Path Planning of Autonomous Underwater Vehicles for Adaptive Sampling

DEAS Group Meeting Presentation

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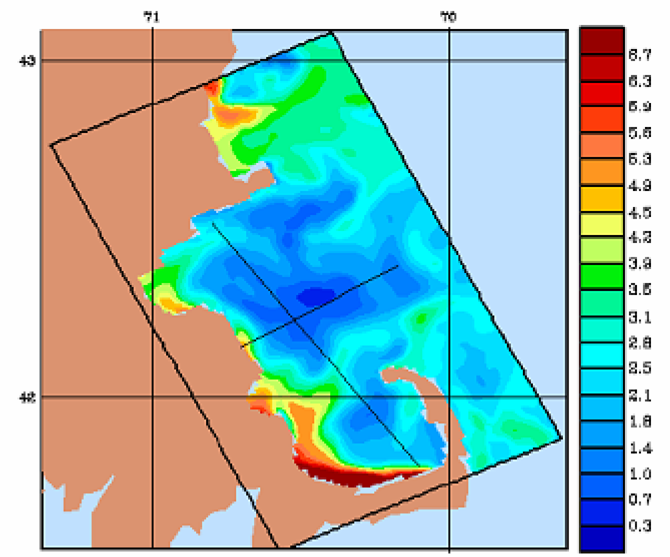
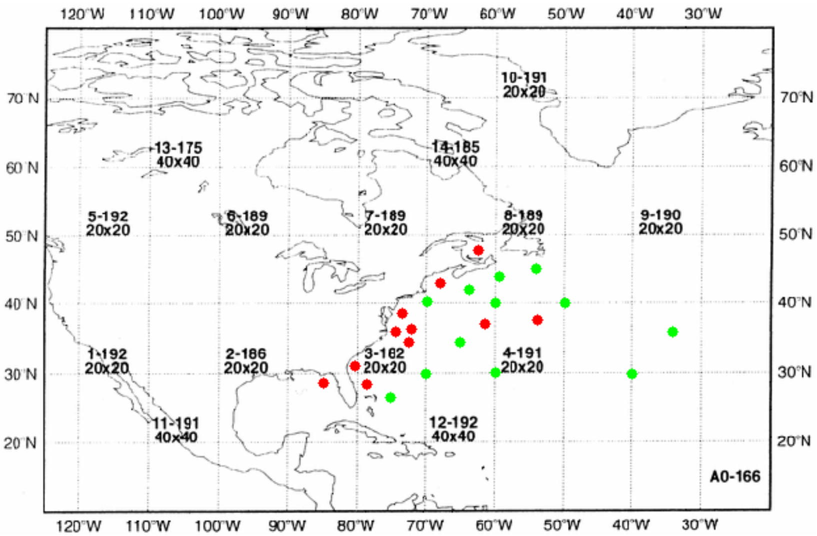
Ocean Forecasting

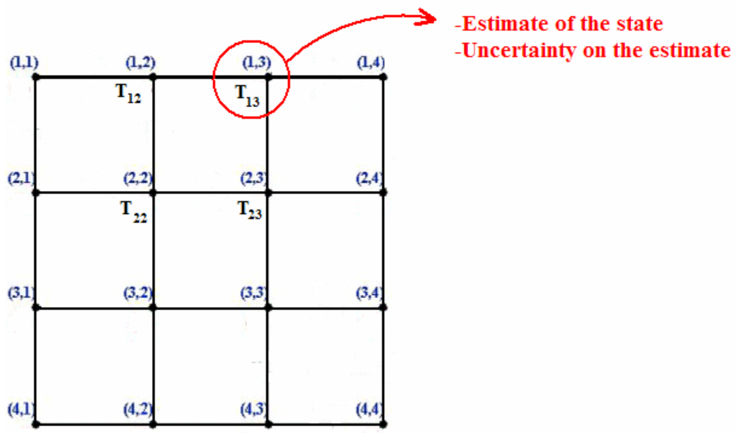
- Ocean forecasting is essential for effective and efficient operations at sea. It is used for:
    - Military operations
    - Coastal zone management
    - Scientific research
- State variables to be forecasted:
    - Temperature
    - Salinity
    - Current velocity
    - Plankton concentration
    - Nutrient concentration
    - Fish concentration
    - Pollution
    - Sound speed



Adaptive Sampling

- There exists a routine component for observations which collects data from a particular region.
- Adaptive sampling is a method which aims to the improvement of the forecast results by deploying some additional assets to gather more accurate data in critical regions.
- The trajectory of this additional component needs to be planned continuously. It needs to adapt to changing conditions, therefore named "adaptive".
- Forecasting systems such as "Error Subspace Statistical Estimation" (ESSE) or "Ensemble Transform Kalman Filter" (ETKF) techniques provide both estimates of the states and the uncertainty on the state estimate.
- Uncertainty fields created by these techniques can be used for the purpose of adaptive sampling.





Adaptive Sampling-Summary Maps

- Summary maps represent the amount of total improvement on a given field as a function of measurement location.
- Path planning is done manually. Paths are either created manually or chosen from a set of pre-designed paths.
- Use of pre-designed paths limit the quality of adaptive path planning.

- The multi-vehicle case is handled by “serial targeting”.
  - Find the best path for first vehicle.
  - *Assimilate the fictitious observations made by the first vehicle using ESSE or ETKF.*
  - *Using the updated summary map, find the path of the second vehicle.*

- The technique does not deal with more complicated scenarios where inter-vehicle interactions and other mission constraints are involved.

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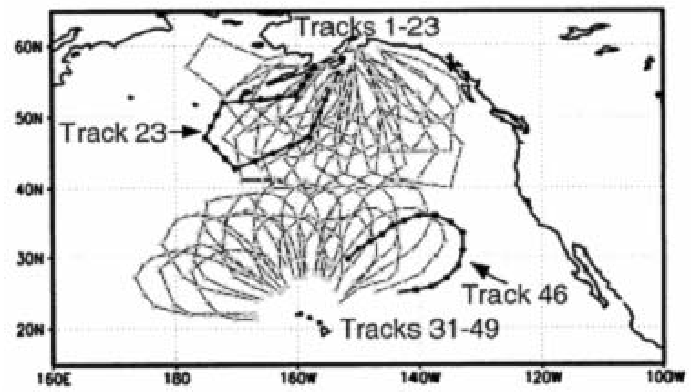
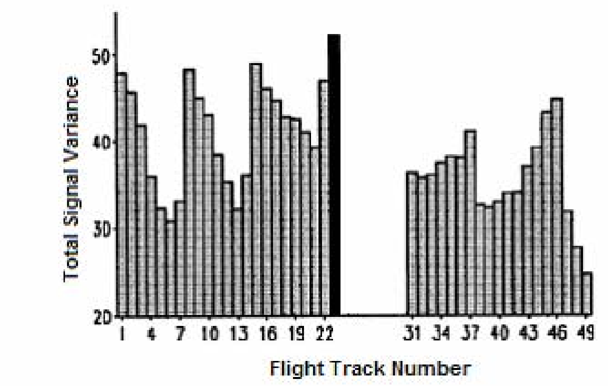




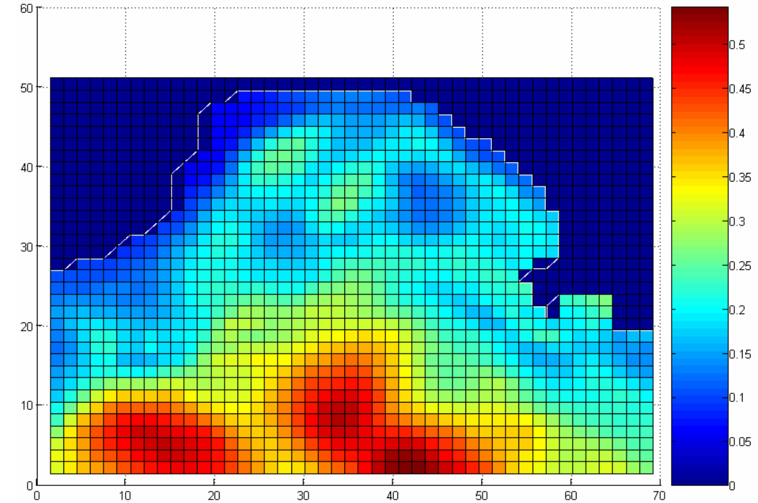
* S. J. Majumdar, C. H. Bishop, and B. J. Etherton. Adaptive sampling with the ensemble transform Kalman filter. Part II: Field program implementation. Monthly Weather Review, 130:1356–1369, 2002.

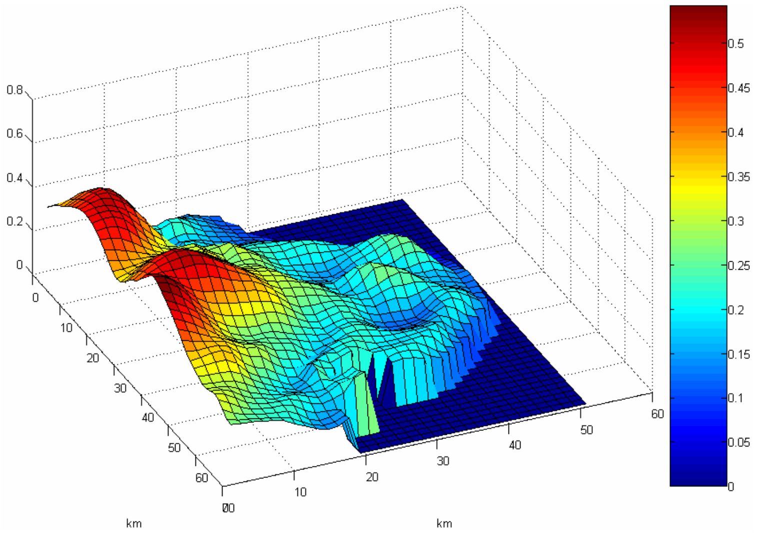
Problem Statement

- Given an uncertainty field, find the paths for the vehicles in the adaptive sampling fleet along which the path integral of uncertainty values will be maximized. Also constraints such as:
  - Vehicle range
  - Desired motion shape
  - Inter-vehicle coordination
  - Collision avoidance
  - Communication needs

must be satisfied.

- Also there might be multiple days in succession involved in the adaptive sampling mission. The optimality must be sought over a time window. This is named the "*time-progressive*" case.
- Global optimality in the spatial and time sense must be satisfied.
  - The fields are neither concave nor convex.
  - Increases the challenge.





A Short Introduction to Optimization Methods

- A generic optimization problem can be written as

$$\min F(x)$$
$$\textit{subject to} \quad g_i(x) = 0 \quad \forall i \in 1,...,m1 \quad m1 > 0$$
$$h_j(x) \geq 0 \quad \forall j \in m1,...,m \quad m > m1$$
$$x \in \Re^n.$$

Types of different optimization problems:

- Non-linear programming problem

- Linear programming problem

$$\min c'x$$
$$\textit{subject to} \quad Ax=b$$
$$x \geq 0$$

- Integer programming problem

$$\min c'x$$
$$\textit{subject to} \quad Ax=b$$
$$x \geq 0$$
$$x \ \textit{integer}$$

- Mixed integer programming (MIP) problem

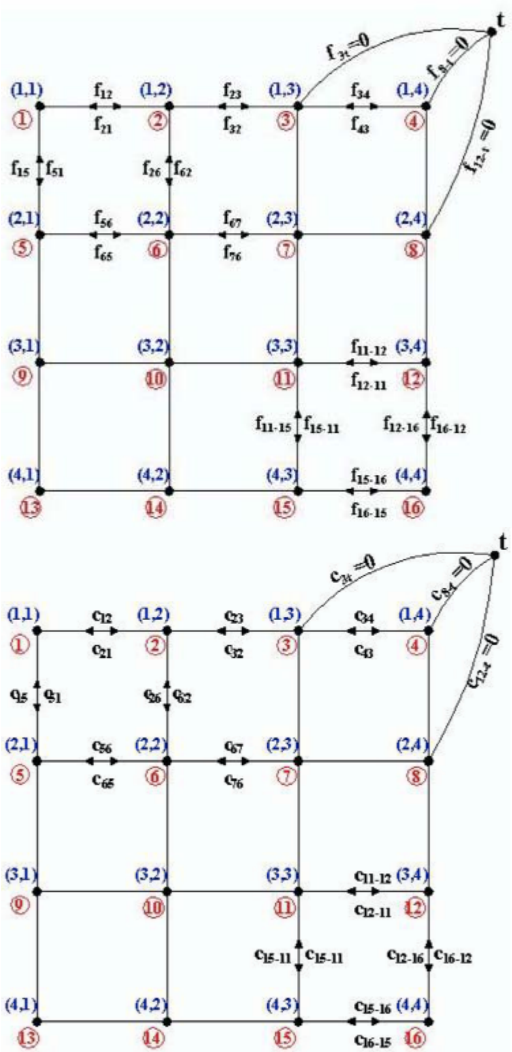
$$\min c'x+d'y$$
$$\textit{subject to} \quad Ax+By=b$$
$$x,y \geq 0$$
$$x \ \textit{integer}$$

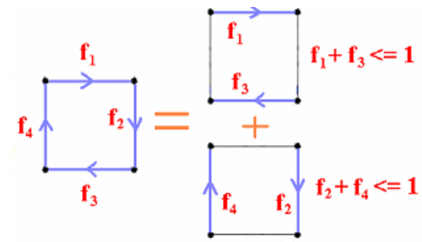
Mixed Integer Programming (MIP) Solution Methods

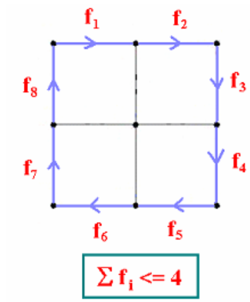
Comparison of some solution methods

Method Name	Globally Optimal	Memory Requirement	Computational Complexity	Ease of Adapting to Modifications
Greedy Method (Local Search)	NO	VERY LOW	LINEAR	MODERATE
Exhaustive Enumeration	YES	VERY HIGH	EXPONENTIAL	HARD
Branch and Bound	YES	MODERATE	EXPONENTIAL	EASY

Network-Based Mixed Integer Programming (MIP) Formulation







$max\ \sum_{p,i,j} c_{ij} \cdot f_{p,ij}$

subject to

$\sum f_{p,ij} - \sum f_{p,ji} = b_{p,i} \quad \forall(i,j) \in N \ \ and \ \ \forall p \in V$, where V is set of all vehicles (or in a equal representation $Af_p = b_p$ where $b_{p,s} = 1 \quad b_{p,t} = -1 \quad b_{p,i} = 0 \quad i \neq s,t$)

$\sum f_{p,ij} \cdot d[i,j] \leq R \quad \forall(i,j) \in N \ \ and \ \ \forall p \in V$ where $d[i,j]$ is a distance modification matrix to take care of diagonal moves if they are allowed to and the curvilinear grid geometry in case it's used.

$f_{p,ij} + f_{p,ji} \leq 1 \quad \forall(i,j) \in N \ \ and \ \ \forall p \in V$ (Opposite flows between two nodes are not allowed)

$\sum_j f_{p,ij} + f_{p,ji} \leq (2 - b_{p,i}) \quad \forall i \in N \ \ and \ \ \forall p \in V$ (To avoid visiting the same node twice)

$\sum f_{p,ij} \leq 1 \quad \forall f_{p,ij} \in S1L_p \ \ and \ \ \forall p \in V$ (To avoid size 1 loops)

$\sum f_{p,ij} \leq 4 \quad \forall f_{p,ij} \in S2L_p \ \ and \ \ \forall p \in V$ (To avoid size 2 loops)

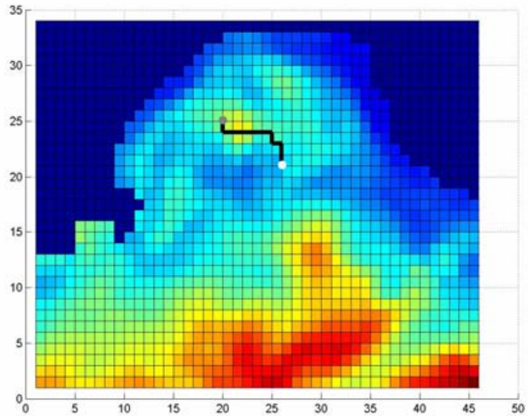
$\sum f_{p,ij} \leq 6 \quad \forall f_{p,ij} \in S3L_p \ \ and \ \ \forall p \in V$ (To avoid size 3 loops)

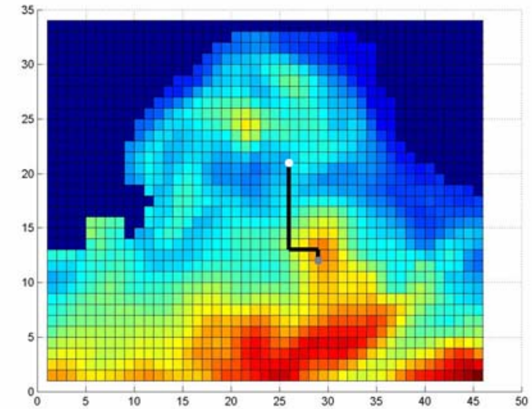
$f_{p,ij} = 0 \quad \forall(i,j) \in (N\text{-}R_p) \ \ and \ \ \forall p \in V$ where R_p stands for the set of nodes within the range of p^{th} vehicle

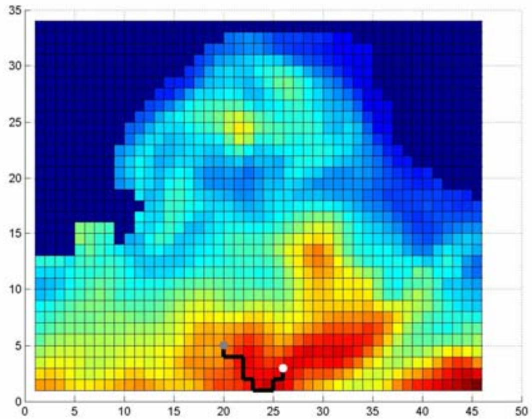
$\sum f_{p,ij} \leq 12(1 - f_{q,kl}) \quad \forall f_{p,ij} \in VCR_p \ \ and \ \ f_{q,ij} \in VCR_q \ \ \forall p,q|p > q \in V$ (Vicinity constraints)

$f_{p,ij} = 0,1 \quad \forall(i,j) \in N \ \ and \ \ \forall p \in V$ (Integrity constraint)

Network-Based MIP Formulation







- Needs processing of huge matrices before each run
- Complicated to include diagonal moves
- Costly to make modifications on the formulation and add new constraints
- Does not handle time-progressive case

New Mixed Integer Programming (MIP) Method

N: Number of path points

P: Total number of vehicles

Variables:

x_{pi} and y_{pi} where i ϵ [1,…,N], and p ϵ [1,…,P].

$1 \leq x_{pi} \leq max_X$

$1 \leq y_{pi} \leq max_Y$

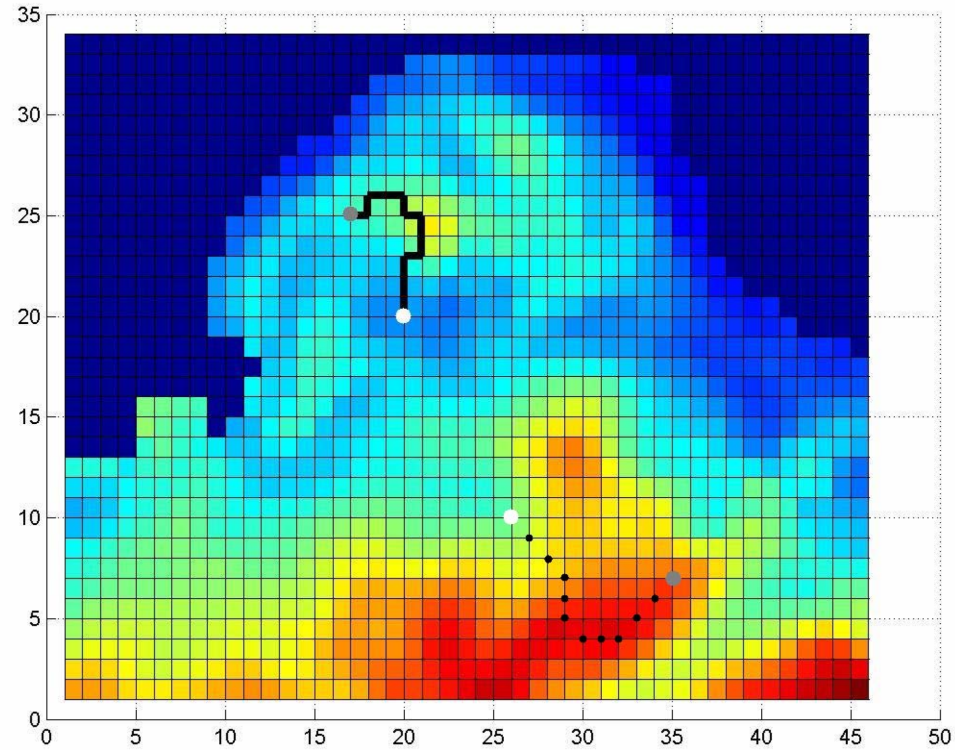
Decision Variables:

b_{pij} ,$t1_{pij}$, $t2_{pij}$, $t3_{pij}$

where i ϵ [1,…,N], and p ϵ [1,…,P],

and j ϵ [1,…,4],

b_{pij} ,$t1_{pij}$,$t2_{pij}$,$t3_{pij}$ ϵ {0,1}



Motion Constraints

N: Number of path points

P: Total number of vehicles

$$\forall p \in [1, ..., P], and \quad \forall i \in [2, ..., N]:$$

$$x_{pi} = x_{p(i-1)} + b_{pi1} - b_{pi2}$$

$$b_{pi1} + b_{pi2} \leq 1$$

$$y_{pi} = y_{p(i-1)} + b_{pi3} - b_{pi4}$$

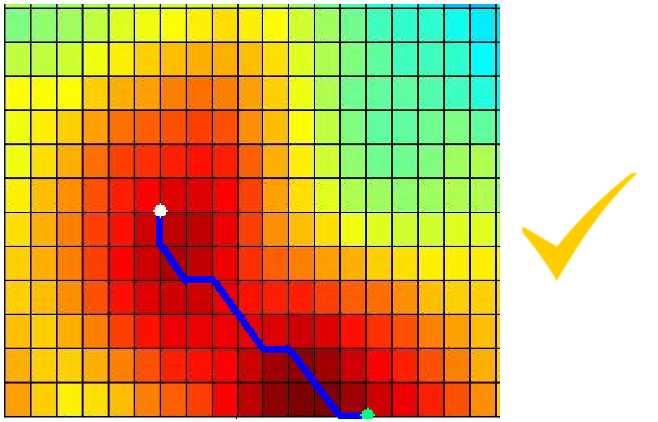
$$b_{pi3} + b_{pi4} \leq 1$$

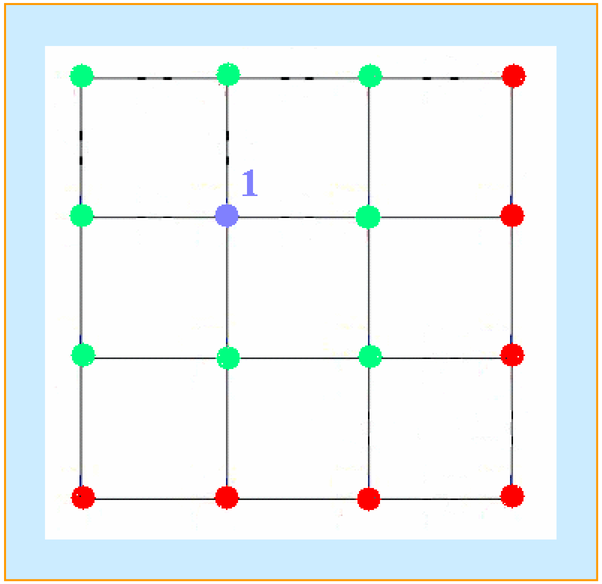
$$\forall p \in [1, ..., P], and \quad \forall i \in [1, ..., N]:$$

$$b_{pi1} + b_{pi2} + b_{pi3} + b_{pi4} \geq 1$$

$$\forall p \in [1, ..., P], \quad \forall i \in [1, ..., N], and \quad \forall j \in [1, ..., 4]:$$

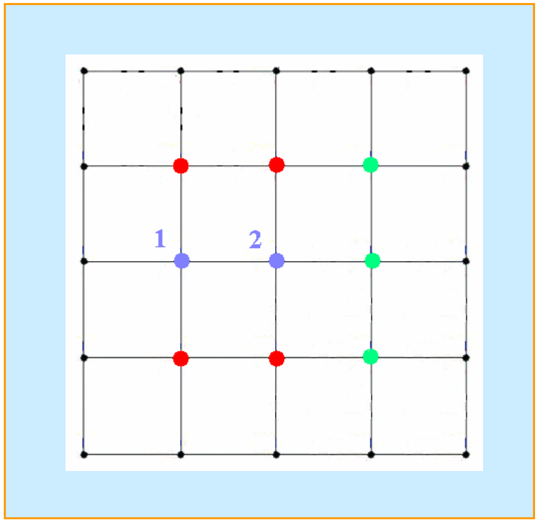
$$b_{pij} \in 0, 1$$



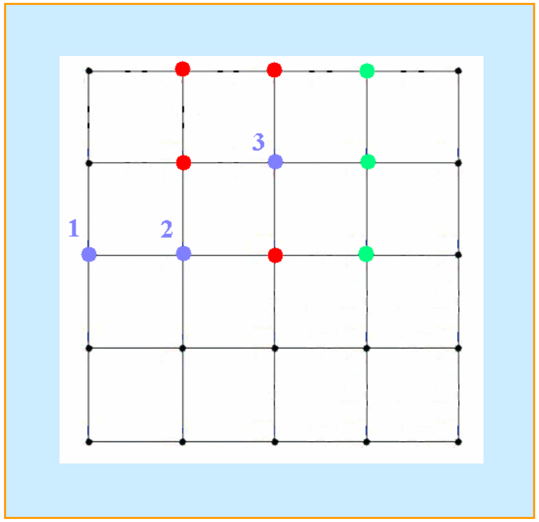


Motion Constraints

$$\forall p \in [1,...,P], and \;\; \forall i \in [3,...,N]:$$
$$|x_{pi} - x_{p(i-2)}| \geq 2 \quad \textbf{OR} \quad |y_{pi} - y_{p(i-2)}| \geq 2$$
$$\Downarrow$$
$$\forall p \in [1,...,P], and \;\; \forall i \in [3,...,N]:$$
$$x_{pi} - x_{p(i-2)} \geq 2 - M * t1_{pi1}$$
$$x_{p(i-2)} - x_{pi} \geq 2 - M * t1_{pi2}$$
$$y_{pi} - y_{p(i-2)} \geq 2 - M * t1_{pi3}$$
$$y_{p(i-2)} - y_{pi} \geq 2 - M * t1_{pi4}$$
$$t1_{pi1} + t1_{pi2} + t1_{pi3} + t1_{pi4} \leq 3$$
$$\forall p \in [1,...,P], \;\; \forall i \in [1,...,N], and \;\; \forall j \in [1,...,4]:$$
$$t1_{pij} \in 0,1$$



$$\forall p \in [1,...,P], and \;\; \forall i \in [4,...,N]:$$
$$|x_{pi} - x_{p(i-3)}| \geq 2.5 \quad \textbf{OR} \quad |y_{pi} - y_{p(i-3)}| \geq 2.5$$
$$\Downarrow$$
$$\forall p \in [1,...,P], and \;\; \forall i \in [4,...,N]:$$
$$x_{pi} - x_{p(i-3)} \geq 2.5 - M * t2_{pi1}$$
$$x_{p(i-3)} - x_{pi} \geq 2.5 - M * t2_{pi2}$$
$$y_{pi} - y_{p(i-3)} \geq 2.5 - M * t2_{pi3}$$
$$y_{p(i-3)} - y_{pi} \geq 2.5 - M * t2_{pi4}$$
$$t2_{pi1} + t2_{pi2} + t2_{pi3} + t2_{pi4} \leq 3$$
$$\forall p \in [1,...,P], \;\; \forall i \in [1,...,N], and \;\; \forall j \in [1,...,4]:$$
$$t2_{pij} \in 0,1$$



Vicinity Constraints for Multi-Vehicle Case

$\forall p \in [1,...,P], and \;\; \forall q \in [1,...,P]: \;\; \forall p,q|p>q \;\; and \;\; \forall i,j \in [1,...,N] \;\; \forall i \in [1,...,N]:$

$|x_{pi} - x_{qj}| \geq 2 \quad \mathbf{OR} \quad |y_{pi} - y_{qj}| \geq 2$

$$\Downarrow$$

$\forall p \in [1,...,P], and \;\; \forall q \in [1,...,P]: \;\; \forall p,q|p>q \;\; and \;\; \forall i,j \in [1,...,N] \;\; \forall i \in [1,...,N]:$

$x_{pi} - x_{qj} \geq 2 - M * v1_{pqi1}$

$x_{qj} - x_{pi} \geq 2 - M * v1_{pqi2}$

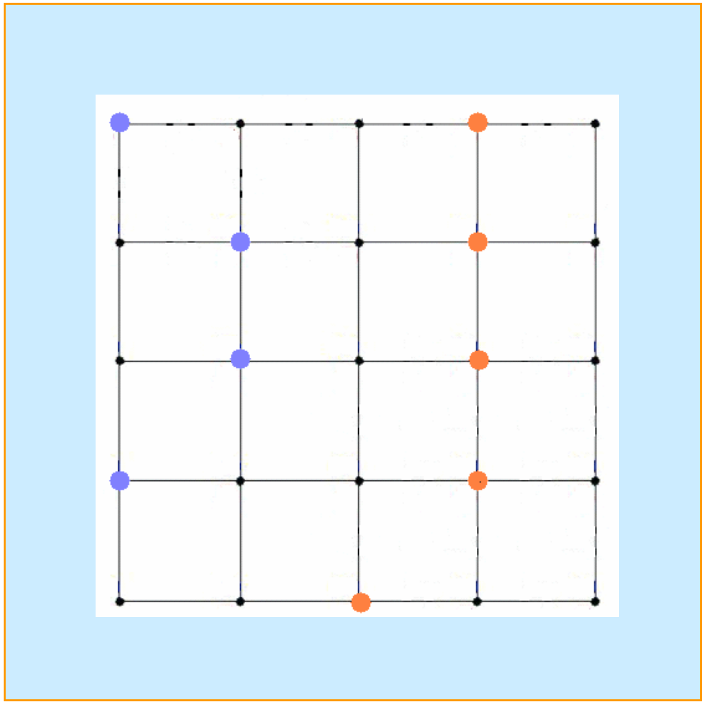
$y_{pi} - y_{qj} \geq 2 - M * v1_{pqi3}$

$y_{qj} - y_{pi} \geq 2 - M * v1_{pqi4}$

$v1_{pqi1} + v1_{pqi2} + v1_{pqi3} + v1_{pqi4} \leq 3$

$\forall p,q \in [1,...,P], \;\; \forall i \in [1,...,N], and \;\; \forall j \in [1,...,4]:$

$v1_{pqij} \in 0,1$



Autonomous Ocean Sampling Network (AOSN) & Communication Constraints

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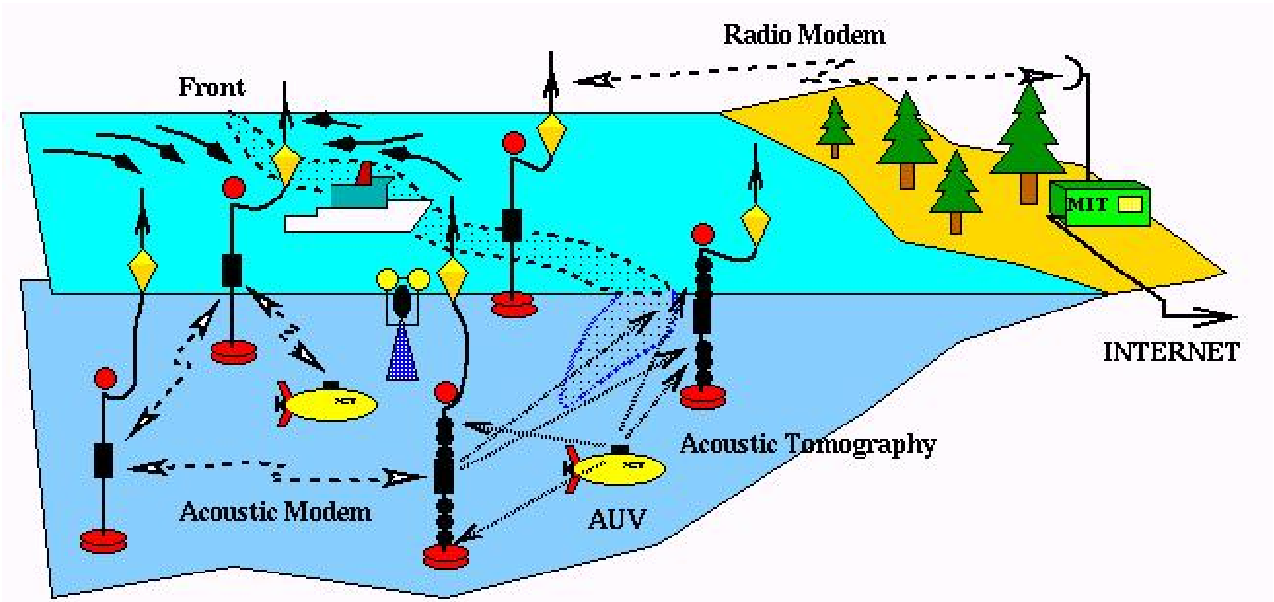


Different Scenarios Based on Communication Needs:

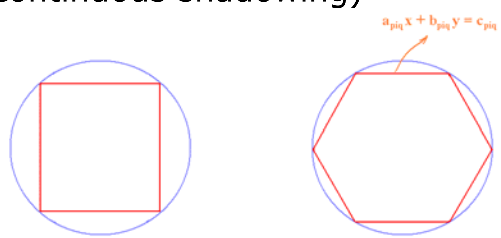
1. Communication with a ship.
2. Communication with a shore station.
3. Communication with buoys.

* Adapted from Tom B. Curtin, James G. Bellingham, J. Catipovic, and D. Webb. "Autonomous Oceanographic Sampling Networks". Oceanography, 6(3), 1993.

Communication with a Ship

1. ***Communication via acoustic link***

- AUV must always lie within a defined vicinity of the ship (continuous shadowing)



$$\forall p \in [1,...,P] \quad \forall i \in [1,...,N_p]:$$

$$|x_{pi} - ship\_x_{pi}| \leq \Delta x_{ship\_vicinity} \quad \textbf{AND} \quad |y_{pi} - ship\_y_{pi}| \leq \Delta y_{ship\_vicinity}$$

⬇

$$x_{pi} - ship\_x_{pi} \leq \Delta x_{ship\_vicinity} + M * s1x_{pi1}$$

$$and \quad ship\_x_{pi} - x_{pi} \leq \Delta x_{ship\_vicinity} + M * s1x_{pi2}$$

$$and \quad y_{pi} - ship\_y_{pi} \leq \Delta y_{ship\_vicinity} + M * s1y_{pi1}$$

$$and \quad ship\_y_{pi} - y_{pi} \leq \Delta y_{ship\_vicinity} + M * s1y_{pi2}$$

$$and \quad \sum_{w=1}^{2} s1x_{piw} \leq 1$$

$$and \quad \sum_{w=1}^{2} s1y_{piw} \leq 1$$

$$s1x_{piw}, s1y_{piw} \in 0,1 \quad \forall w \in [1,2]$$

- Constraints for collision avoidance between the ship and the AUVs

$$\forall p \in [1,...,P] \quad \forall i \in [1,...,N_p]:$$

$$|x_{pi} - ship\_x_{pi}| \geq \Delta x_{ship\_safety} \quad \textbf{OR} \quad |y_{pi} - ship\_y_{pi}| \geq \Delta y_{ship\_safety}$$

⬇

$$and \quad ship\_x_{pi} - x_{pi} \geq \Delta x_{ship\_safety} - M * s2_{pi2}$$

$$and \quad y_{pi} - ship\_y_{pi} \geq \Delta y_{ship\_safety} - M * s2_{pi3}$$

$$and \quad ship\_y_{pi} - y_{pi} \geq \Delta y_{ship\_safety} - M * s2_{pi4}$$

$$and \quad \sum_{w=1}^{4} s2_{piw} \leq 3$$

$$s2_{piw} \in 0,1 \quad \forall w \in [1,...,4]$$

- If the vehicle needs to return to the ship

$$x_{pN_p} = ship\_x_{pN_p} \quad \forall p \in [1,...,P]$$

$$y_{pN_p} = ship\_y_{pN_p} \quad \forall p \in [1,...,P]$$

Communication with a Ship

•If the vehicle needs to lie in a tighter vicinity of the ship at the terminal path point

$$\forall p \in [1, ..., P]:$$

$$x_{pN_p} - ship\_x_{pN_p} \leq \Delta x_{ship\_vicinity\_TP} + M * s1xTP_{pN_p1}$$

$$and \quad ship\_x_{pN_p} - x_{pN_p} \leq \Delta x_{ship\_vicinity\_TP} + M * s1xTP_{pN_p2}$$

$$and \quad y_{pN_p} - ship\_y_{pN_p} \leq \Delta y_{ship\_vicinity\_TP} + M * s1yTP_{pN_p1}$$

$$and \quad ship\_y_{pN_p} - y_{pN_p} \leq \Delta y_{ship\_vicinity\_TP} + M * s1yTP_{pN_p2}$$

$$and \quad \sum_{w=1}^{2} s1xTP_{pN_pw} = 1$$

$$and \quad \sum_{w=1}^{2} s1yTP_{pN_pw} = 1$$

$$s1xTP_{pN_pw}, s1yTP_{pN_pw} \in 0,1 \quad \forall w \in [1, ..., 2]$$

2. *Communication via radio link*

- AUV must come within a vicinity of the ship only at the end of the mission to transfer data.

Only these equations apply !

Communication with a Shore Station and AOSN

- **Communication with a shore station**

$$\forall p \in [1,...,P]:$$

$$x_{pN_p} - shore\_x \leq \Delta x_{shore\_vicinity} + M * s3x_{p1}$$

$$and \quad shore\_x - x_{pN_p} \leq \Delta x_{shore\_vicinity} + M * s3x_{p2}$$

$$and \quad y_{pN_p} - shore\_y \leq \Delta y_{shore\_vicinity} + M * s3y_{p1}$$

$$and \quad shore\_y - y_{pN_p} \leq \Delta y_{shore\_vicinity} + M * s3y_{p2}$$

$$and \quad \sum_{w=1}^{2} s3x_{pw} \leq 1$$

$$and \quad \sum_{w=1}^{2} s3y_{pw} \leq 1$$

$$s3x_{pw}, s3y_{pw} \in 0,1 \quad \forall w \in [1,...,2]$$

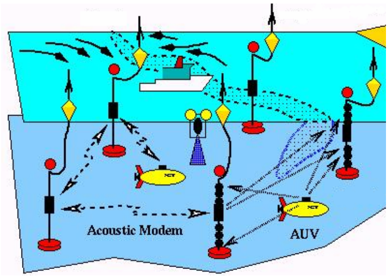
If the AUVs need to return to the shore station:

$$x_{pN_p} = shore\_x \quad \forall p \in [1,...,P]$$

$$y_{pN_p} = shore\_y \quad \forall p \in [1,...,P]$$

- **Communication with an AOSN**

•We have “M” buoys and only one AUV can dock at a given buoy.



$$x_{pN_p} = \sum_{h=1}^{M} buoy\_x_h * bv_{ph} \quad \forall p \in [1,...,P]:$$

$$y_{pN_p} = \sum_{h=1}^{M} buoy\_y_h * bv_{ph} \quad \forall p \in [1,...,P]:$$

$$\sum_{h=1}^{M} bv_{ph} = 1 \quad \forall p \in [1,...,P]$$

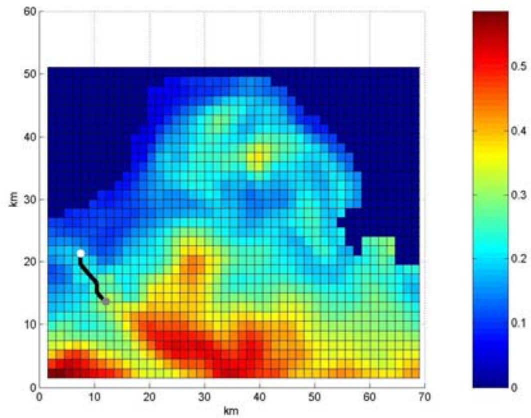
$$bv_{ph} \in 0,1 \quad \forall h \in [1,...,M]$$

$$\sum_{p=1}^{N} bv_{ph} \leq 1 \quad \forall h \in [1,...,M]$$

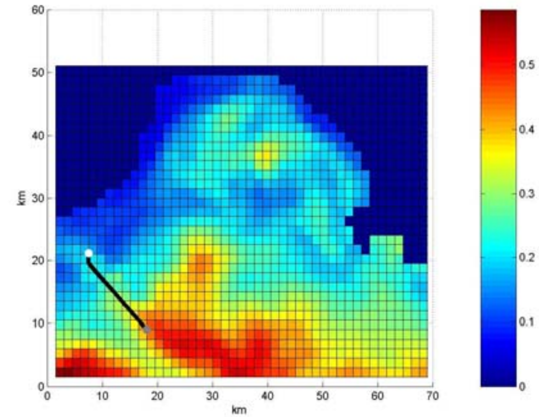
At most one AUV can dock at a given buoy

Results for a Single Vehicle

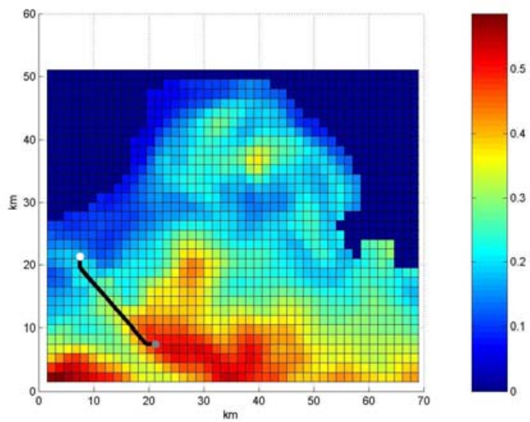
Range:10 km



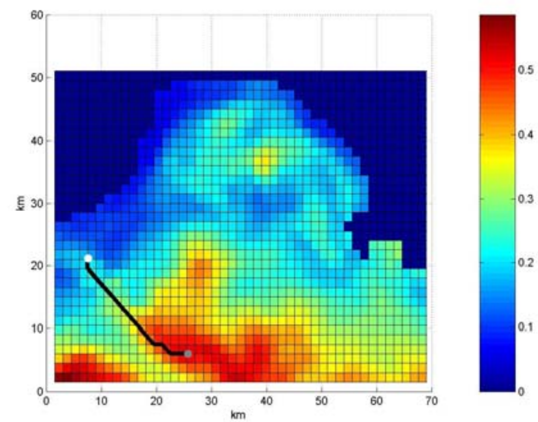
Range:15 km



Range:20 km



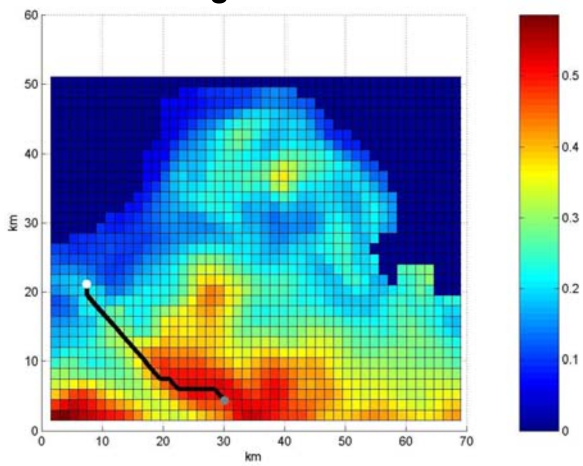
Range:25 km



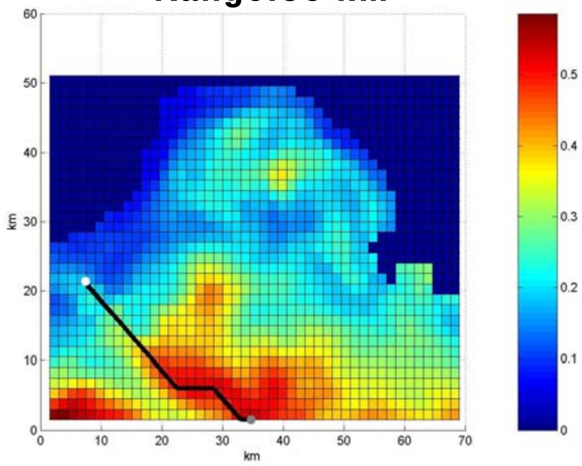
Starting Coordinates: x=7.5km; y=21km

Results for a Single Vehicle

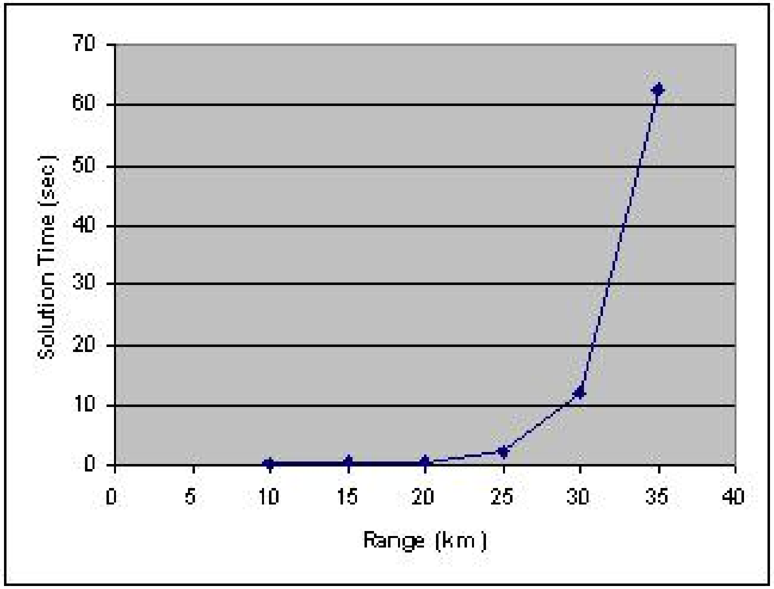
Range:30 km



Range:35 km

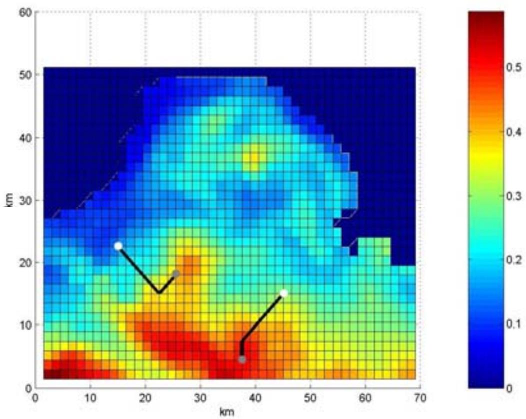


Solution time as a function of range on a Pentium 4- 2.8Ghz computer with 1GB RAM.

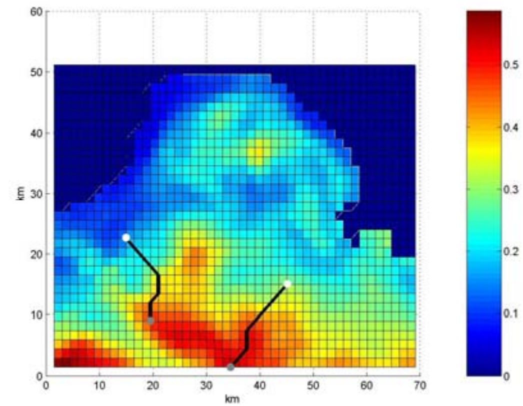


- Exponential growth is observed as expected
- The solution times are acceptable

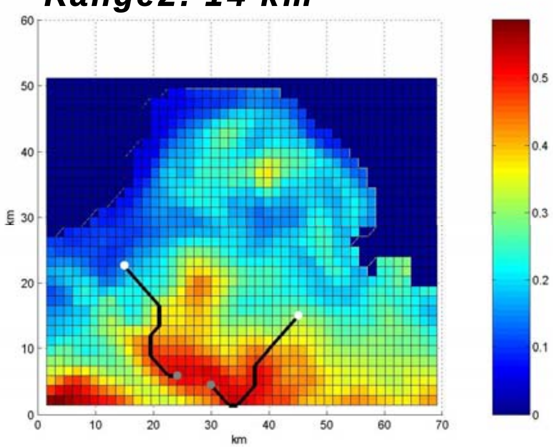
Results for Two Vehicles



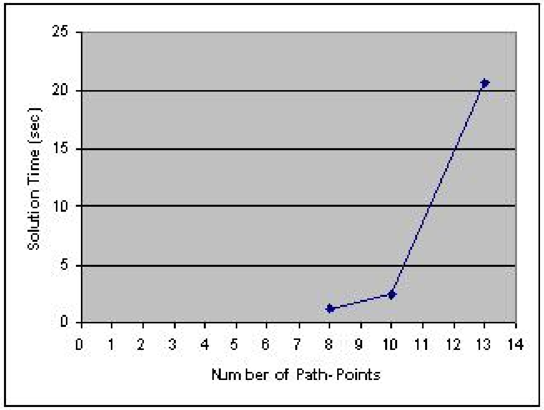
Number of path points: 8
Range1: 15 km
Range2: 14 km



Number of path points: 10
Range1: 16.5 km
Range2: 18 km



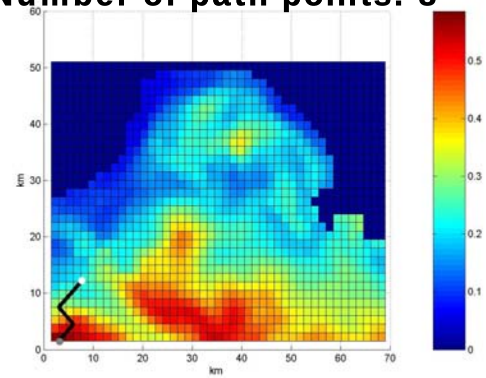
Number of path points: 13
Range1: 22.5 km
Range2: 24 km



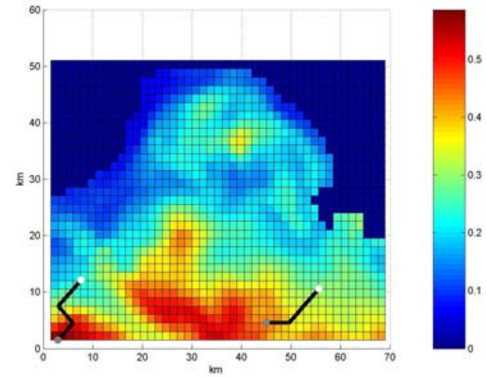
Solution time as a function of path-points on a Pentium 4- 2.8Ghz computer with 1GB RAM

Results for Vehicle Number Sensitivity

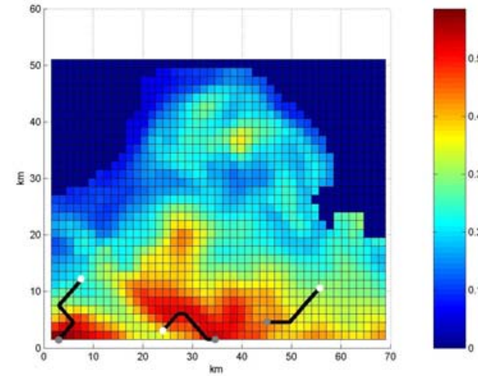
Number of path points: 8



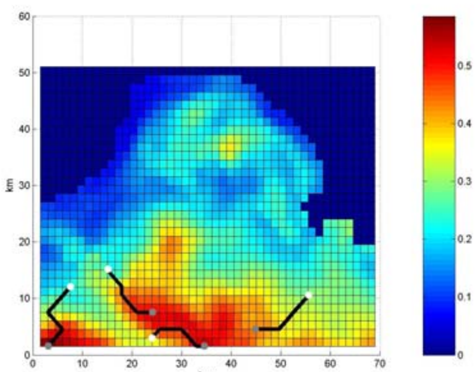
1 vehicle



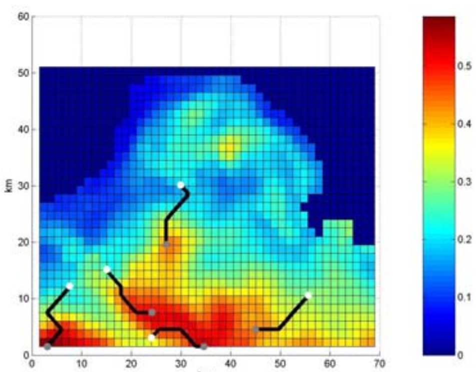
2 vehicles



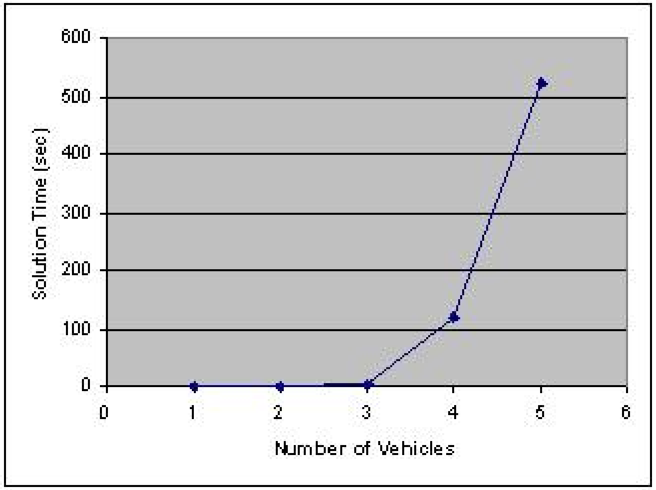
3 vehicles



4 vehicles



5 vehicles



Solution time as a function of vehicle number on a Pentium 4- 2.8Ghz computer with 1GB RAM

Time Progressive Path Planning

- When path planning needs to be performed over multiple days, the formulation must be extended to combine information from all days under consideration.

D: Total Number of Days

Single Day Formulation

$$maximize \quad \sum_{p=1}^{P}\sum_{k=1}^{N_p} f_{pk}$$

$$\forall p \in [1,...,P] \quad and \quad \forall i \in [3,...,N_p]:$$

$$x_{pi} - x_{p(i-2)} \geq \Delta_1 - M * t1_{pi1}$$

$$and \quad x_{p(i-2)} - x_{pi} \geq \Delta_1 - M * t1_{pi2}$$

$$and \quad y_{pi} - y_{p(i-2)} \geq \Delta_1 - M * t1_{pi3}$$

$$and \quad y_{p(i-2)} - y_{pi} \geq \Delta_1 - M * t1_{pi4}$$

$$and \quad \sum_{w=1}^{4} t1_{piw} \leq 3$$

Time Progressive Formulation

$$maximize \quad \sum_{p=1}^{P}\sum_{d=1}^{D}\sum_{k=1}^{N_p} f_{pdk}$$

$$\forall p \in [1,...,P], \quad \forall d \in [1,...,D] \quad and \quad \forall i \in [3,...,N_p]:$$

$$x_{pdi} - x_{pd(i-2)} \geq \Delta_1 - M * t1_{pdi1}$$

$$and \quad x_{pd(i-2)} - x_{pdi} \geq \Delta_1 - M * t1_{pdi2}$$

$$and \quad y_{pdi} - y_{pd(i-2)} \geq \Delta_1 - M * t1_{pdi3}$$

$$and \quad y_{pd(i-2)} - y_{pdi} \geq \Delta_1 - M * t1_{pdi4}$$

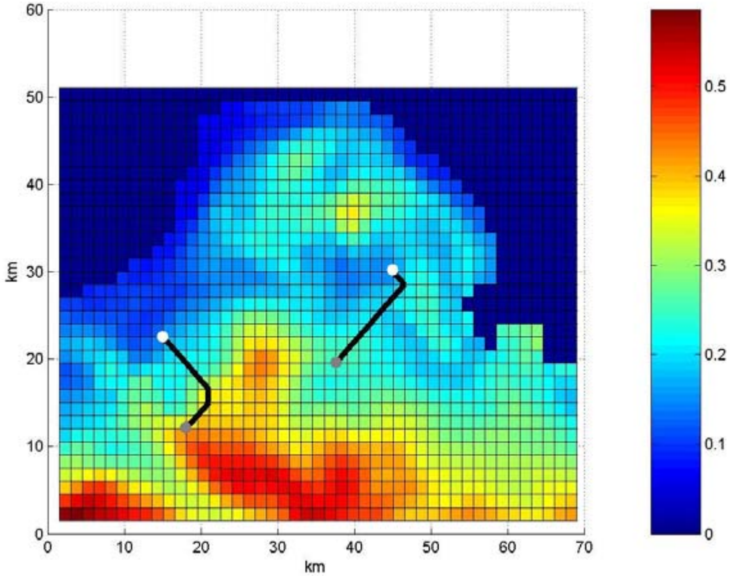
$$and \quad \sum_{w=1}^{4} t1_{pdiw} \leq 3$$

$$\forall p \in [1,...,P], \quad \forall d \in [2,...,D]:$$

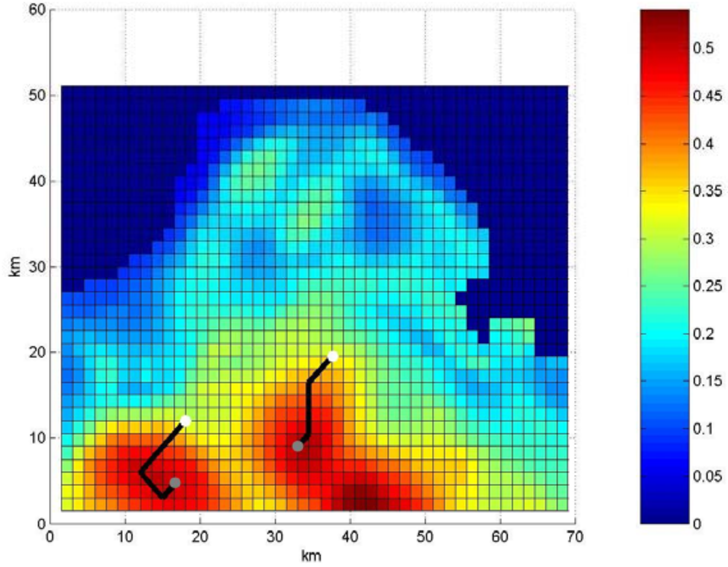
$$x_{pd1} = x_{p(d-1)N_p}$$

$$y_{pd1} = y_{p(d-1)N_p}$$

Illustrative Results for Time-Progressive Path Planning



Paths Generated for Day 1



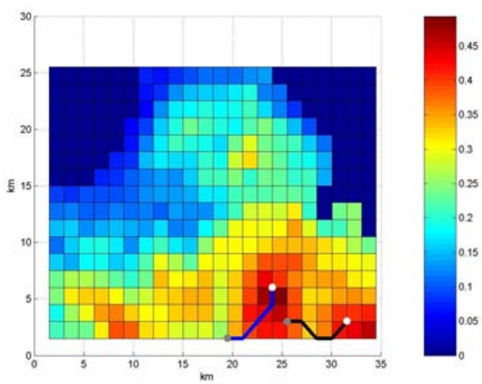
Paths Generated for Day 2

Number of Path Points= 8
Solution Time: 179 sec

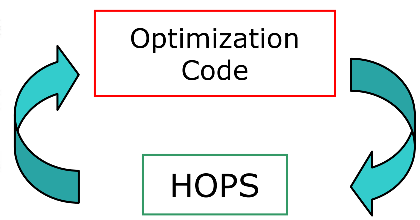
An Example Solved by the Dynamic Method

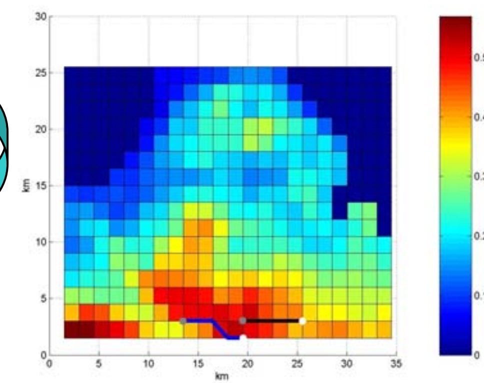
- It is a 3 day long mission: August 26-28, 2003.
- 2 consecutive 2 day time-progressive and 1 single day problems are solved.
- Measurements are assimilated using Harvard Ocean Prediction System (HOPS).

Phase 1
August 26&27

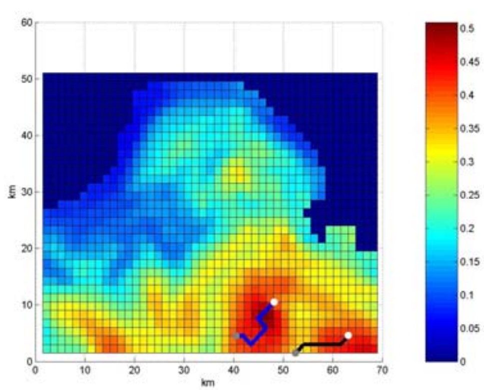


Solution on Coarse (Half-Size) Grid for Day 1 (August 26)

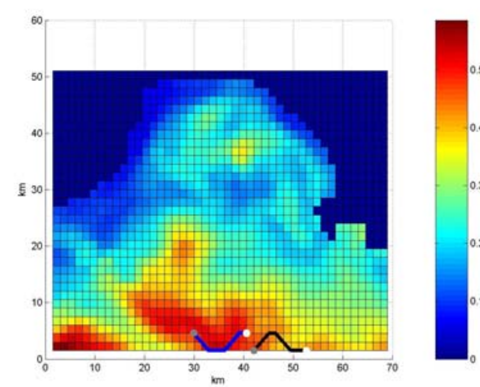




Solution on Coarse (Half-Size) Grid for Day 2 (August 27)



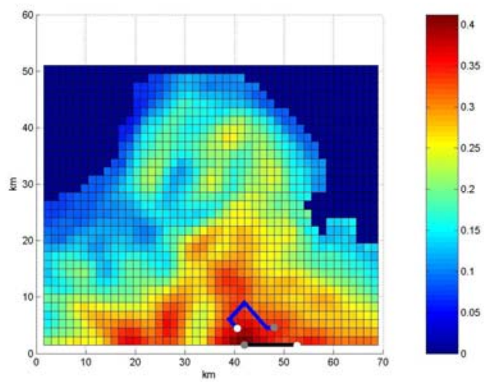
Solution on Full Size Grid for Day 1 (August 26)



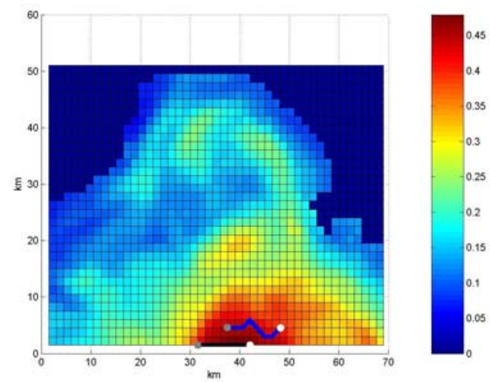
Solution on Full Size Grid for Day 2 (August 27)

An Example Solved by the Dynamic Method

Phase 2
August 27&28

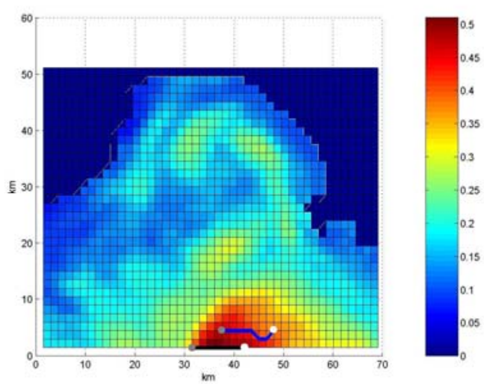


Solution on Full Size Grid for Day 1 (August 27)



Solution on Full Size Grid for Day 2 (August 28)

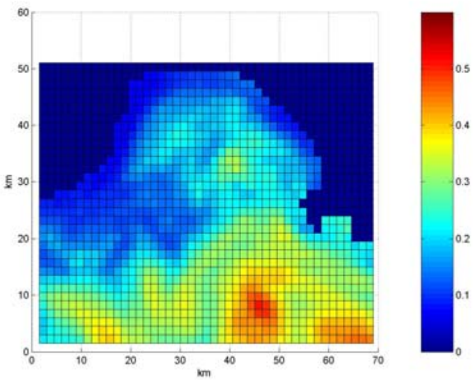
Phase 3
August 28



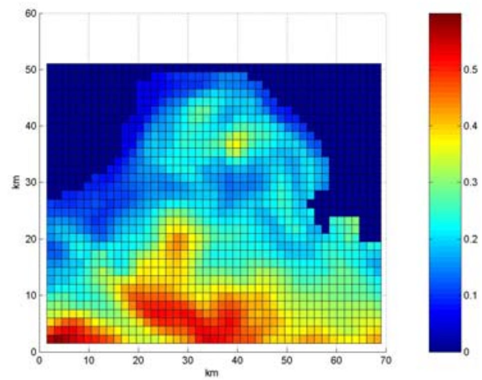
Solution on Full Size Grid for Day 1 (August 28)

Comparison of Results of the Dynamic Method with Results *without* Adaptive Sampling

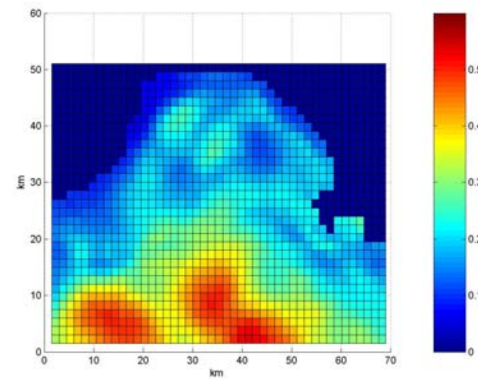
Results Without Adaptive Sampling:



August 26

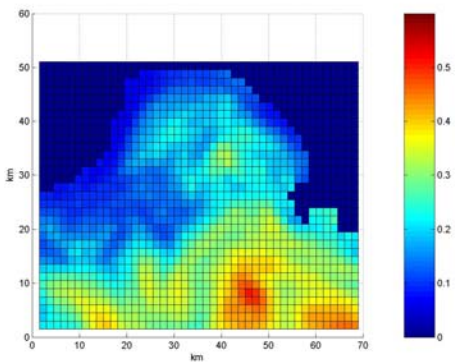


August 27

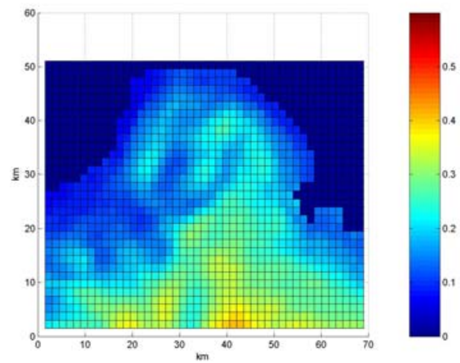


August 28

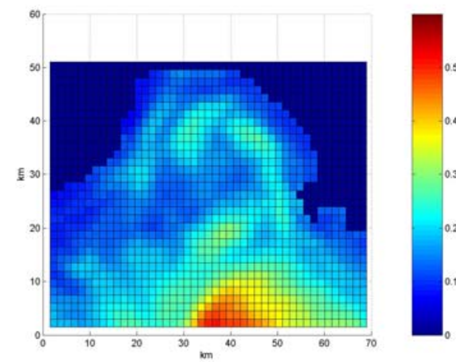
Dynamic Case Results:



August 26



August 27



August 28

Recommendations for Future Work

- Devise methods to automatically determine the optimal values of some of the problem parameters based on a given uncertainty field, e.g. inter-vehicle distance, starting point seperation etc..

- Approximate the uncertainty prediction/assimilation by a linear operator and embed this operator into the optimization code. It will help:
  - To establish a link between the choice of measurement locations and its effect on the following day's uncertainty field.
  - To improve the quality of path planning as a result.

Current Work:

- Working on two journal and one conference papers.
- Investigating the feasibility of implementing a (simplified) version of the MILP formulation on MATLAB.