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Dependency of tsunami simulations on advection scheme, grid resolution, bottom friction und topography

C. Wekerle, S. Harig, W. Pranowo, A. Androsov, A. Fuchs,
N. Rakowsky, J. Schröter, S. Danilov and J. Behrens

Outline

- The tsunami model TSUNAWI
  - Numerical concepts and inundation scheme
- The Okushiri tsunami 1993
  - Influence of advection scheme, grid resolution, bottom friction on simulation results
- A worst case scenario for Padang
  - Influence of topography data on inundation
- Conclusion

Shallow water equations

Continuity equation:

$$\partial_t \eta + \nabla \cdot (\mathbf{v} H) = 0$$

Momentum equation:

$$\partial_t \mathbf{v} + \underset{\substack{\uparrow \\ \text{advection}}}{(\mathbf{v} \cdot \nabla) \mathbf{v}} + \underset{\substack{\uparrow \\ \text{Coriolis}}}{\mathbf{f} \times \mathbf{v}} + \underset{\substack{\uparrow \\ \text{pressure} \\ \text{gradient}}}{g \nabla \eta} + \underset{\substack{\uparrow \\ \text{bottom} \\ \text{friction}}}{\frac{g n^2 \mathbf{v} |\mathbf{v}|}{H^{4/3}}} - \underset{\substack{\uparrow \\ \text{viscosity}}}{\nabla \cdot (A_h \nabla \mathbf{v})} = 0$$

where

$\mathbf{v} = (u(t,x,y), v(t,x,y))$: horizontal velocity

$H = h(x,y) + \eta(t,x,y)$: total water depth

Boundary Conditions:

$$\mathbf{v} \cdot \mathbf{n} = \sqrt{\frac{g}{H}}\eta, \quad (x,y) \in \partial\Omega_1$$

$$\mathbf{v} \cdot \mathbf{n} = 0, \quad (x,y) \in \partial\Omega_2$$

Initial Conditions:

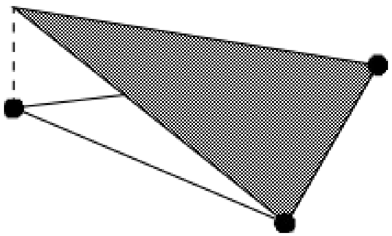
$$\mathbf{v}\big|_{t=0} = 0$$

$$\eta\big|_{t=0} = \eta_0$$

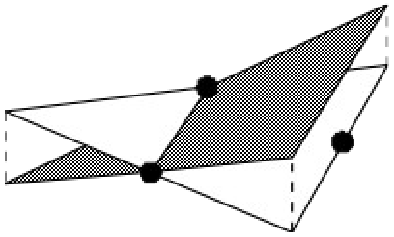
Discretization

Finite element spatial discretization:

non-conforming mixed P_1-P_1^{nc} (Hanert et al., 2005)



Linear conforming shape functions for η



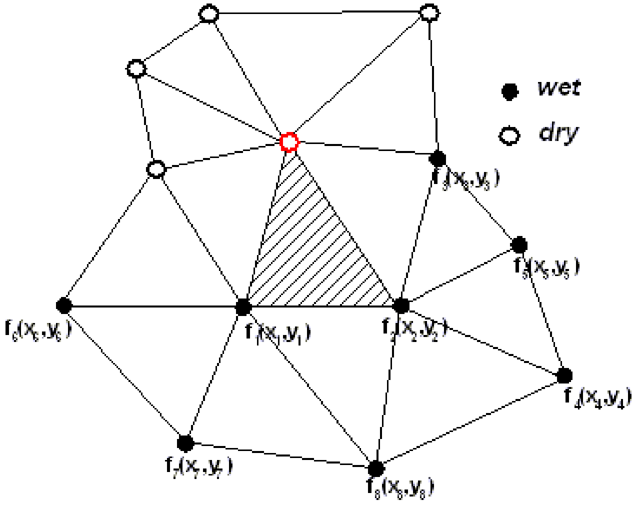
Linear non-conforming shape functions for v

Explicit time stepping scheme:

Leap frog with Robert-Asselin filter

Inundation: Extrapolation scheme

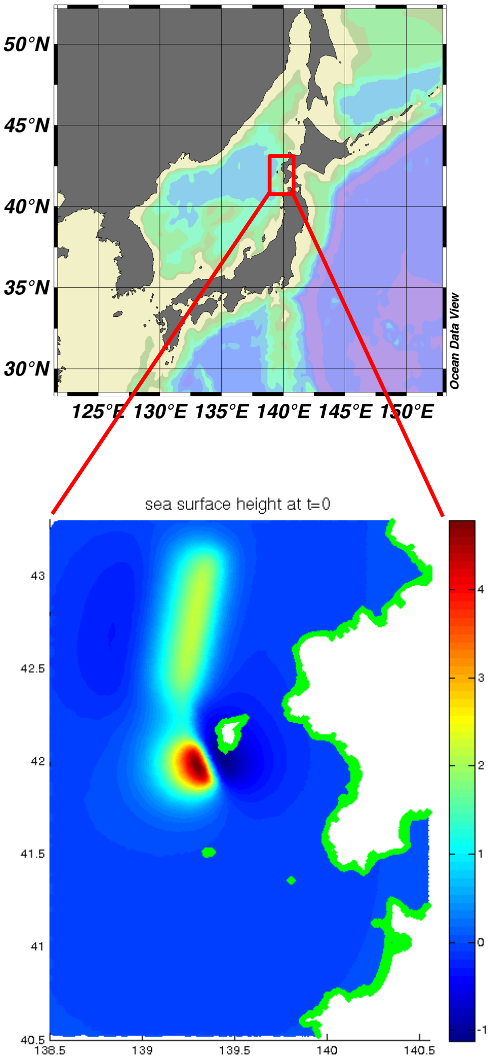
„Dry node concept“ by Lynett et al., 2002



The Okushiri Tsunami 1993 (M_w 7.8)

Field benchmark for the validation of tsunami models (Synolakis, NOAA, 2007)

- Initial condition, tide gauge data and bathymetry provided by NOAA
- Very high runup up to 30m at Monai (west coast of Okushiri island)

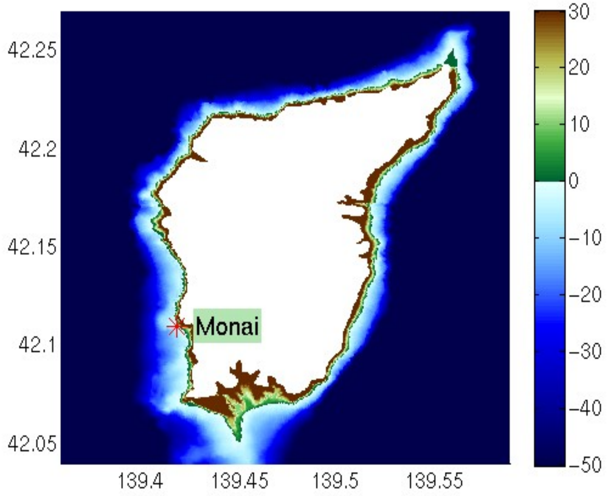


Initial uplift distribution

Takahashi et al, 1995

max. uplift: 4.87m
max. depression: -1.12m

Okushiri island



Mesh Generation

Mesh refinement is based on the CFL criterion and bathymetry:

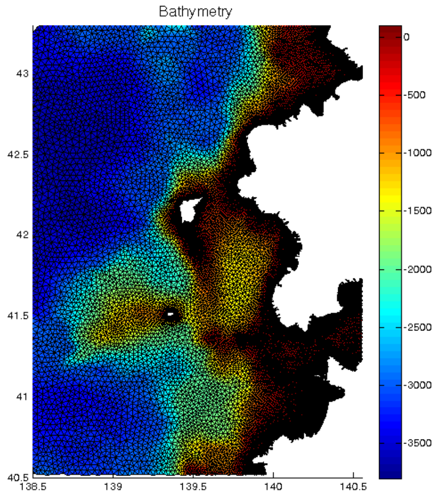
$$\Delta x \leq \min\left(k_1\sqrt{gh},\; k_2\frac{h}{\nabla h}\right)$$

→ fine resolution at the shoreline and at regions of steep bathymetry, coarse mesh in the deep ocean

For the Okushiri testcase, four meshes with different resolution are used:

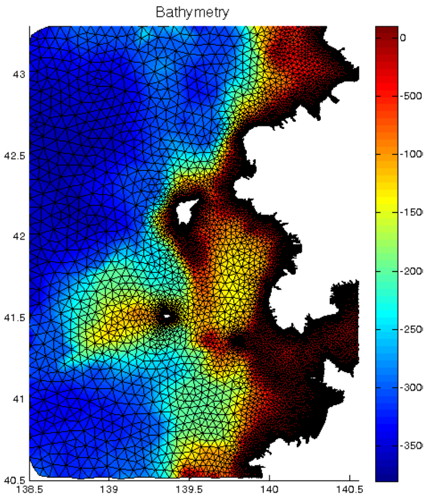
Mesh 1 (fine_mesh):

nodes: 309 410
min. res. 50m
max. res. 3km



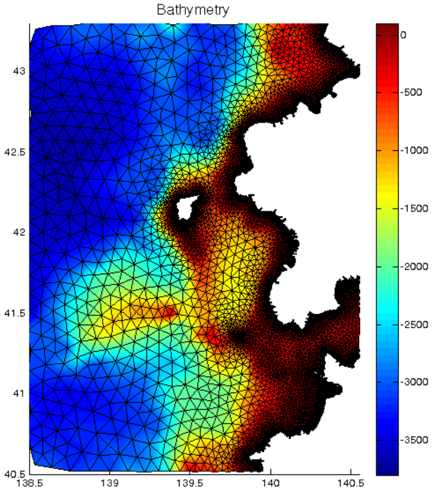
Mesh 2 (medium_mesh):

nodes: 103 361
min. res. 100m
max. res. 6km



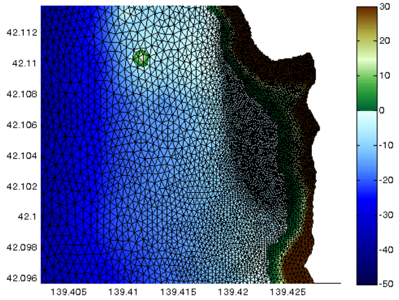
Mesh 3 (coarse_mesh):

nodes: 45 028
min. res. 150m
max. res. 9km



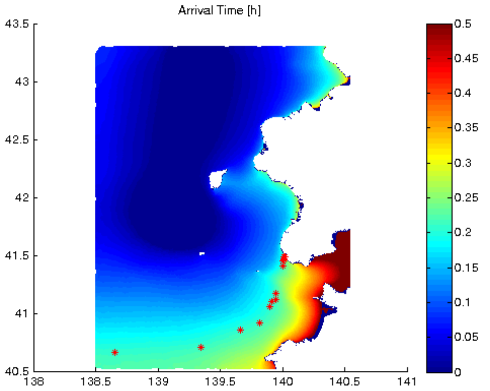
Mesh 4:

nodes: 214 124
local refinement in the Monai area:
min. res. 10m
max.res. 6km

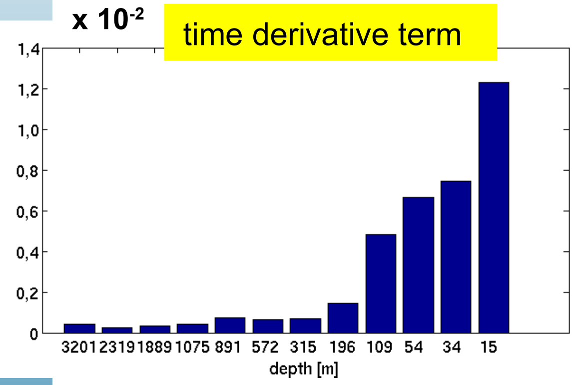


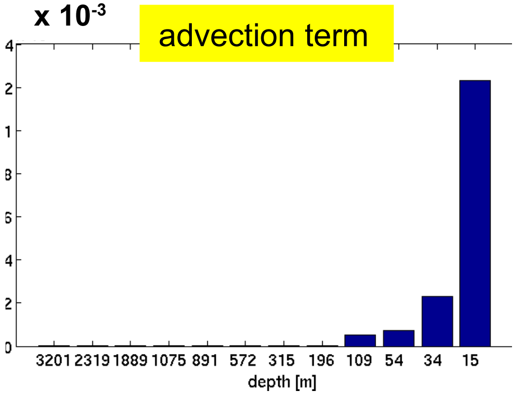
Fractions of terms in the momentum equation dependent on depth

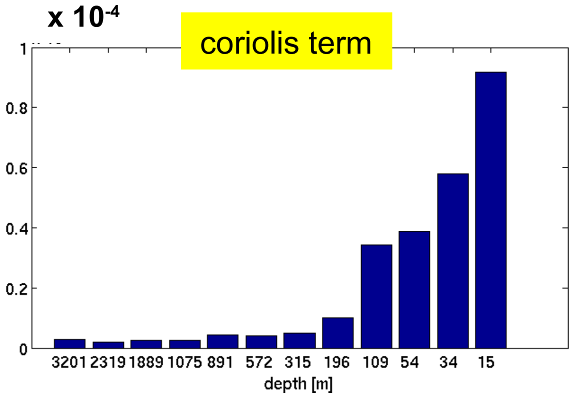
Arrival time

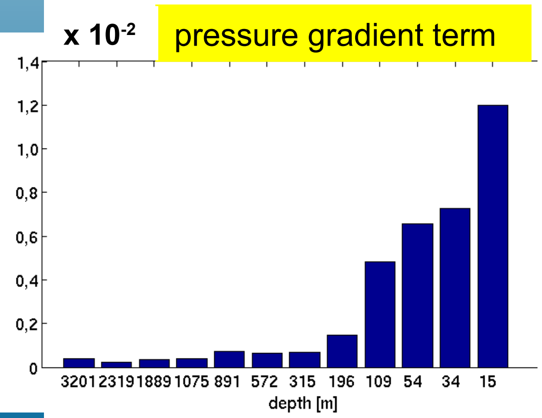


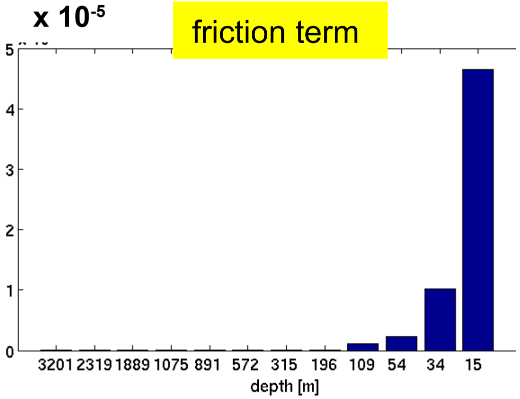
Locations of different depth on 12 min isochrone

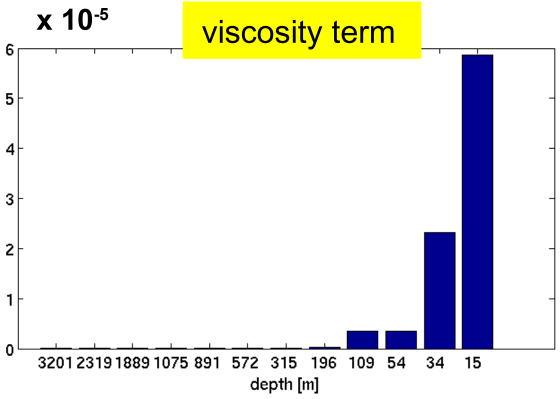








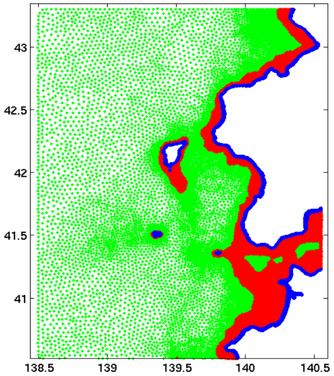




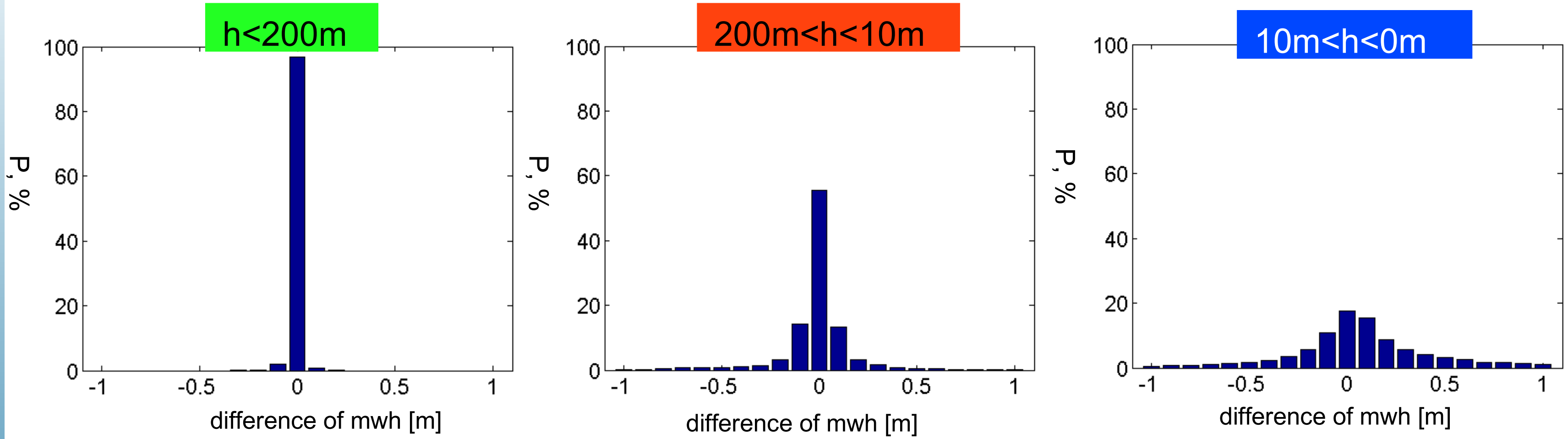
Momentum eq. with and without advection

Divison of nodes into
3 categories:

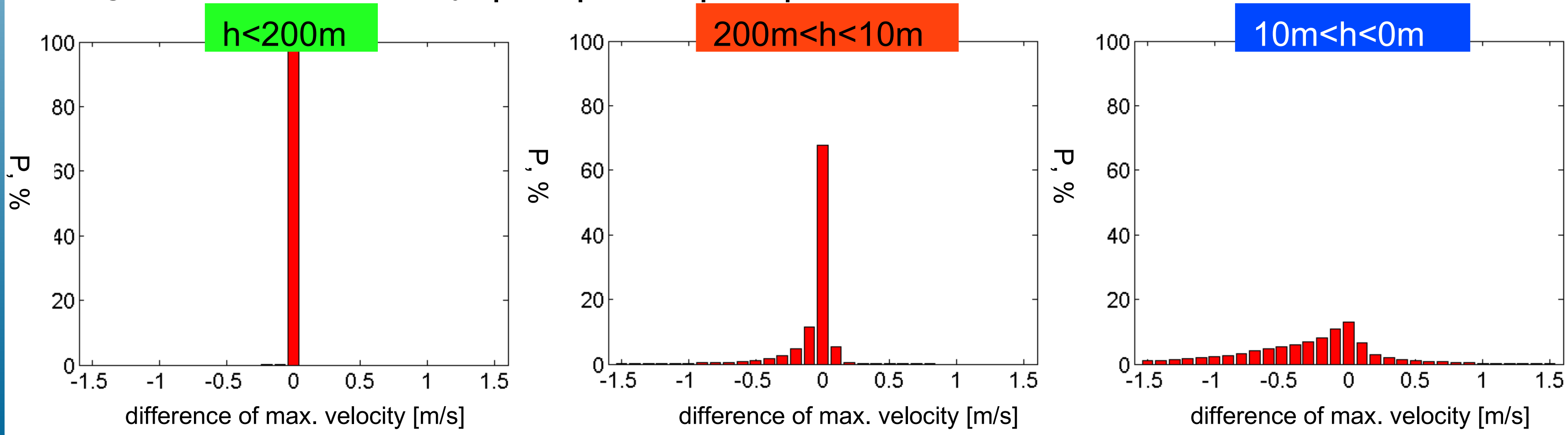
🟢 depth<200m
🔴 200m<depth<10m
🔵 10m<depth<0m



Histograms of mwh: η_{max} linear - η_{max} non-linear

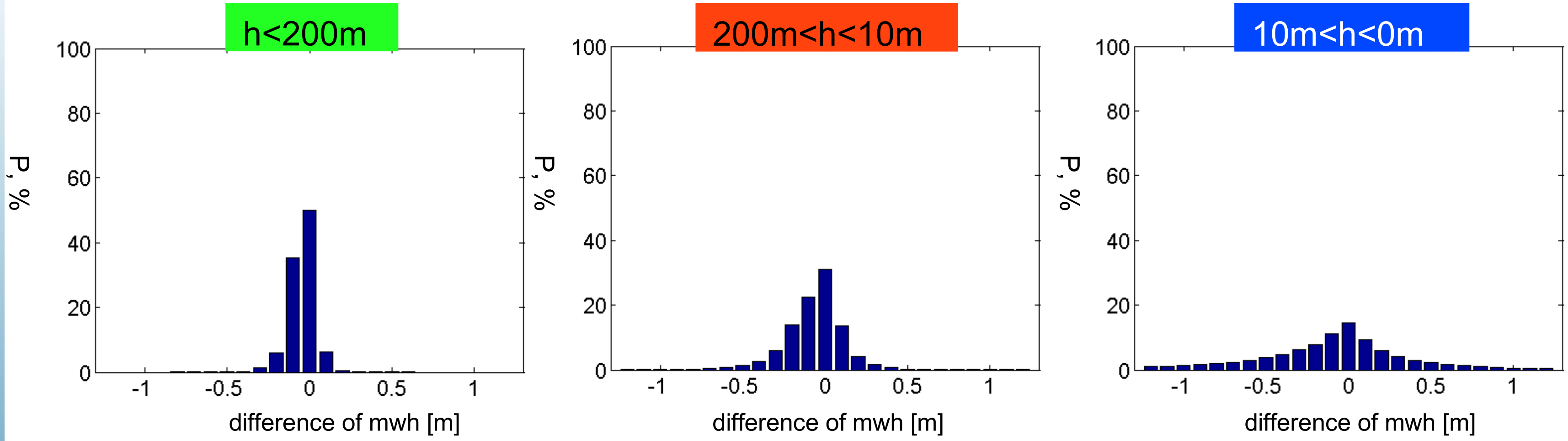


Histograms of max. velocity: $|v_{max}|$linear - $|v_{max}|$non-linear

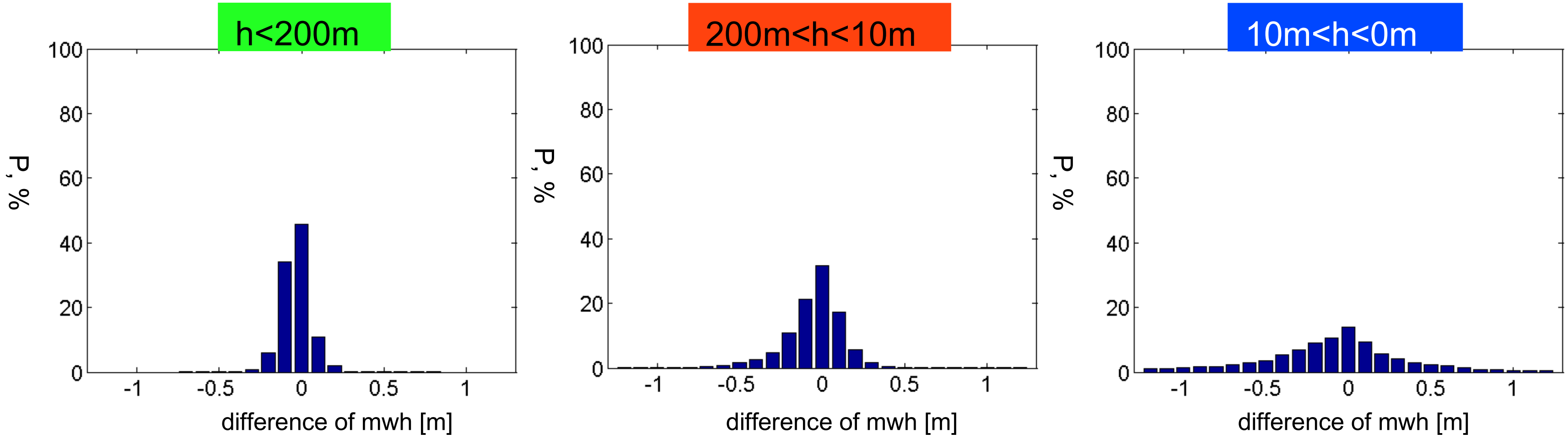


Influence of mesh resolution on mwh

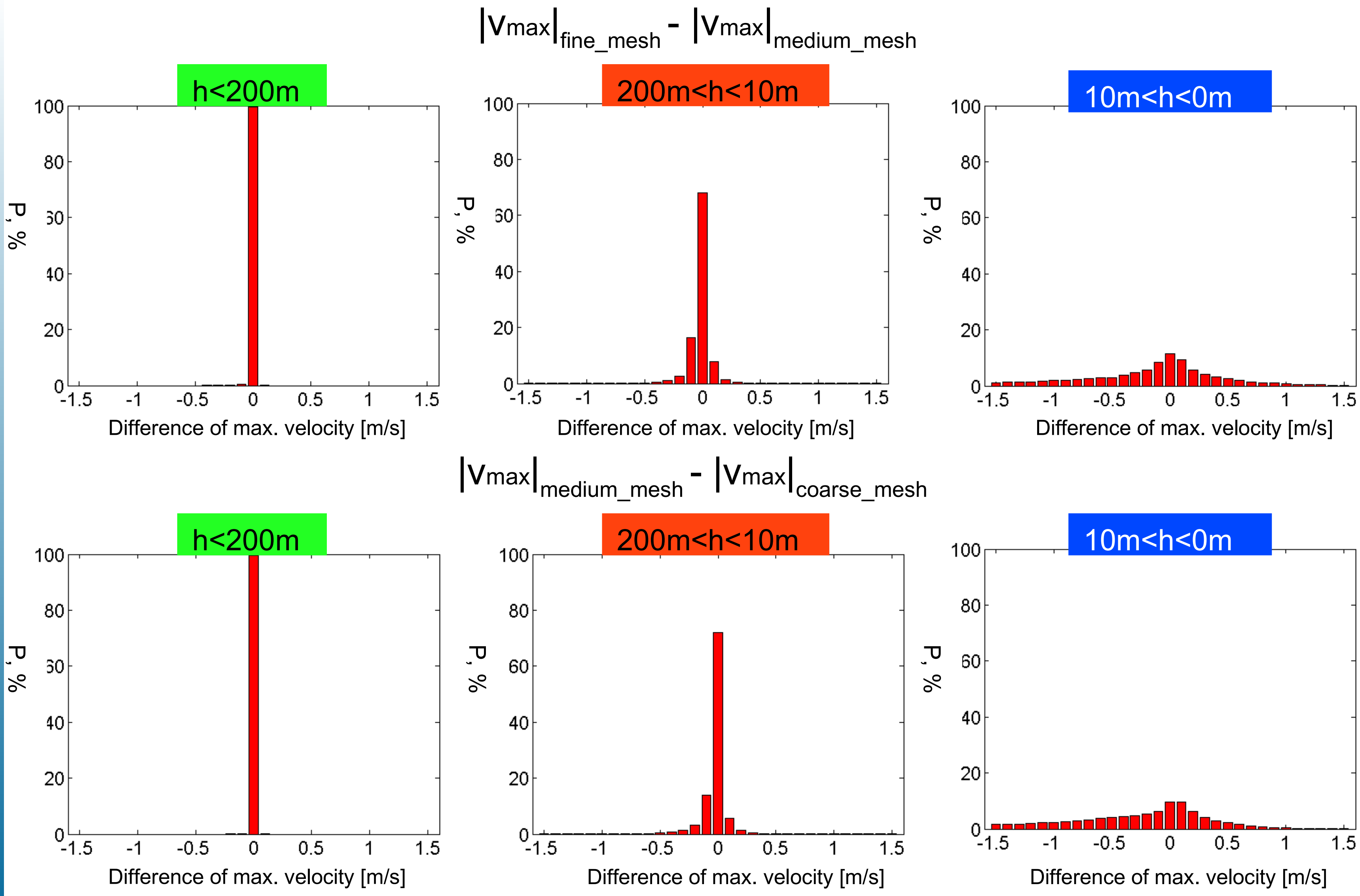
$\eta \mathrm{max}_{\mathrm{fine\_mesh}} - \eta \mathrm{max}_{\mathrm{medium\_mesh}}$



$\eta \mathrm{max}_{\mathrm{medium\_mesh}} - \eta \mathrm{max}_{\mathrm{coarse\_mesh}}$

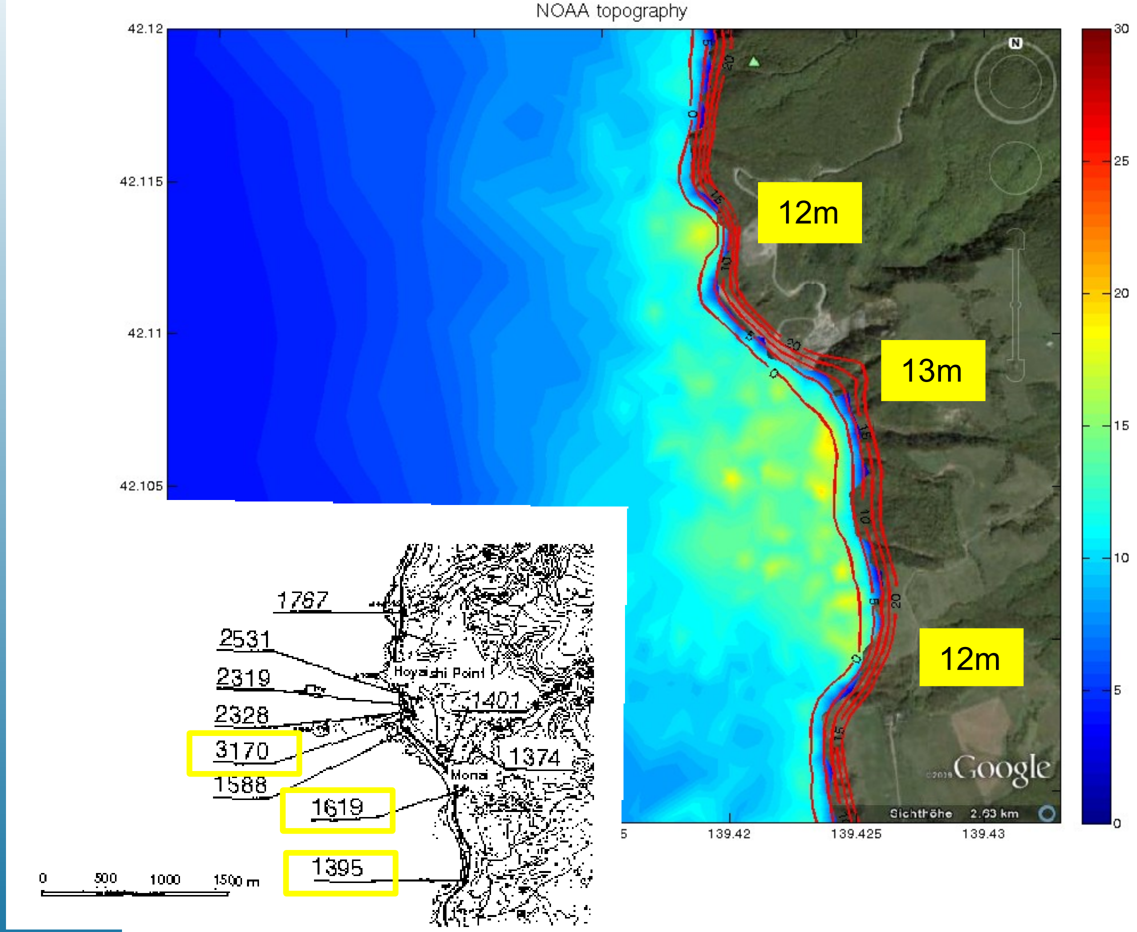


Influence of mesh resolution on max. velocity



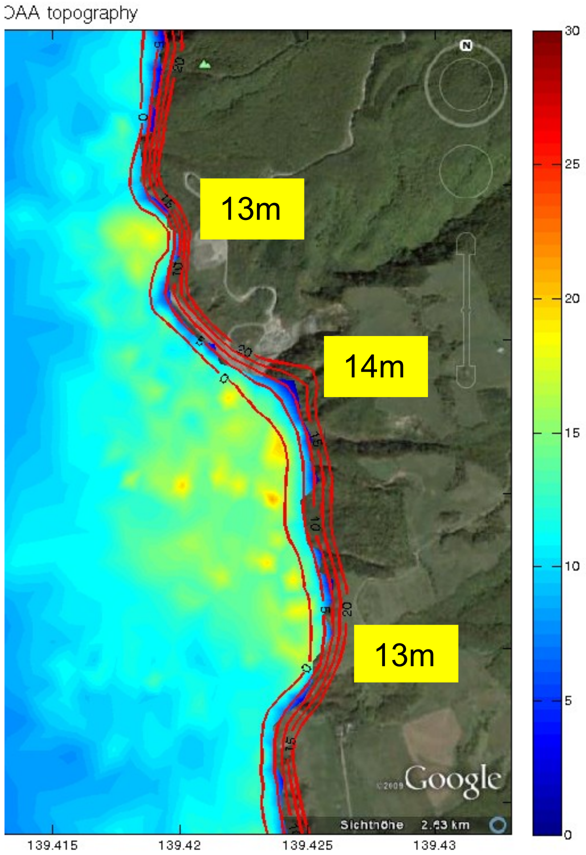
Inundation of the Monai area – with and without friction

friction parameter: n=0.02



Runup distribution in the Monai area (in cm)

without friction

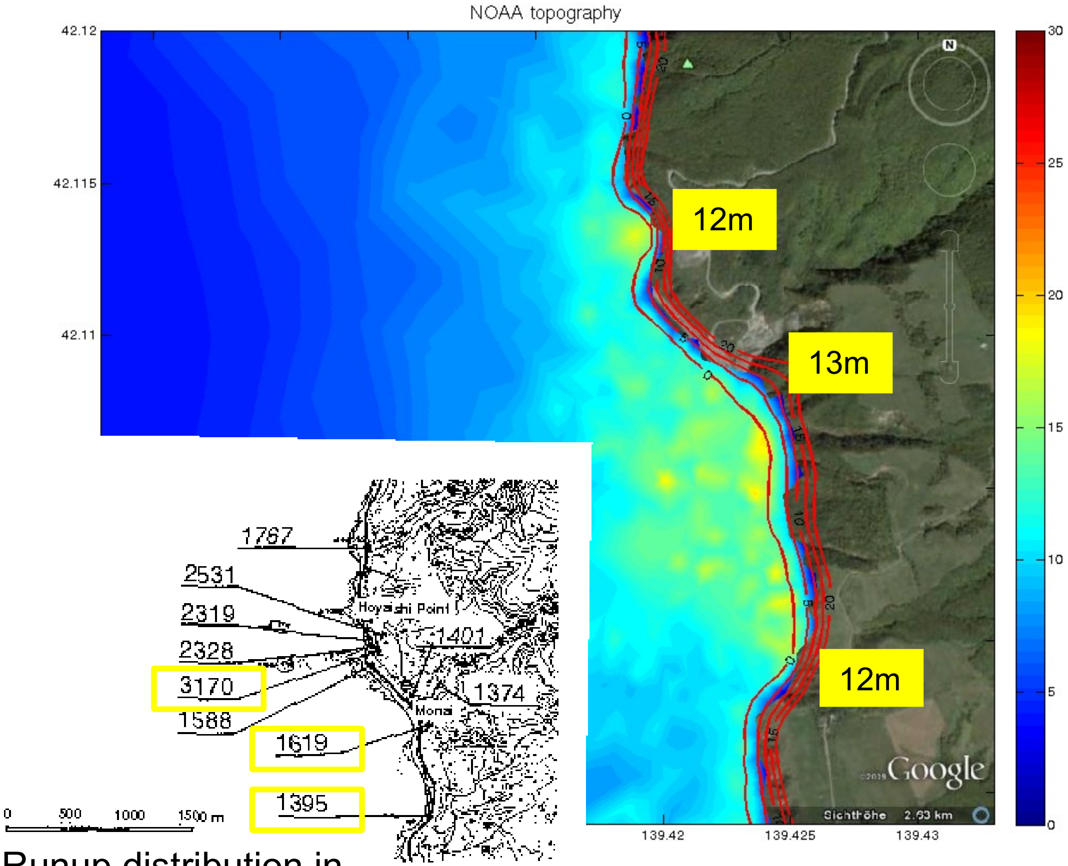


Max. wave height

— isolines of topography (0m,5m,10m,15m,20m)

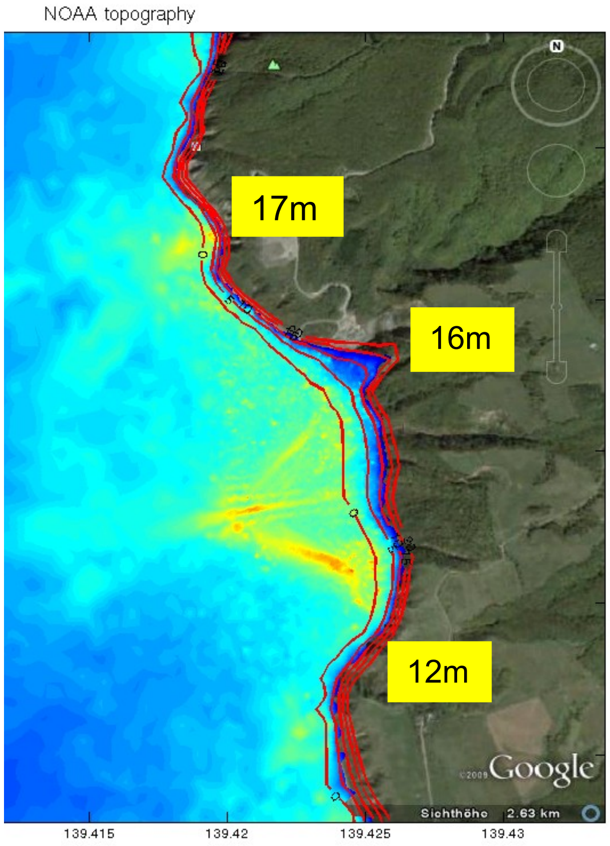
Inundation of the Monai area – depending on mesh resolution

50 m res. at the coast



Runup distribution in the Monai area (in cm)

10 m res. at the coast

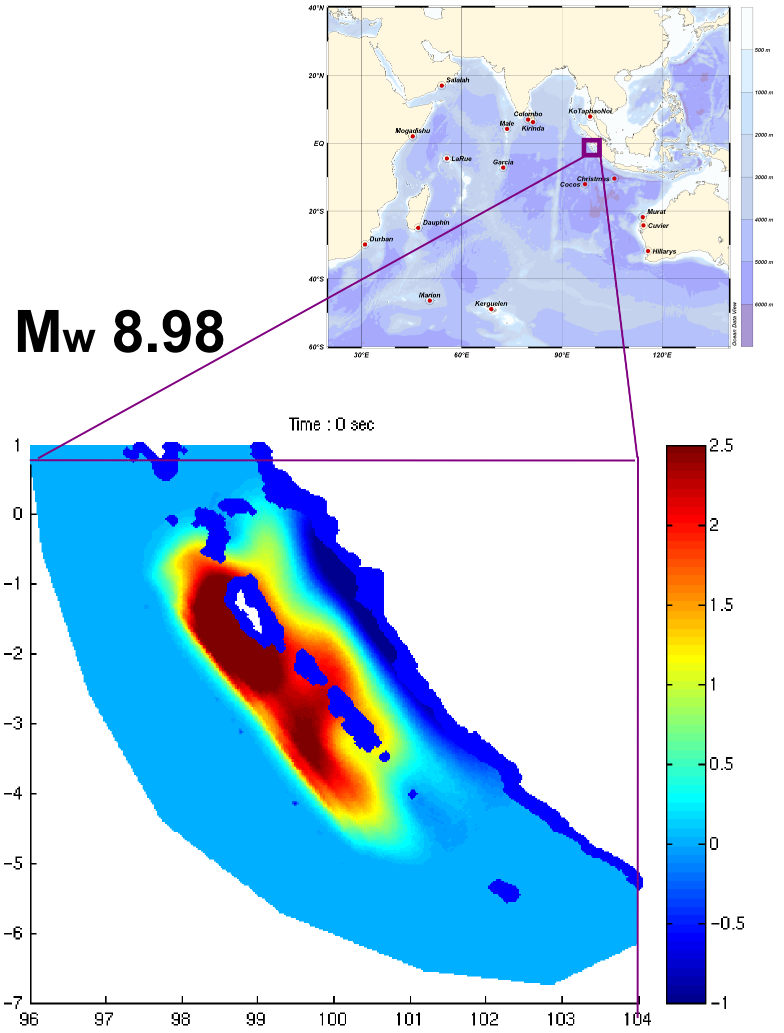


Max. wave height

— isolines of topography (0m,5m,10m,15m,20m)

Worst case tsunami scenario for Padang, Sumatra

Mw 8.98



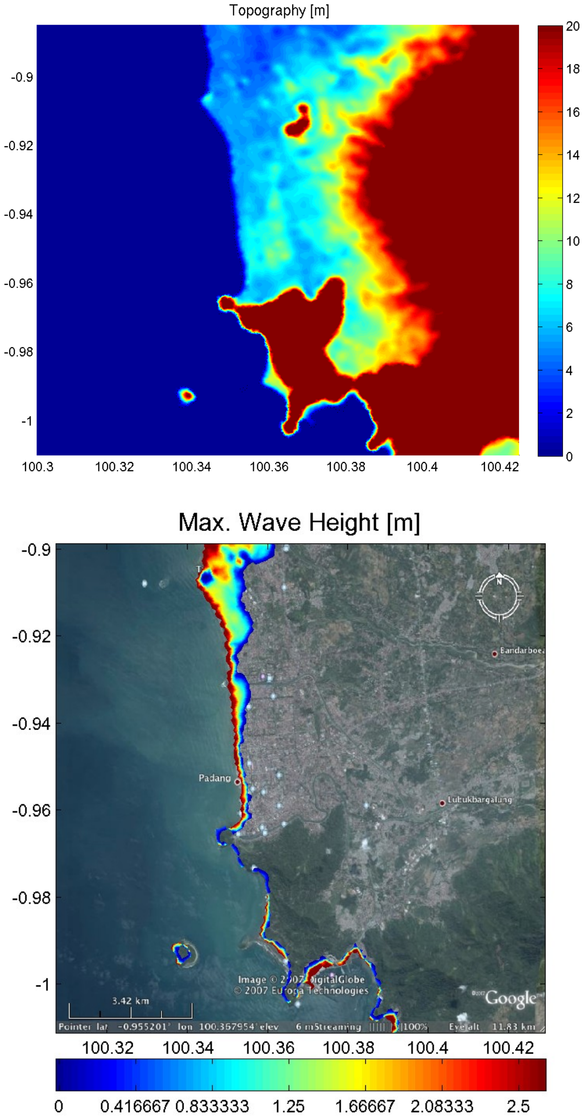
Max. Uplift = 3.73 m
Max. Depression = -1.60 m

Variable resolution of the mesh:
~57 m in Padang region
~7 km in deep sea

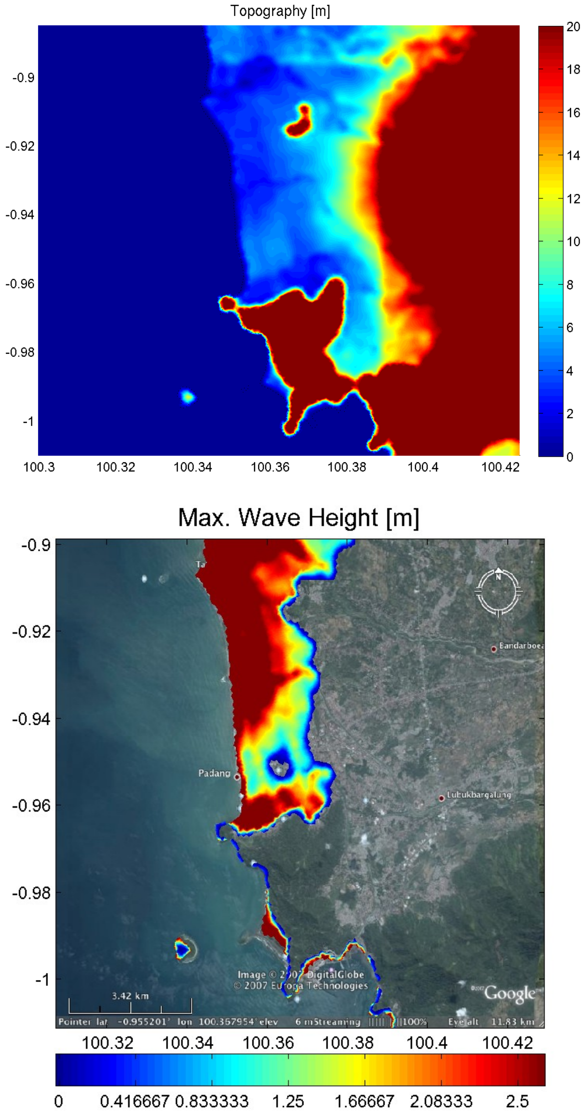
Worst case tsunami scenario for Padang

Topography and inundation results

SRTM
(90 m res.)



HRSC
(50 m res.)



Conclusion

- Advection is important in shallow water
- Grid resolution has effect on mwh and velocity in coastal regions
- To simulate runup successfully, a fine mesh resolution is needed
- Good topography data is crutial for reliable inundation results