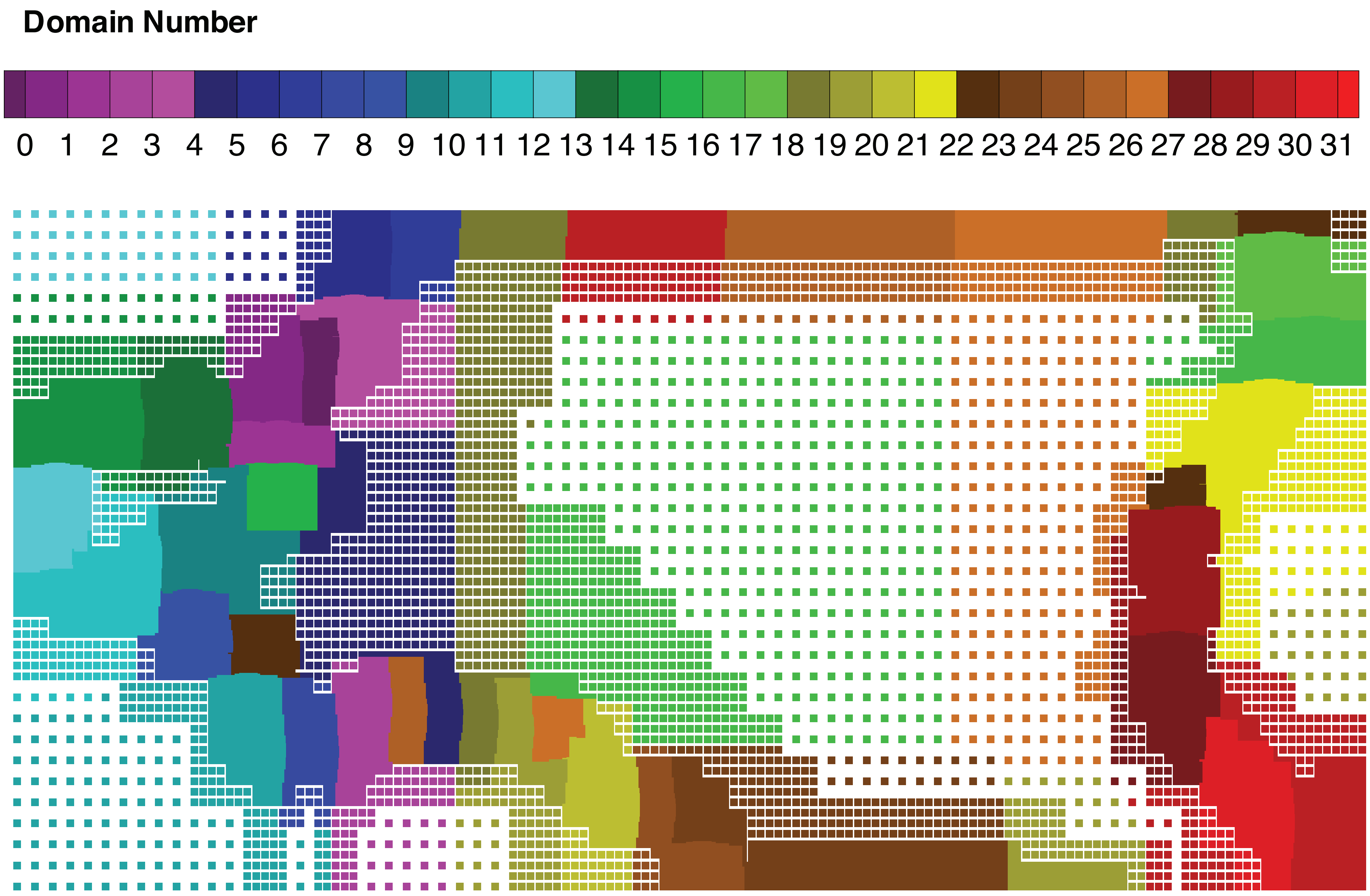

Parallel Speed up vs. Number of Processors
08 cpu
16 cpu
32 cpu
64 cpu
64
32
16
8
4
2
Speed up using CPU Timers
34.2857
34.2857
17.0616
8
2
4
8
16
32
64
Number of Processors

Domain Decomposition



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In conclusion,

- Adaptive wavelet collocation method and Brinkman penalization work well at simulating North Atlantic circulation

Future Work:

- 2D Shallow Water Model
  - Finish high resolution simulations
  - North Atlantic simulations
- 3D Primitive Equations Model
  - Continue work on non-hydrostatic primitive equations

Thank you!