Harvard Feb. 17 Contribution
Adaptive Sampling and Prediction (ASAP)

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Multiple Facets of Adaptive Sampling

Foci	- Optimal ocean science (Physics, Acoustics and/or Biology) - Demonstration of adaptive sampling value, etc.
Objective Fields	i. Maintain synoptic accuracy (e.g. upwelling, BL or CUC/CCS coverage) ii. Minimize uncertainties (e.g. uncertain ocean estimates), or iii. Maximize the sampling of expected events (e.g. start of upwelling/ relaxation, dynamics of upwelling filament, small scales/model errors) Multidisciplinary or not Local, regional or global, etc.
Time and Space Scales	i. Tactical scales (e.g. minutes-to-hours adaptation by each glider) ii. Strategic scales (e.g. hours-to-days adaptation for glider group/cluster) iii. Experiment scales
Assumptions	- Fixed or variable environment (w.r.t. asset speeds) - Objective field depends on the predicted data values or not, etc. - Operational, time and cost constraints, or not, etc.
Methods	Bayesian-based, Nonlinear programming, (Mixed)-integer programming, Simulated Annealing, Genetic algorithms, Neural networks, Fuzzy logics

For each of the 5 categories, there are multiple choices (only a few listed here)
Choices set the type of adaptive sampling research

1. Adaptive sampling via ESSE

- Objective: Minimize predicted trace of full error covariance (T,S,U,V error std Dev).
- Scales: Strategic/Experiment (not tactical yet). Day to week.
- Assumptions: Small number of pre-selected tracks/regions (based on quick look on error forecast and constrained by operation)
- Problem solved: e.g. Compute today, the tracks/regions to sample tomorrow, that will most reduce uncertainties the day after tomorrow.

- Objective field changes during computation and is affected by data to-be-collected

- Model errors ***Q*** can account for coverage term

Dynamics:	$dx = M(x)dt + d\eta$	$\eta \sim N(0, Q)$
Measurement:	$y = H(x) + \varepsilon$	$\varepsilon \sim N(0, R)$

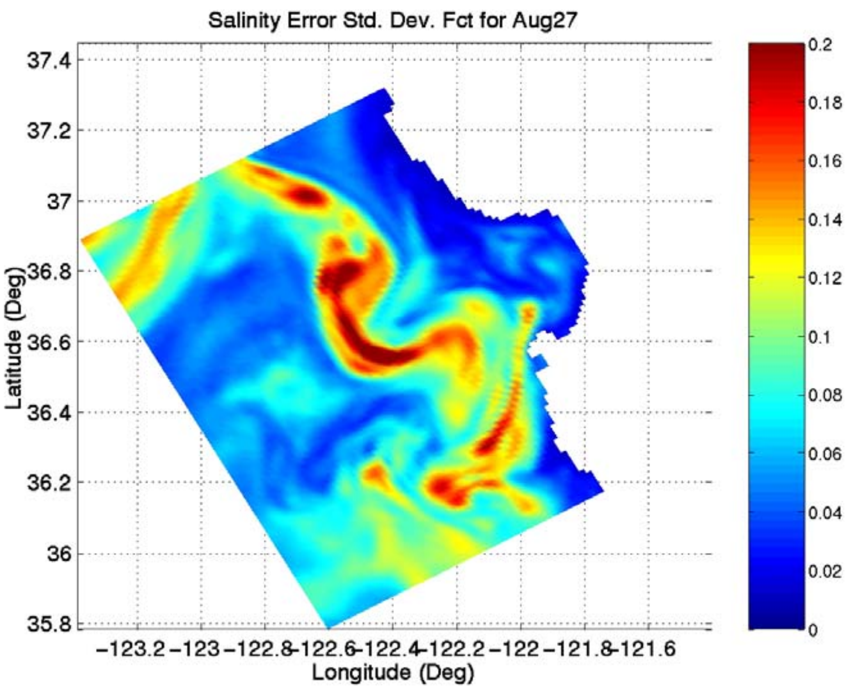
Non-lin. Err. Cov.:

$$dP/dt = \langle (x-\hat{x})(M(x)-M(\hat{x}))^T \rangle + \langle (M(x)-M(\hat{x})(x-\hat{x})^T \rangle + Q$$

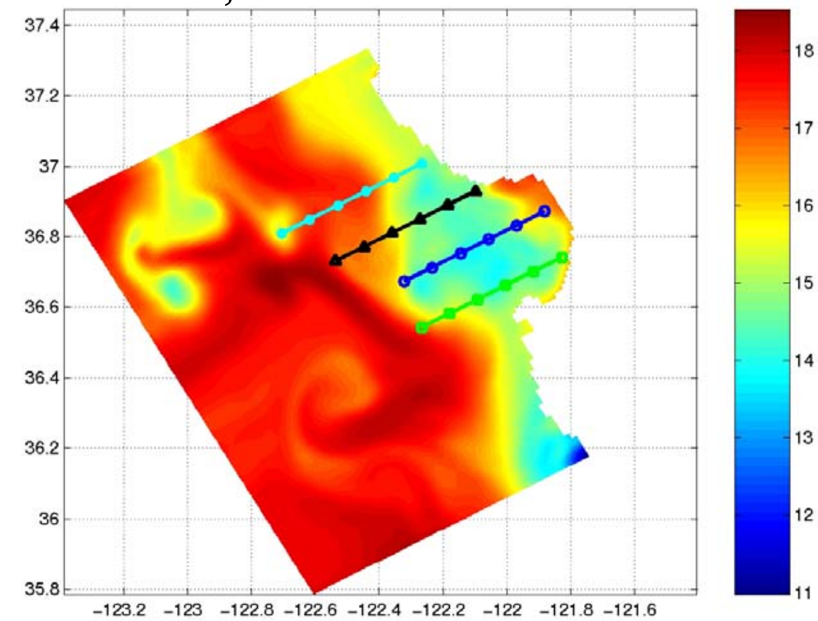
Metric or Cost function: e.g. Find future H_i and R_i such that

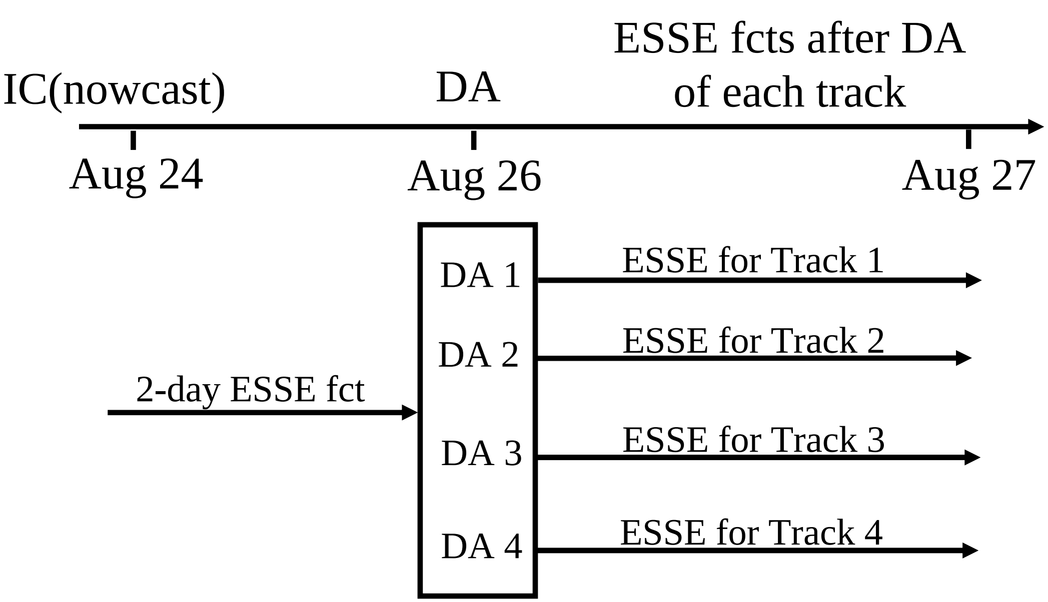
$$\underset{H_i,R_i}{Min} \; tr(P(t_f)) \quad or \quad \underset{H_i,R_i}{Min} \int_{t_0}^{t_f} tr(P(t)) \, dt$$

Which sampling on Aug 26 optimally reduces uncertainties on Aug 27?

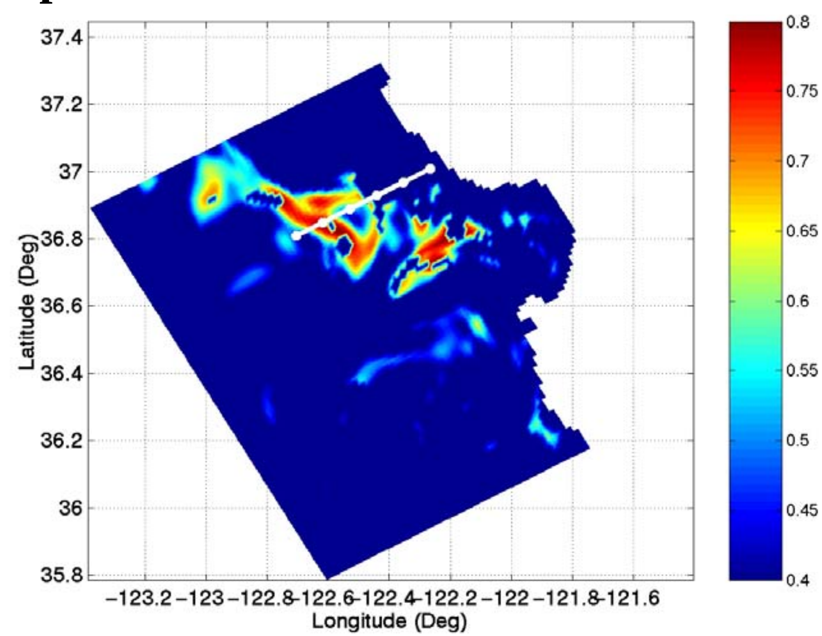


4 candidate tracks, overlaid on surface T fct for Aug 26





Best predicted relative error reduction: track 1

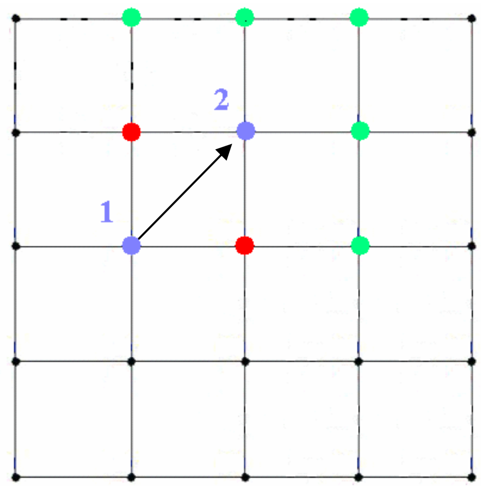


2. Optimal Paths Generation for a “fixed” objective field

- Objective: Minimize error standard deviation of temperature field
- Scales: Strategic/Tactical
- Assumptions
  - Speed of platforms >> time-rate of change of environment
  - Objective field fixed during the computation of the path and is not affected by new data
  - Problem solved: assuming the error is like that now and will remain so for the next few hours, where do I send my gliders/AUVs?
- Methods (global optimization) vary with type of cost function/problem size:
  - *Combinatorial problems*:
    - Objective function is linear or nonlinear, defined over large but finite set of possible solutions (networking, scheduling problems, etc).
    - If cost function piecewise linear, solved *exactly* by Mixed-Integer Programming (MIP)
  - *General unconstrained problems:*
    - Nonlinear function over real numbers with no/simple bounds
    - Partitioning strategies for exact solution, brute force for approx. (simul. annealing, etc)
  - *General constrained problems:*
    - Nonlinear function over real numbers with complex bounds/constraints

Generation of Paths that minimize ESSE uncertainties using MIP (Namik K. Yilmaz, P. Lermusiaux and N. Patrikalakis)

- MIP method is often used to solve modified ``traveling salesman'' problems. Here, towns to be visited are hot-spots in discretized fields and salesmen are the gliders
- Represent ESSE error stand. dev. field as a piecewise-linear cost function
- Possible paths defined on discrete grid: set of possible path is thus finite (but large)
- Constraints on displacements dx, dy, dz:
    - No-Return constraints for single vehicle   e.g. ⇒
    - No-Vicinity constraints for multiple vehicles
    - Both can be set by dominant ocean length-scale



- Optimization carried-out by commercial optimization tool Xpress-MP from dash optimization

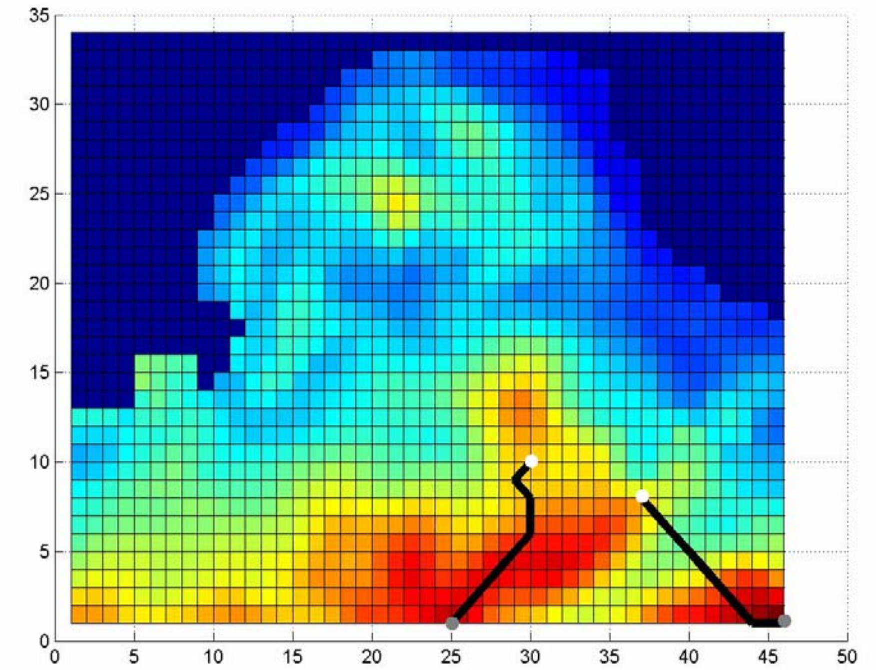
Example for Two and Three Vehicles, 2D objective field

Two Vehicles

Starting Coordinates:
Vehicle#1:x=37;y=8 Range1: 19 km
Vehicle#2:x=20;y=10 Range2: 19 km

Total reward: 1185

Vicinity constraint such that two vehicles are away from each other by at least 7 units (11 km).



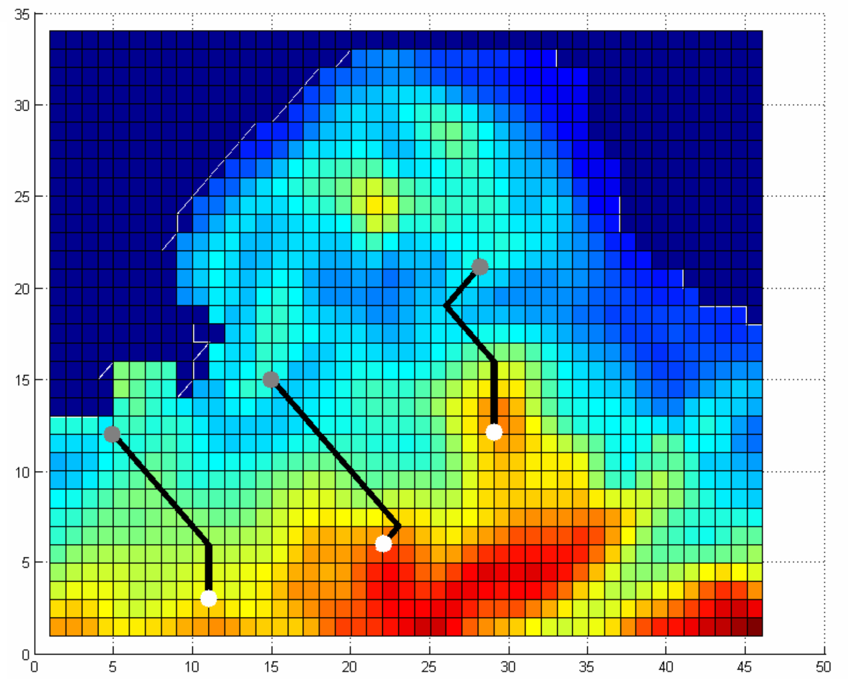
Three Vehicles

Starting Coordinates:

Vehicle #1 : x=5, y=12 Range=17 km

Vehicle #2 : x=15, y=15 Range=19 km

Vehicle #3 : x=28, y=21 Range=17 km



Legend
Grey dots: starting points
White dots: MIP optimal termination points

Example for Two Vehicles and 3D objective field

$$\forall p \in [1, ..., P], and \;\; \forall i \in [2, ..., N]:$$

$$x_{pi} = x_{p(i-1)} + b_{pi1} - b_{pi2}$$

$$b_{pi1} + b_{pi2} \le 1$$

$$y_{pi} = y_{p(i-1)} + b_{pi3} - b_{pi4}$$

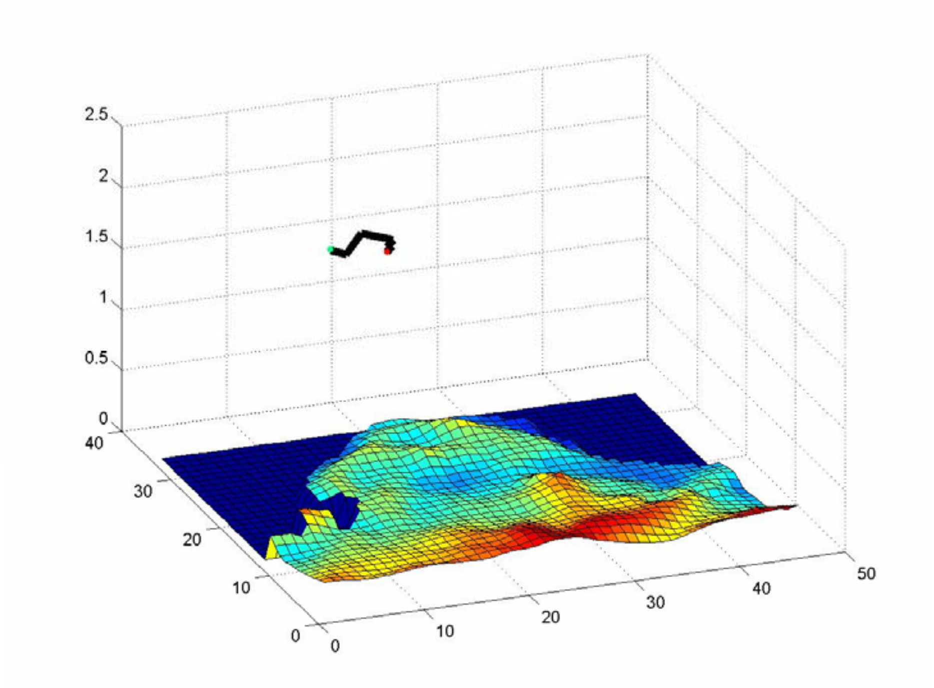
$$b_{pi3} + b_{pi4} \le 1$$

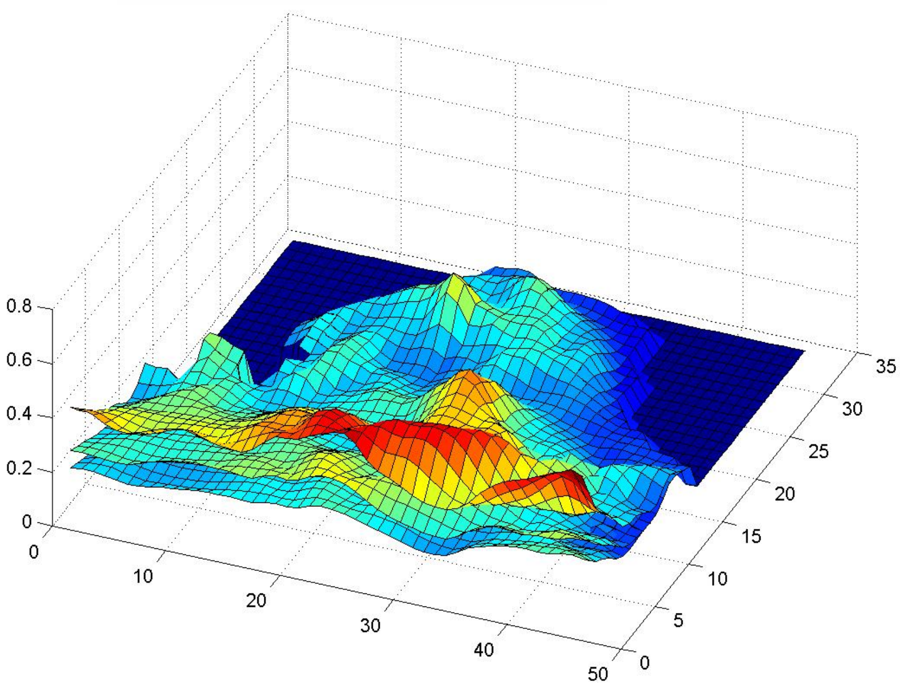
$$z_{pi} = z_{p(i-1)} + b_{pi5} - b_{pi6}$$

$$b_{pi5} + b_{pi6} \le 1$$

$$\forall p \in [1, ..., P], and \;\; \forall i \in [1, ..., N]:$$

$$b_{pi1} + b_{pi2} + b_{pi3} + b_{pi4} \ge 1$$





Starting Coordinates: x=12;y=21

Range: 10 km

Complete Formulation for 3D Case

N: Number of path points

P: Total number of vehicles

R: Reward matrix designating the 2D data field

$max \sum_{pt} f_{pt} = \sum_{pt} R[x_{pt}, y_{pt}, z_{pt}]$

subject to

$\forall p \in [1, ..., P], and \ \ \forall t \in [2, ..., N]:$

$x_{pt} = x_{p(t-1)} + b_{pt1} - b_{pt2}$

$b_{pt1} + b_{pt2} \leq 1$

$y_{pt} = y_{p(t-1)} + b_{pt3} - b_{pt4}$

$b_{pt3} + b_{pt4} \leq 1$

$z_{pt} = z_{p(t-1)} + b_{pt5} - b_{pt6}$

$b_{pt5} + b_{pt6} \leq 1$

$\forall p \in [1, ..., P], and \ \ \forall t \in [1, ..., N]:$

$b_{pt1} + b_{pt2} + b_{pt3} + b_{pt4} \geq 1$

$\forall p \in [1, ..., P], \ \ \forall t \in [1, ..., N], and \ \ \forall j \in [1, ..., 6]:$

$b_{ptj} \in 0, 1$

$\forall p \in [1, ..., P], and \ \ \forall t \in [3, ..., N]:$

$x_{pt} - x_{p(t-2)} \geq 2 - M * t1_{pt1}$

$x_{p(t-2)} - x_{pt} \geq 2 - M * t1_{pt2}$

$y_{pt} - y_{p(t-2)} \geq 2 - M * t1_{pt3}$

$y_{p(t-2)} - y_{pt} \geq 2 - M * t1_{pt4}$

$z_{pt} - z_{p(t-2)} \geq 2 - M * t1_{pt5}$

$z_{p(t-2)} - z_{pt} \geq 2 - M * t1_{pt6}$

$t1_{pt1} + t1_{pt2} + t1_{pt3} + t1_{pt4} + t1_{pt5} + t1_{pt6} \leq 5$

$\forall p \in [1, ..., P], \ \ \forall t \in [1, ..., N], and \ \ \forall j \in [1, ..., 6]:$

$t1_{ptj} \in 0, 1$

$\forall p \in [1, ..., P], and \ \ \forall t \in [4, ..., N]:$

$x_{pt} - x_{p(t-3)} \geq 2.5 - M * t2_{pt1}$

$x_{p(t-3)} - x_{pt} \geq 2.5 - M * t2_{pt2}$

$y_{pt} - y_{p(t-3)} \geq 2.5 - M * t2_{pt3}$

$y_{p(t-3)} - y_{pt} \geq 2.5 - M * t2_{pt4}$

$z_{pt} - z_{p(t-3)} \geq 2.5 - M * t2_{pt5}$

$z_{p(t-3)} - z_{pt} \geq 2.5 - M * t2_{pt6}$

$t2_{pt1} + t2_{pt2} + t2_{pt3} + t2_{pt4} + t2_{pt5} + t2_{pt6} \leq 5$

$\forall p \in [1, ..., P], \ \ \forall t \in [1, ..., N], and \ \ \forall j \in [1, ..., 6]:$

$t2_{ptj} \in 0, 1$

$\forall p \in [1, ..., P], and \ \ \forall t \in [5, ..., N]:$

$x_{pt} - x_{p(t-4)} \geq 3 - M * t3_{pt1}$

$x_{p(t-4)} - x_{pt} \geq 3 - M * t3_{pt2}$

$y_{pt} - y_{p(t-4)} \geq 3 - M * t3_{pt3}$

$y_{p(t-4)} - y_{pt} \geq 3 - M * t3_{pt4}$

$z_{pt} - z_{p(t-4)} \geq 3 - M * t3_{pt5}$

$z_{p(t-4)} - z_{pt} \geq 3 - M * t3_{pt6}$

$t3_{pt1} + t3_{pt2} + t3_{pt3} + t3_{pt4} + t3_{pt5} + t3_{pt6} \leq 5$

$\forall p \in [1, ..., P], \ \ \forall t \in [1, ..., N], and \ \ \forall j \in [1, ..., 6]:$

$t3_{ptj} \in 0, 1$

$\forall p \in [1, ..., P], and \ \ \forall q \in [1, ..., P]: \ \ \forall p, q | p > q \ \ and \ \ \forall t, j \in [1, ..., N] \ \ \forall t \in [1, ..., N]:$

$x_{pt} - x_{qj} \geq 2 - M * v1_{pqt1}$

$x_{qj} - x_{pt} \geq 2 - M * v1_{pqt2}$

$y_{pt} - y_{qj} \geq 2 - M * v1_{pqt3}$

$y_{qj} - y_{pt} \geq 2 - M * v1_{pqt4}$

$z_{pt} - z_{qj} \geq 2 - M * v1_{pqt5}$

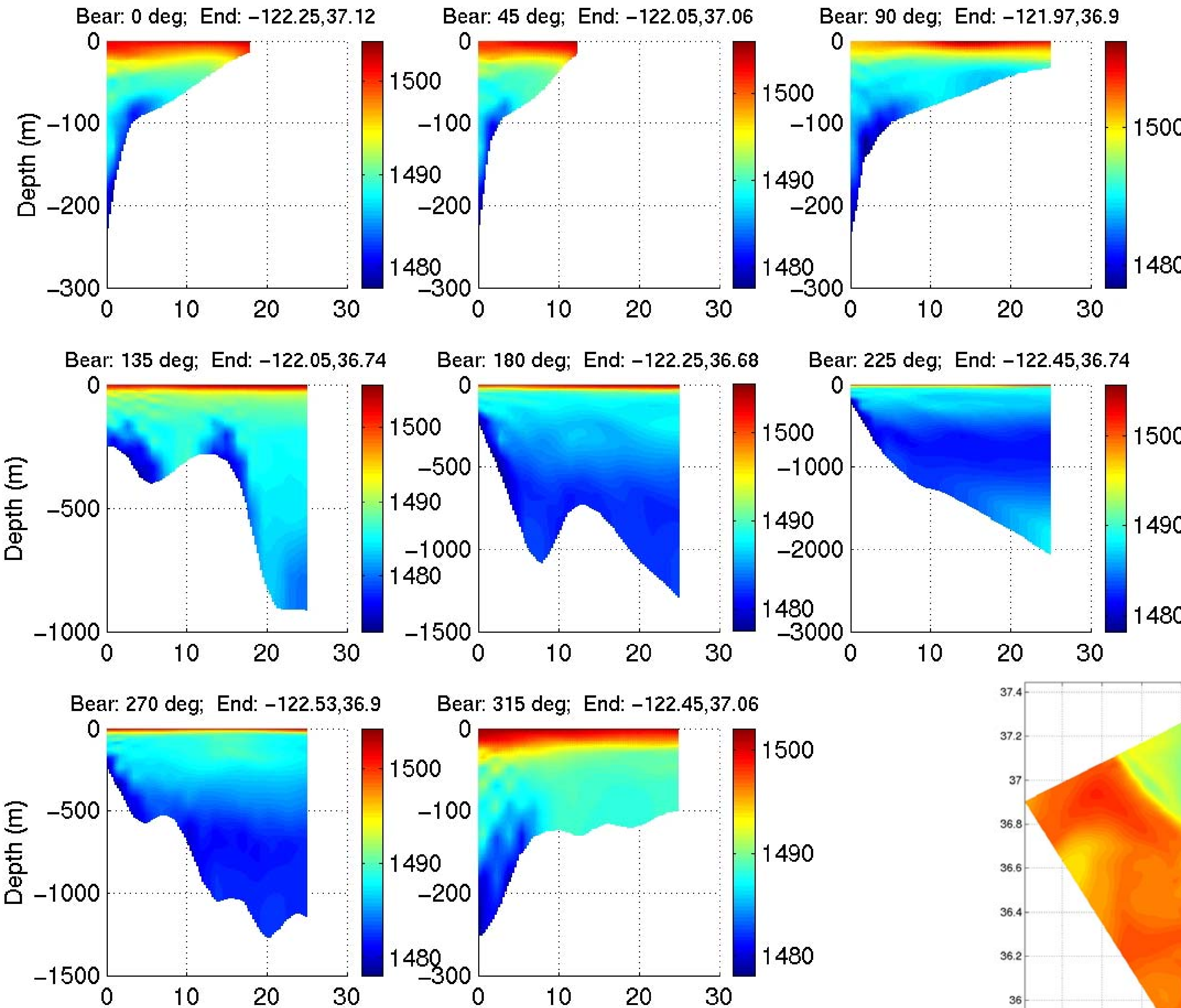
$z_{qj} - z_{pt} \geq 2 - M * v1_{pqt6}$

$v1_{pqt1} + v1_{pqt2} + v1_{pqt3} + v1_{pqt4} + v1_{pqt5} + v1_{pqt6} \leq 5$

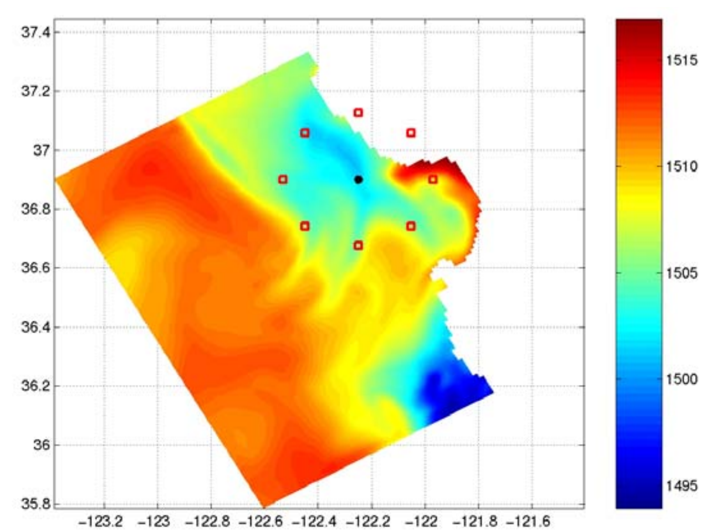
$\forall p, q \in [1, ..., P], \ \ \forall t \in [1, ..., N], and \ \ \forall j \in [1, ..., 6]:$

$v1_{pqtj} \in 0, 1$

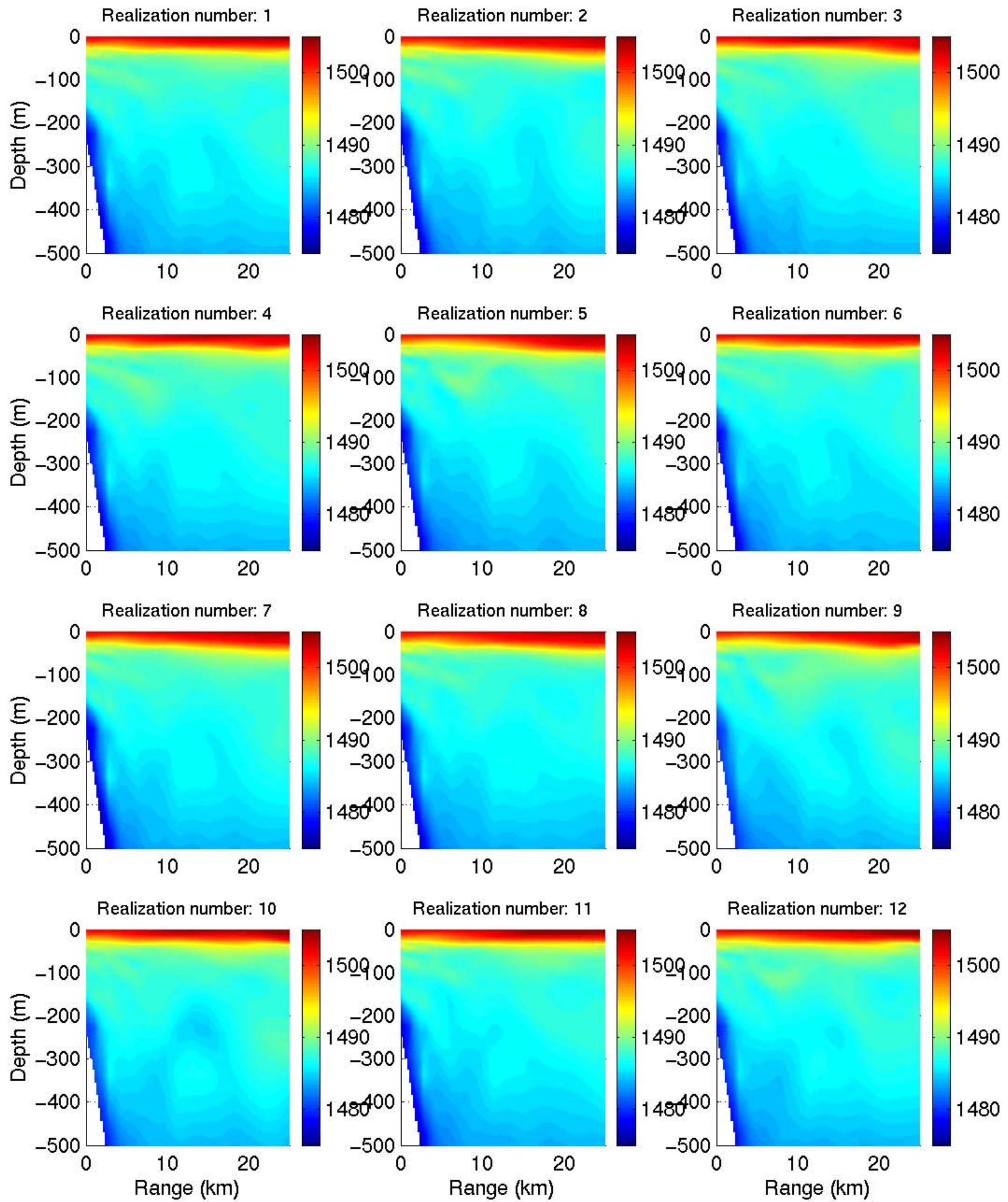
3. Initiate Merging of ESSE/AREA, here for ocean science



All 8 sections of Aug 28 ESSE realization # 1

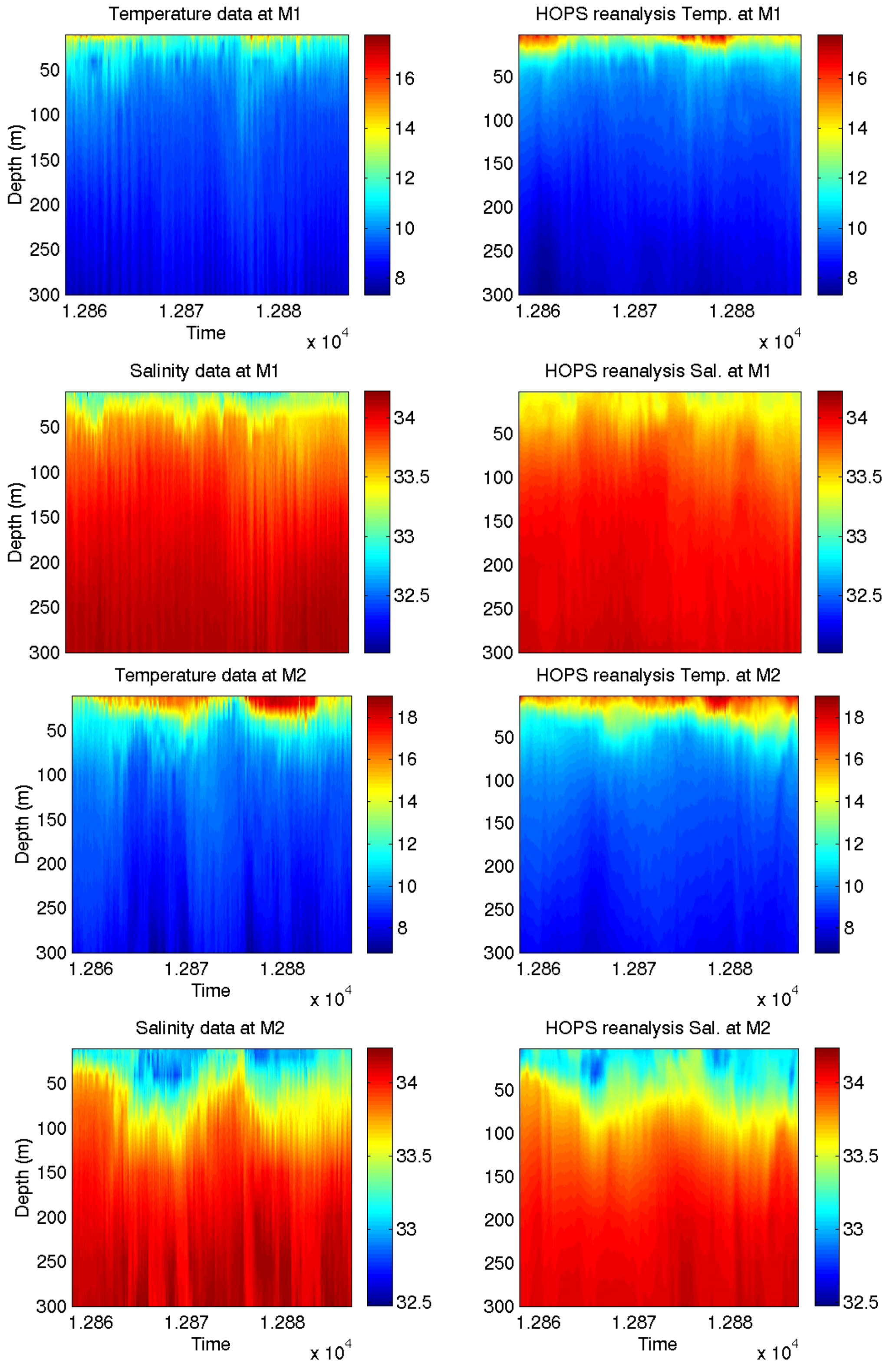


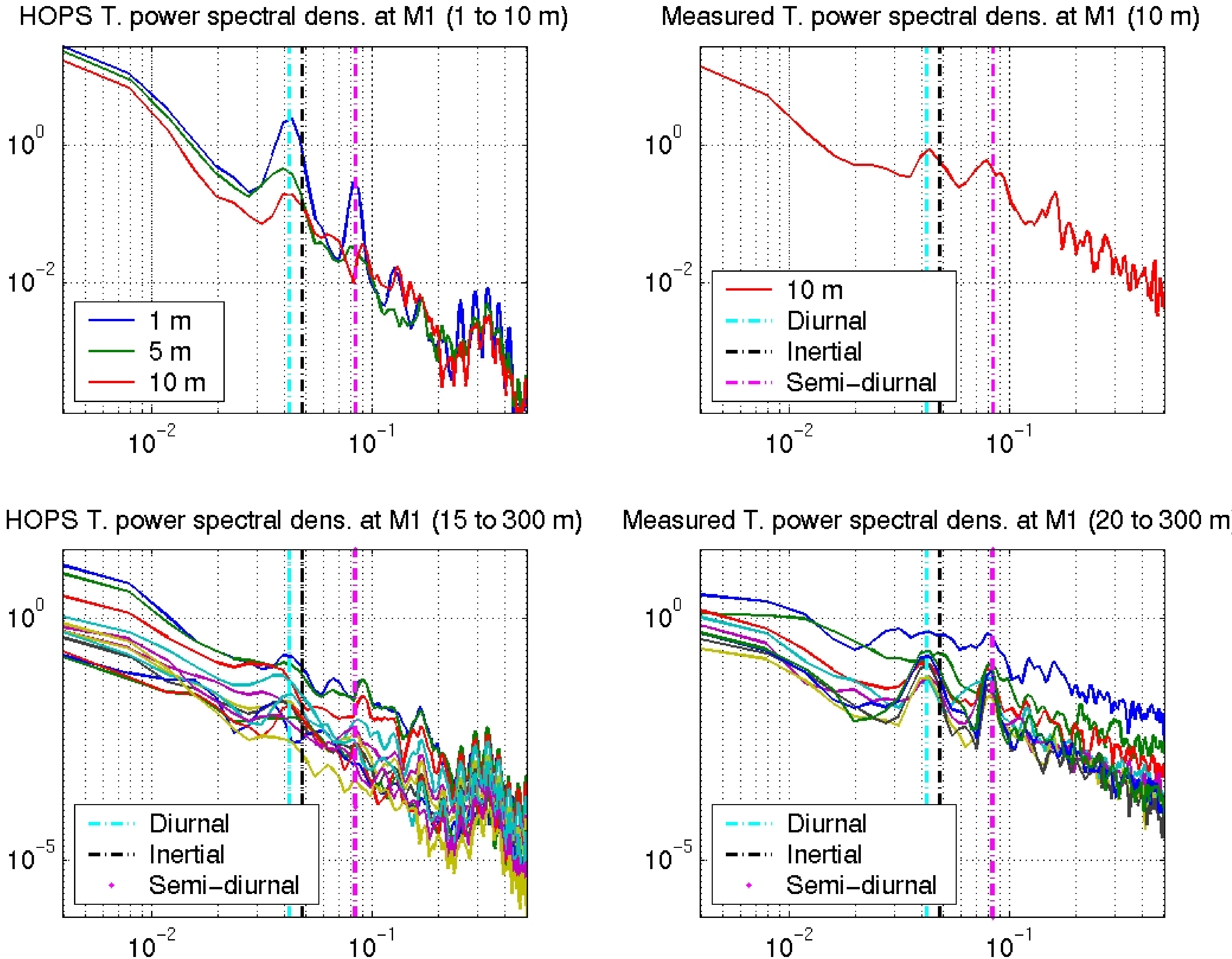
Aug 28 ESSE realizations 1-12 of Section 5 (Bear: 180 deg)



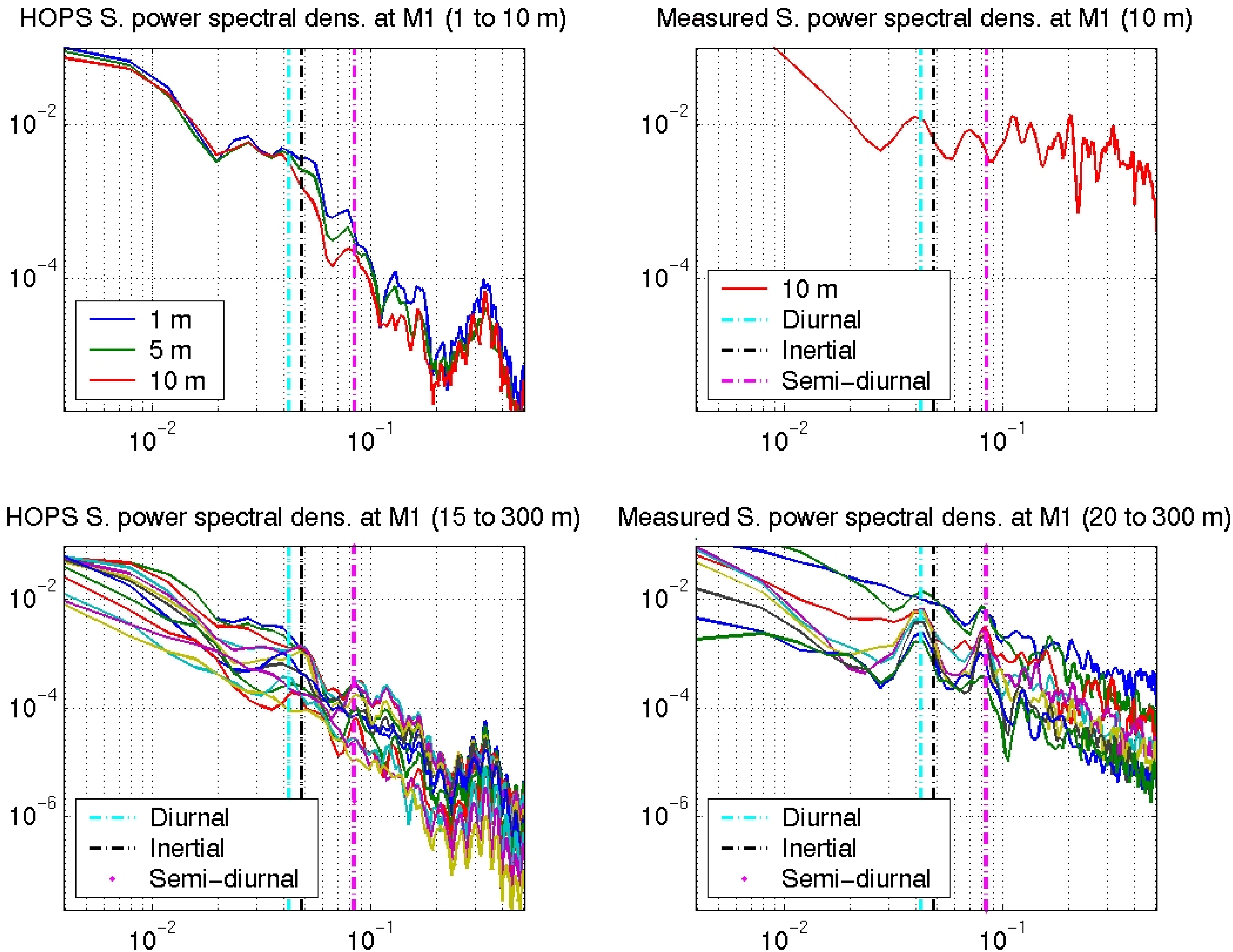
II. Progress towards Models of "Model errors"

- **HOPS/ESSE stochastic forcings**
  - 3D random noise
  - Amplitude(z) = ε *O*(Geos. Bal.)
  - Exponentially decorrelated in time
  - 2 grid pts correlation in space
- **Need to estimate parameters of stochastic model from data**
- **Here, look at near-inertial and tidal scales**
  - Compare model and data at M1/M2
  - Initiate research towards:
    - Stochastic models of these "smaller" scales
    - Optimal gliders patterns for sampling/filtering missing scales

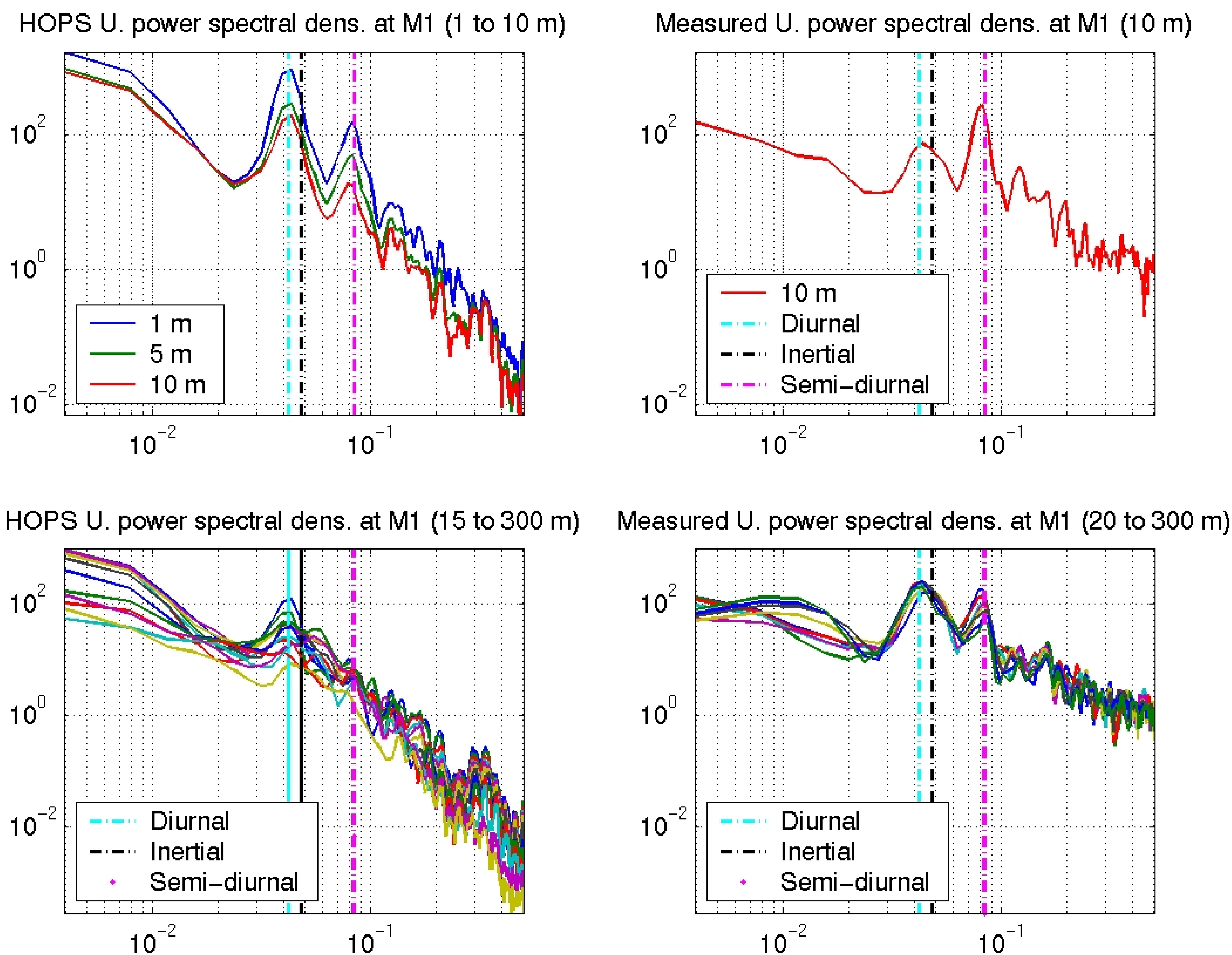




HOPS T. power spectral dens. at M1 (1 to 10 m)
1 m
5 m
10 m
Measured T. power spectral dens. at M1 (10 m)
10 m
Diurnal
Inertial
Semi−diurnal
HOPS T. power spectral dens. at M1 (15 to 300 m)
Diurnal
Inertial
Semi−diurnal
Measured T. power spectral dens. at M1 (20 to 300 m)
Diurnal
Inertial
Semi−diurnal



HOPS S. power spectral dens. at M1 (1 to 10 m)
1 m
5 m
10 m
Measured S. power spectral dens. at M1 (10 m)
10 m
Diurnal
Inertial
Semi−diurnal
HOPS S. power spectral dens. at M1 (15 to 300 m)
Diurnal
Inertial
Semi−diurnal
Measured S. power spectral dens. at M1 (20 to 300 m)
Diurnal
Inertial
Semi−diurnal

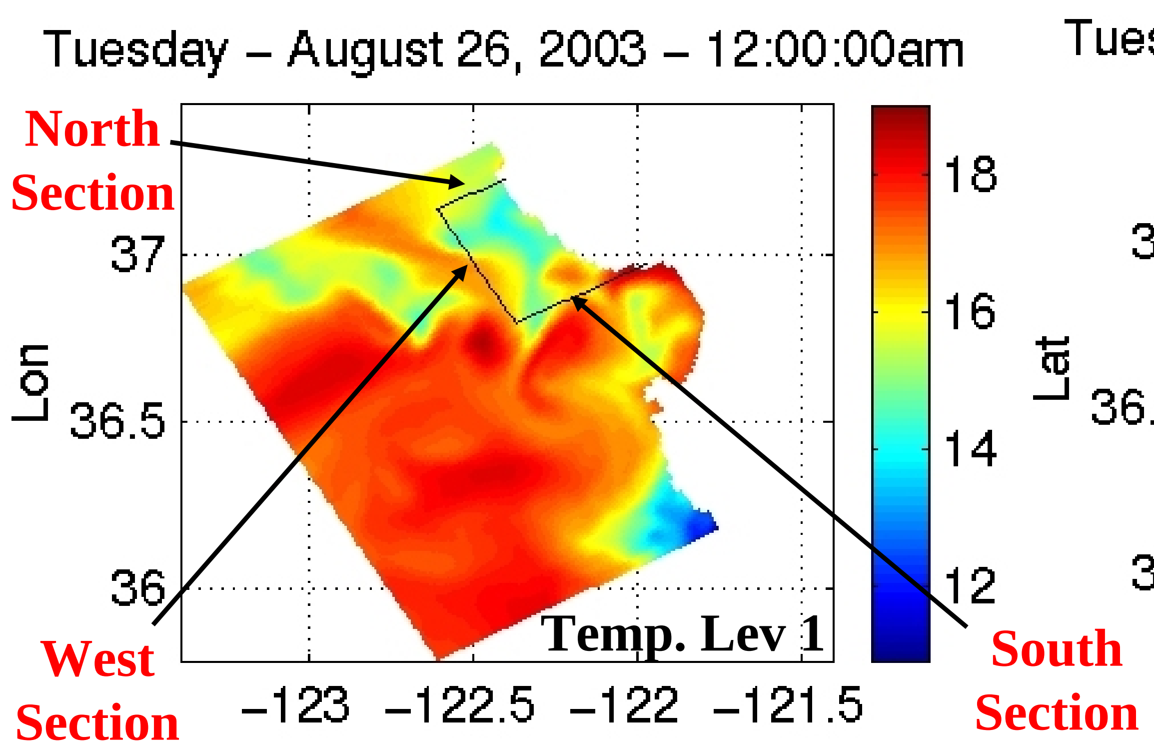


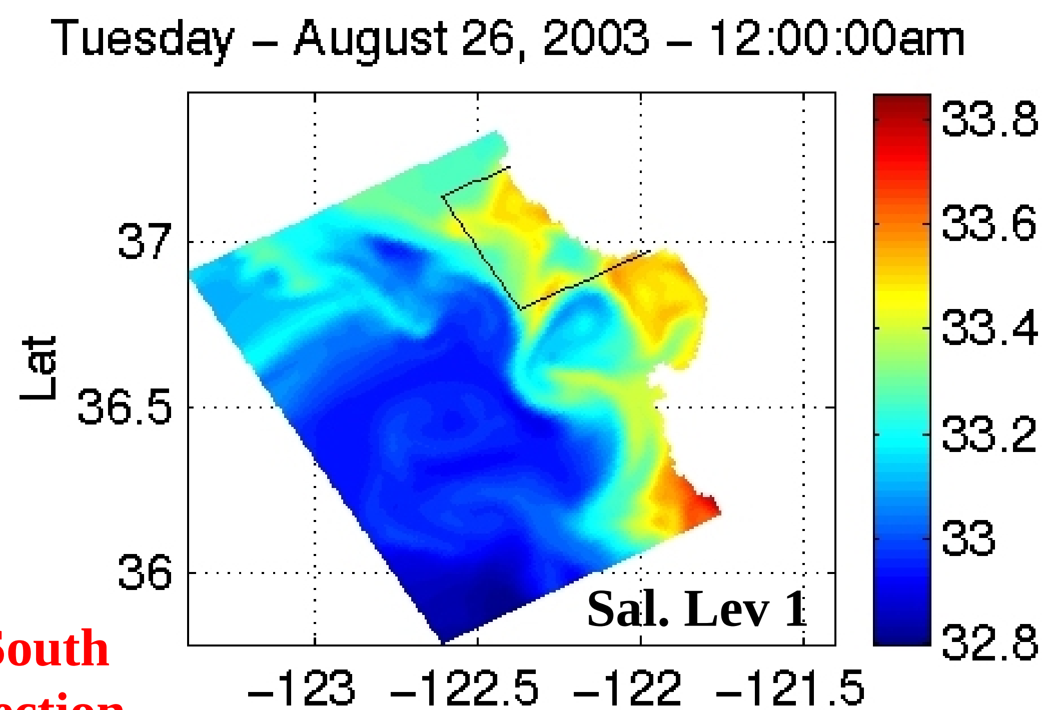
HOPS U. power spectral dens. at M1 (1 to 10 m)
Measured U. power spectral dens. at M1 (10 m)
HOPS U. power spectral dens. at M1 (15 to 300 m)
Measured U. power spectral dens. at M1 (20 to 300 m)
1 m
5 m
10 m
Diurnal
Inertial
Semi−diurnal
10^{2}
10^{0}
10^{-2}
10^{-1}

III. Term by Term Balances and Flux Balances

- **Physical model: Primitive-Equation (PDE, x, y, z, t: HOPS)**

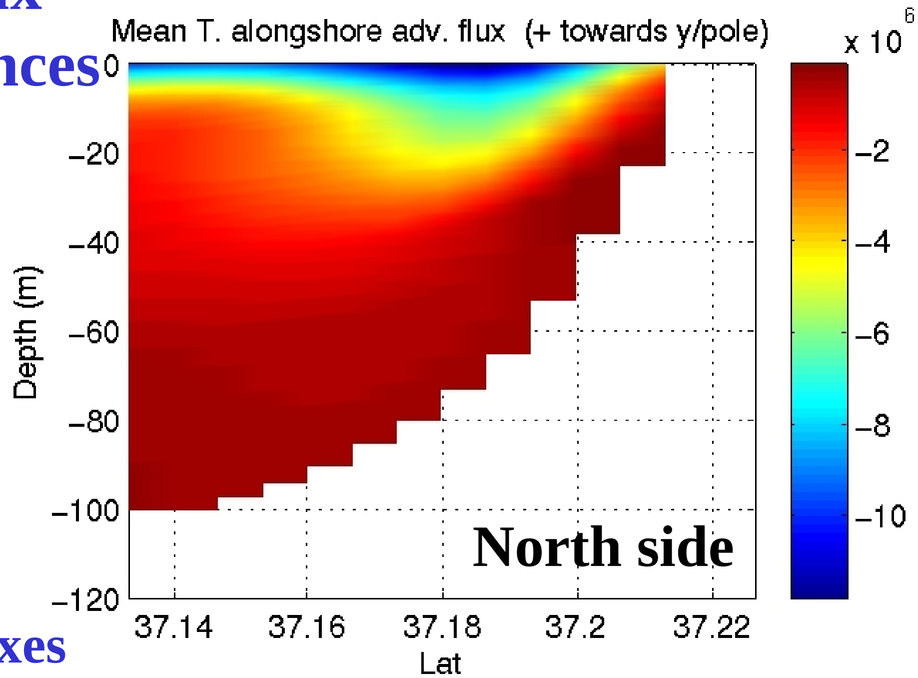
Horiz. Mom.	$\frac{D\mathbf{u_h}}{Dt} + f\,\mathbf{e}_3 \wedge \mathbf{u}_h = -\frac{1}{\rho_0}\nabla_h p_w + \nabla_h \cdot (A_h \nabla_h \mathbf{u}_h) + \frac{\partial\, A_v\, \partial \mathbf{u}_h/\partial z}{\partial z}$
Vert. Mom.	$\rho g + \frac{\partial p_w}{\partial z} = 0$
Thermal en.	$\frac{DT}{Dt} = \nabla_h \cdot (K_h \nabla_h T) + \frac{\partial\, K_v\, \partial T/\partial z}{\partial z}$
Cons. of salt	$\frac{DS}{Dt} = \nabla_h \cdot (K_h \nabla_h S) + \frac{\partial\, K_v\, \partial S/\partial z}{\partial z}$
Cons. of mass	$\nabla \cdot \mathbf{u} = 0$
Eqn. of state	$\rho(\mathbf{r}, z, t) = \rho(T, S, p_w)$

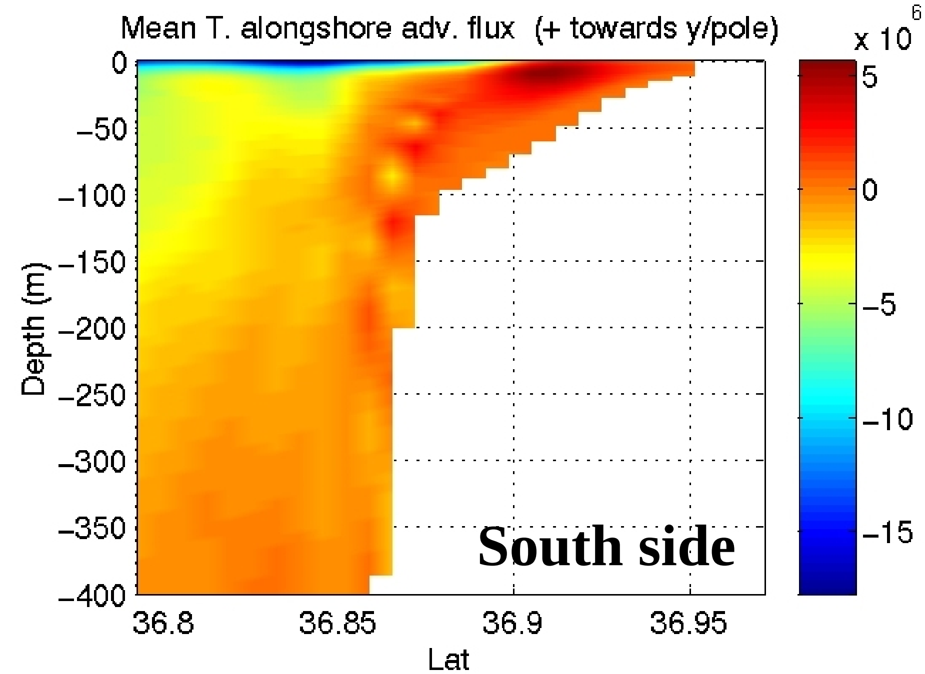




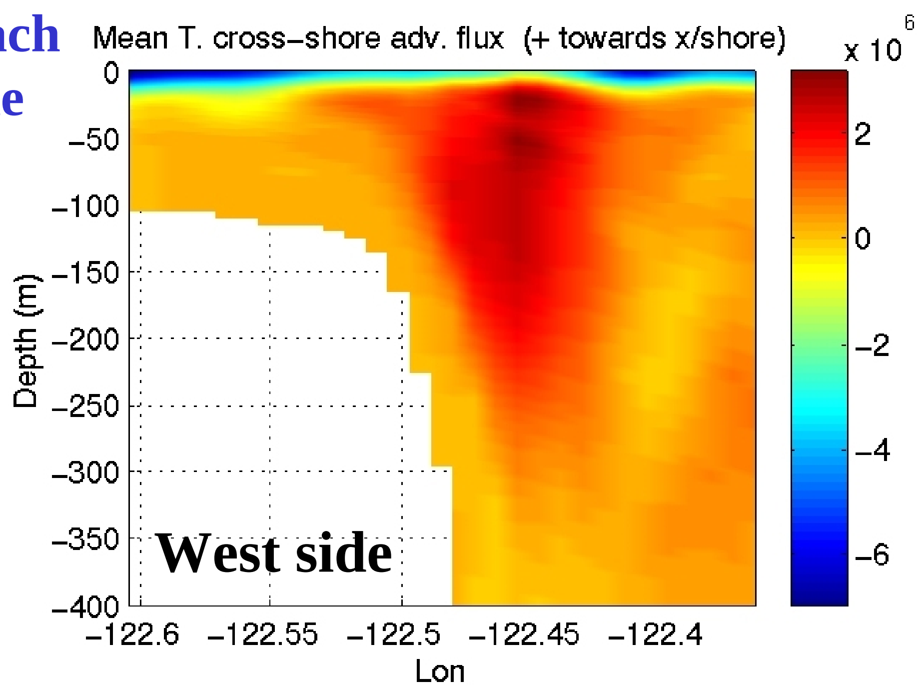
Heat Flux Balances

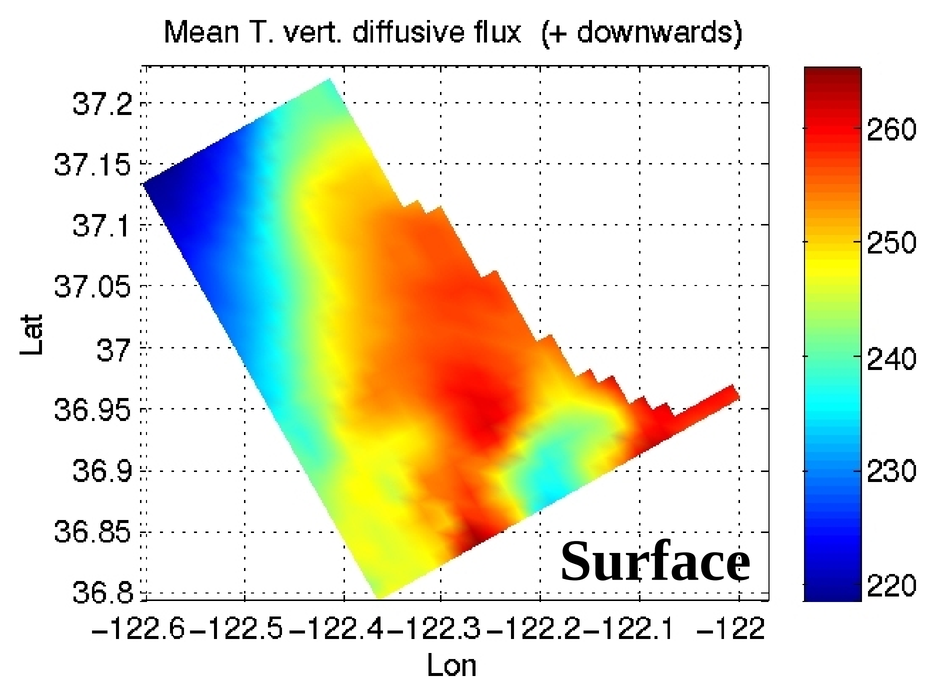
Mean Fluxes (W/m2) over: August 6, 2003 - 10:30:00pm -> August 13, 2003 - 4:30:00am GMT





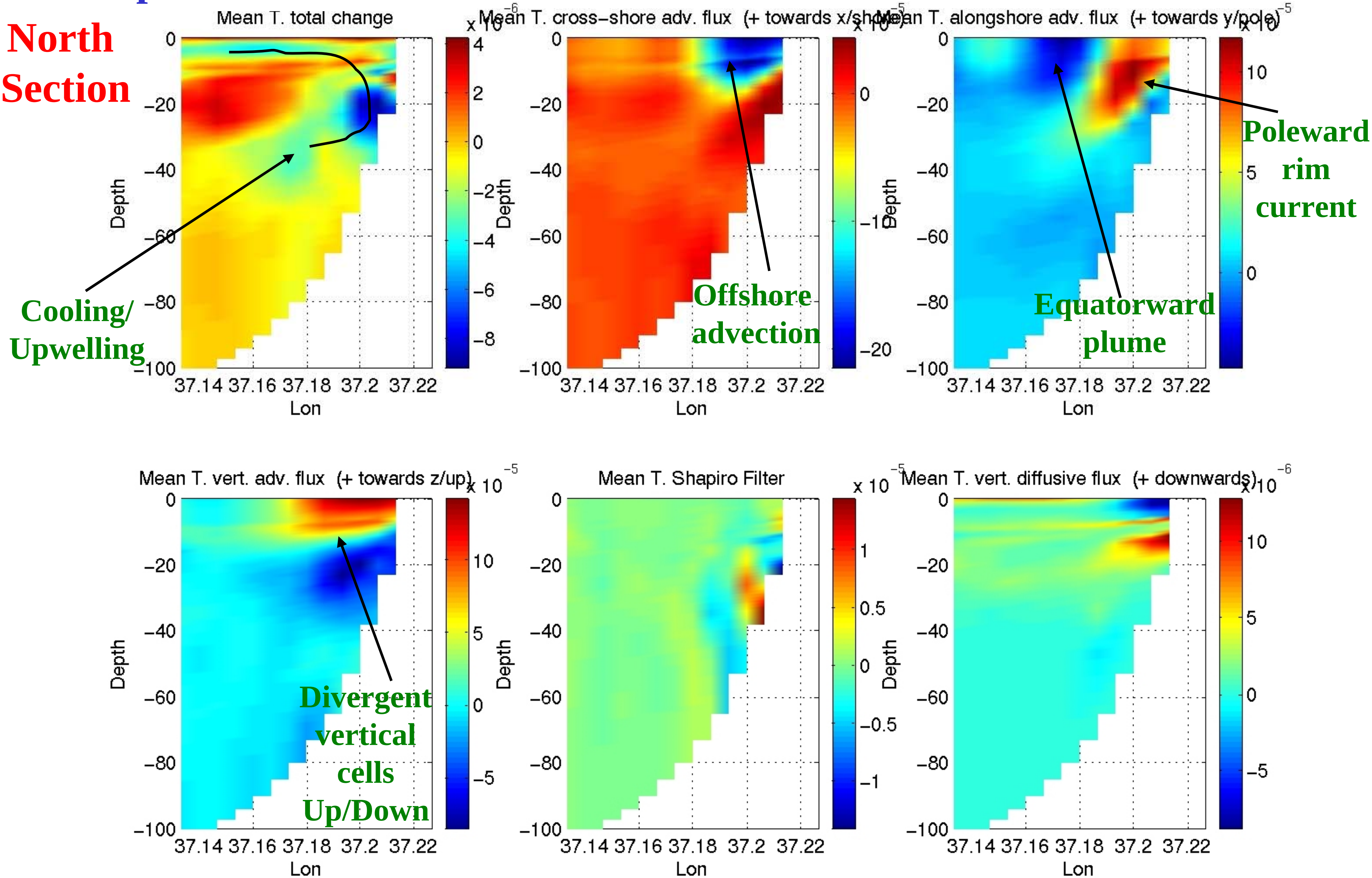
4 fluxes normal to each side





Mean Term-by-Term Temp. balances

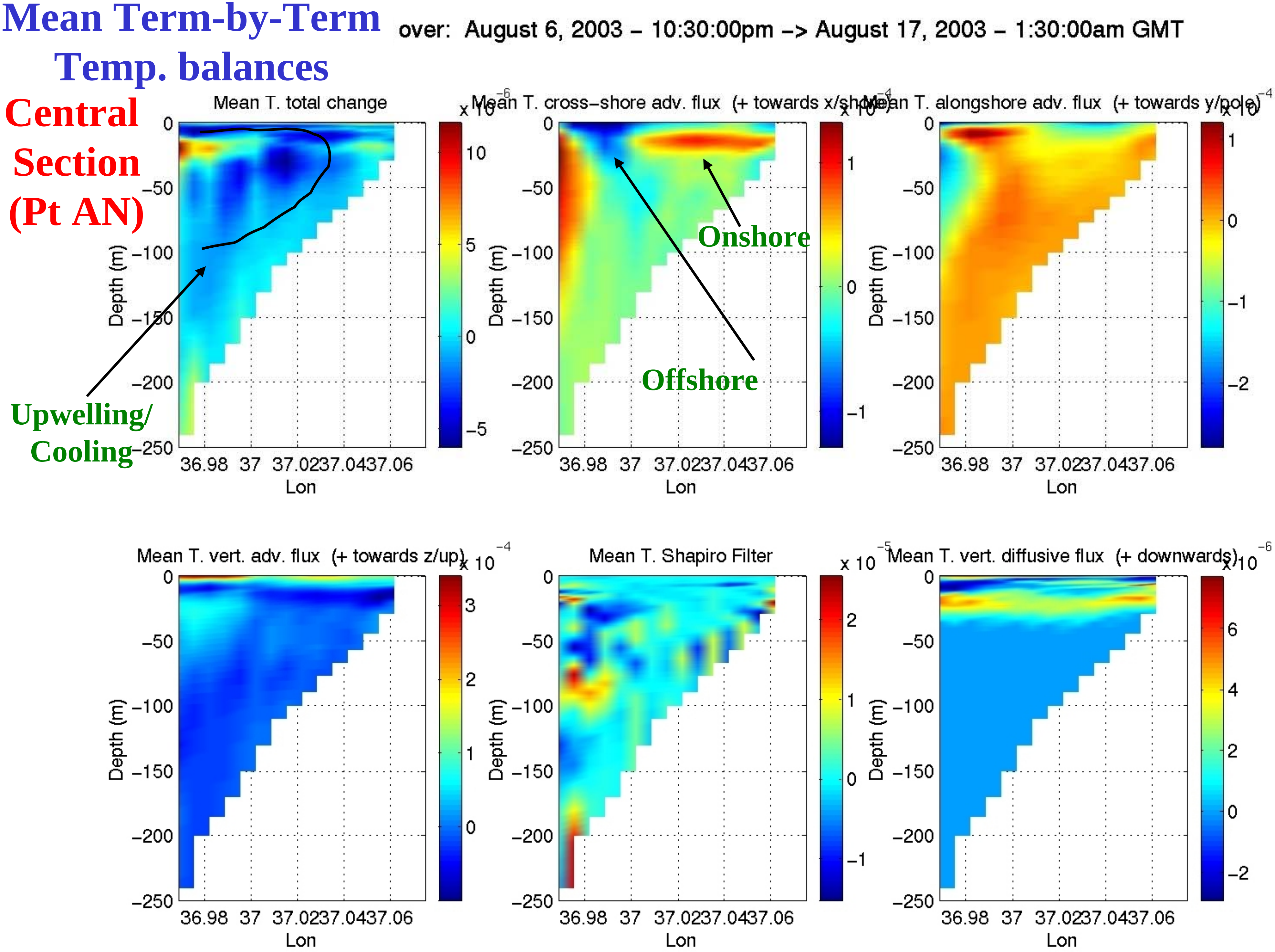
over: August 6, 2003 – 10:30:00pm -> August 17, 2003 – 1:30:00am GMT



Mean Rate of change ≈ (Cross-shore +Alongshore +Vertical) Advection + Vertical. Diff (surf)

Mean Term-by-Term Temp. balances

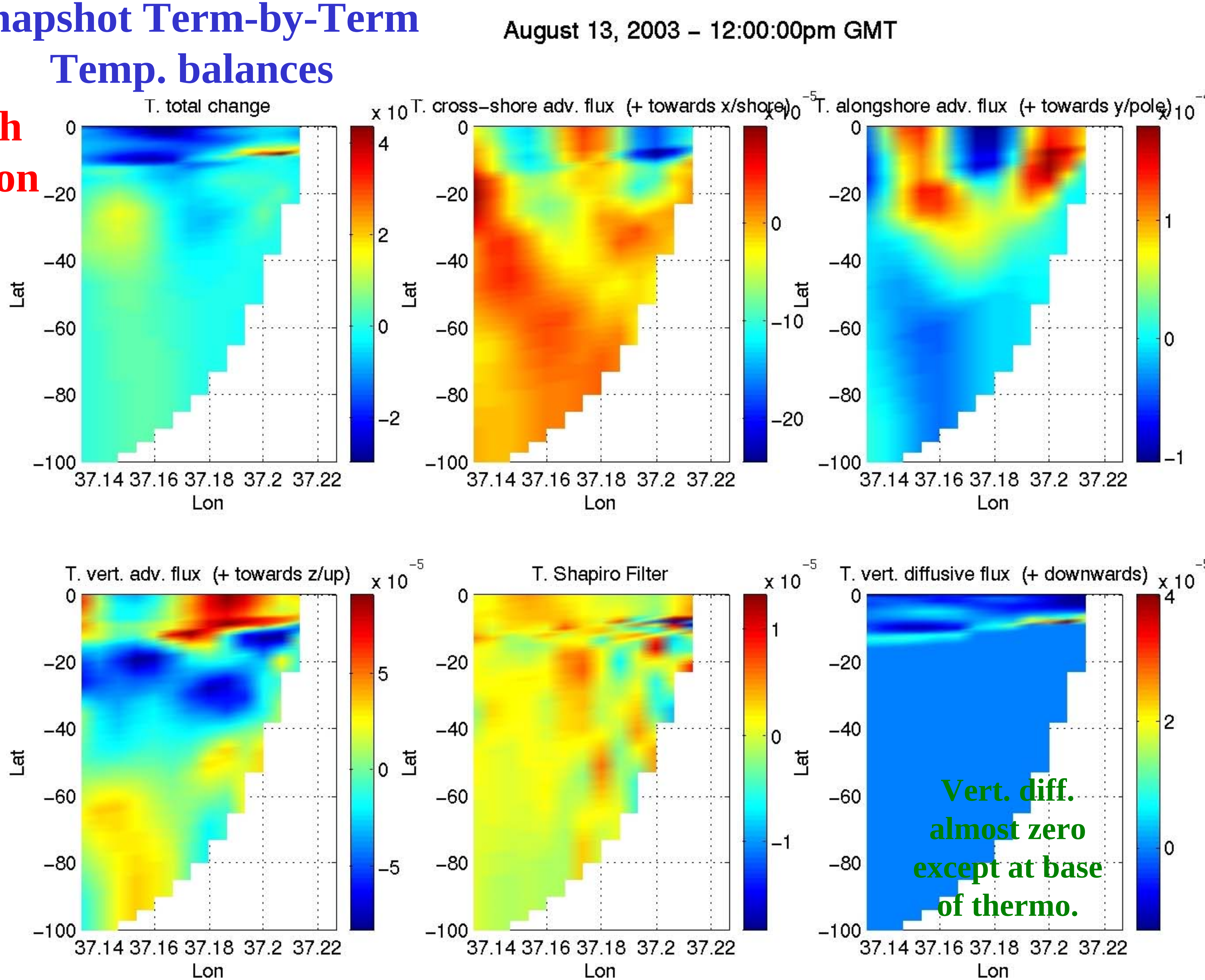
Central Section (Pt AN)



Mean Rate of change ≈ (Cross-shore +Alongshore +Vertical) Advection

Snapshot Term-by-Term Temp. balances

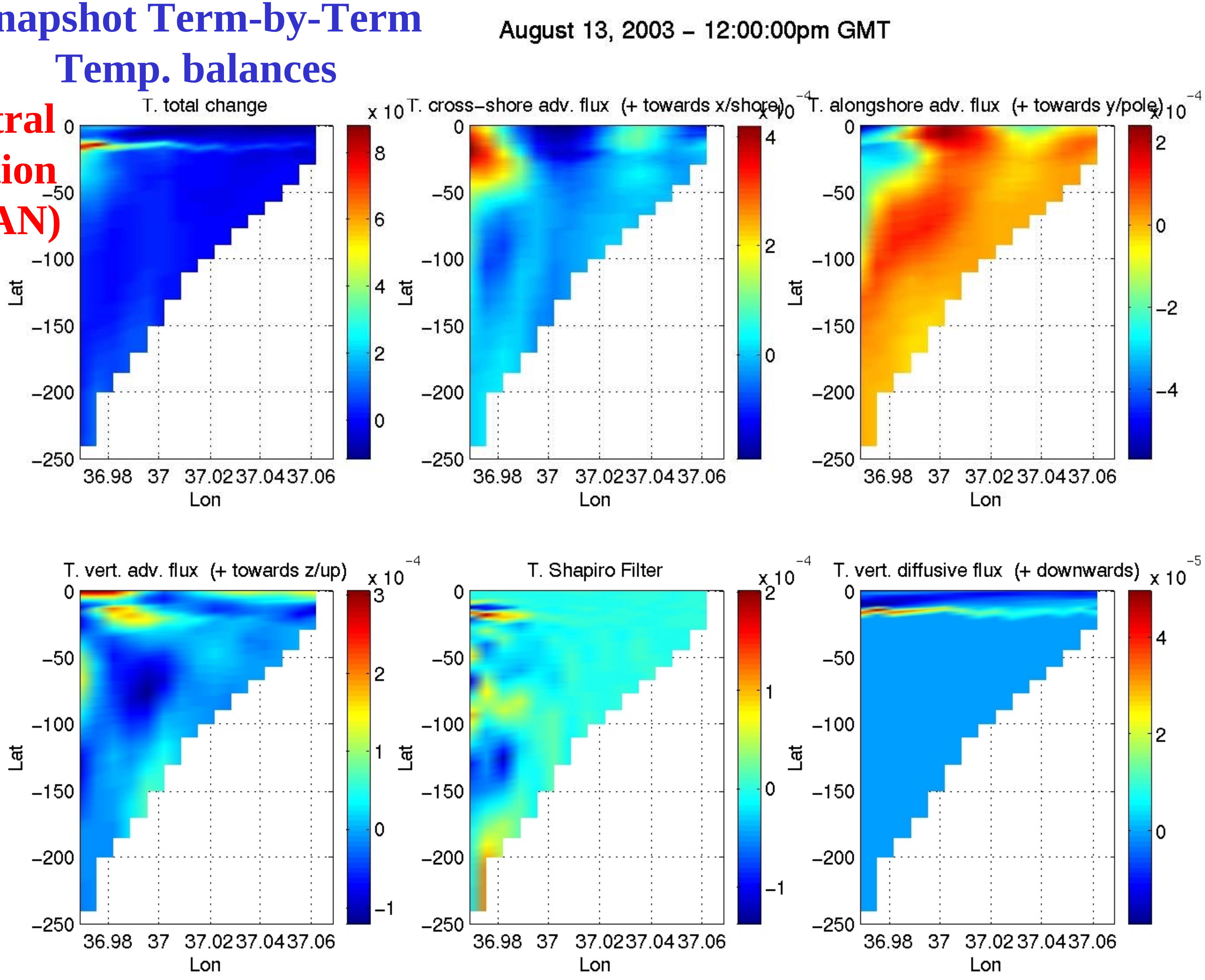
North Section



Mean Rate of change ≈ (Cross-shore +Alongshore +Vertical) Advection

Snapshot Term-by-Term Temp. balances

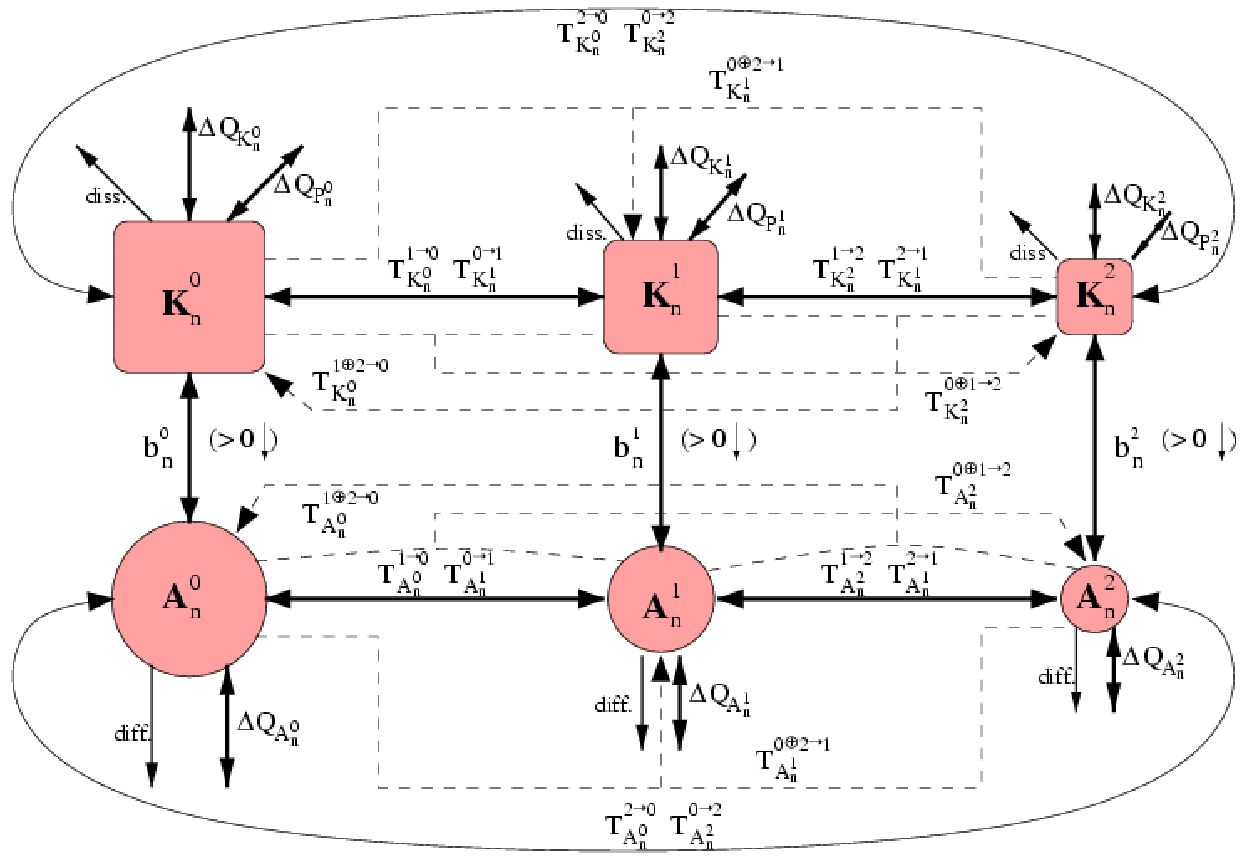
Central Section (Pt AN)



Multi-Scale Energy and Vorticity Analysis

MS-EVA is a new methodology utilizing multiple scale window decomposition in space and time for the investigation of processes which are:

- multi-scale interactive
- nonlinear
- intermittent in space
- episodic in time



Through exploring:

- pattern generation and
- energy and enstrophy
  - transfers
  - transports, and
  - conversions

MS-EVA helps unravel the intricate relationships between events on different scales and locations in phase and physical space.

Dr. X. San Liang

Multi-Scale Energy and Vorticity Analysis

Window-Window Interactions:
MS-EVA-based Localized Instability Theory

Perfect transfer:

A process that exchanges energy among distinct scale windows which does not create nor destroy energy as a whole.

In the MS-EVA framework, the perfect transfers are represented as field-like variables. They are of particular use for real ocean processes which in nature are non-linear and intermittent in space and time.

Localized instability theory:

BC: Total perfect transfer of APE from large-scale window to meso-scale window.

BT: Total perfect transfer of KE from large-scale window to meso-scale window.

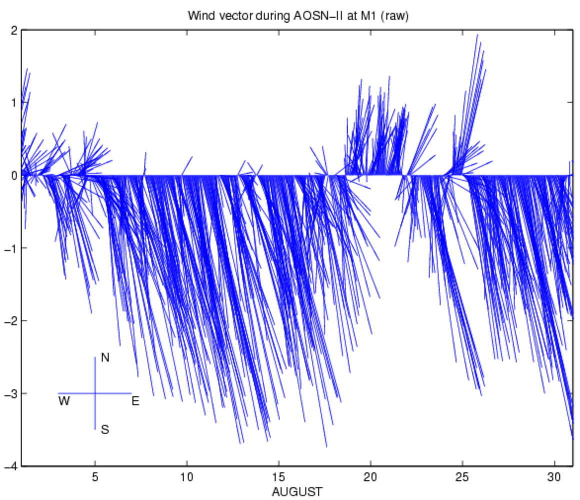
$BT + BC > 0$ => system locally unstable; otherwise stable

If $BT + BC > 0$, and

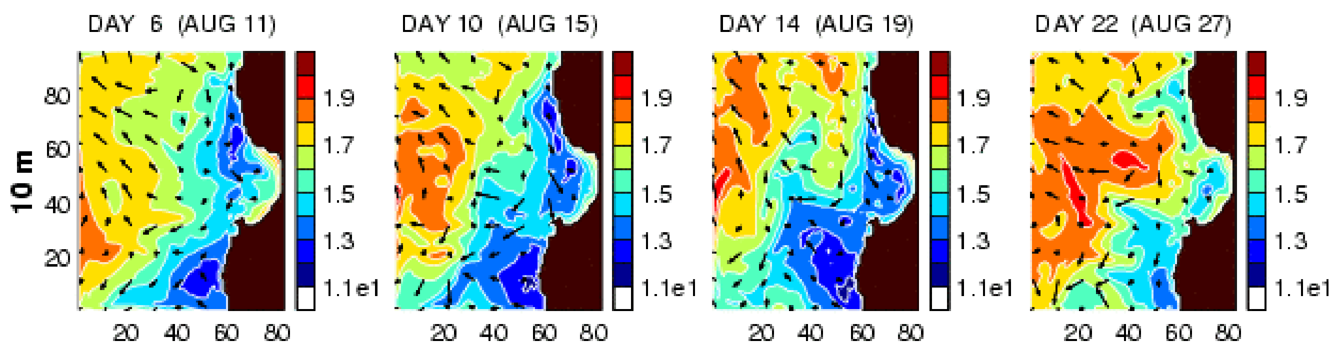
- $BC \leq 0$ => barotropic instability;
- $BT \leq 0$ => baroclinic instability;
- $BT > 0$ and $BC > 0$ => mixed instability

Multi-Scale Energy and Vorticity Analysis

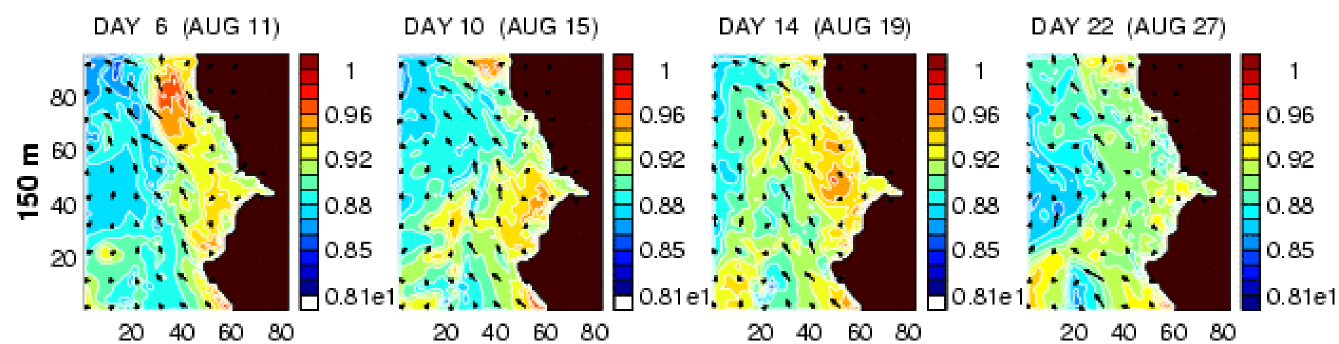
AOSN-II



M1 Winds



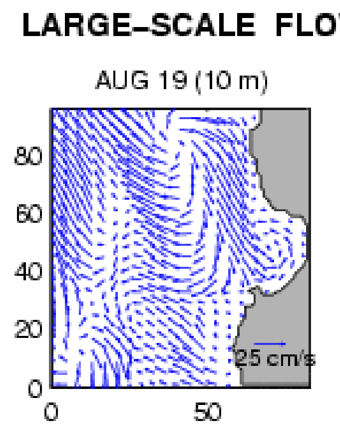
Temperature at 10m



Temperature at 150m

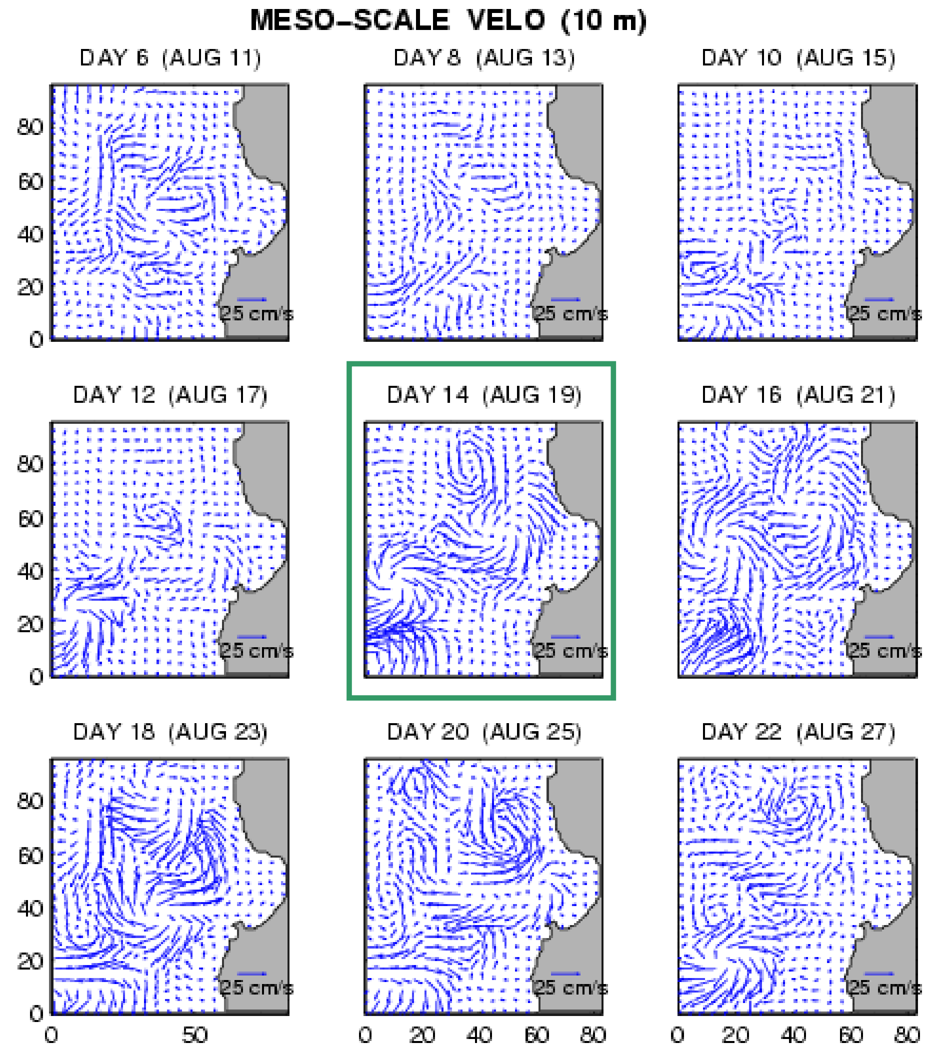
Multi-Scale Energy and Vorticity Analysis

Multi-Scale Window Decomposition in AOSN-II Reanalysis



The reconstructed large-scale and meso-scale fields are filtered in the horizontal with features < 5km removed.

Time windows
Large scale: > 8 days
Meso-scale: 0.5-8 days
Sub-mesoscale: < 0.5 day

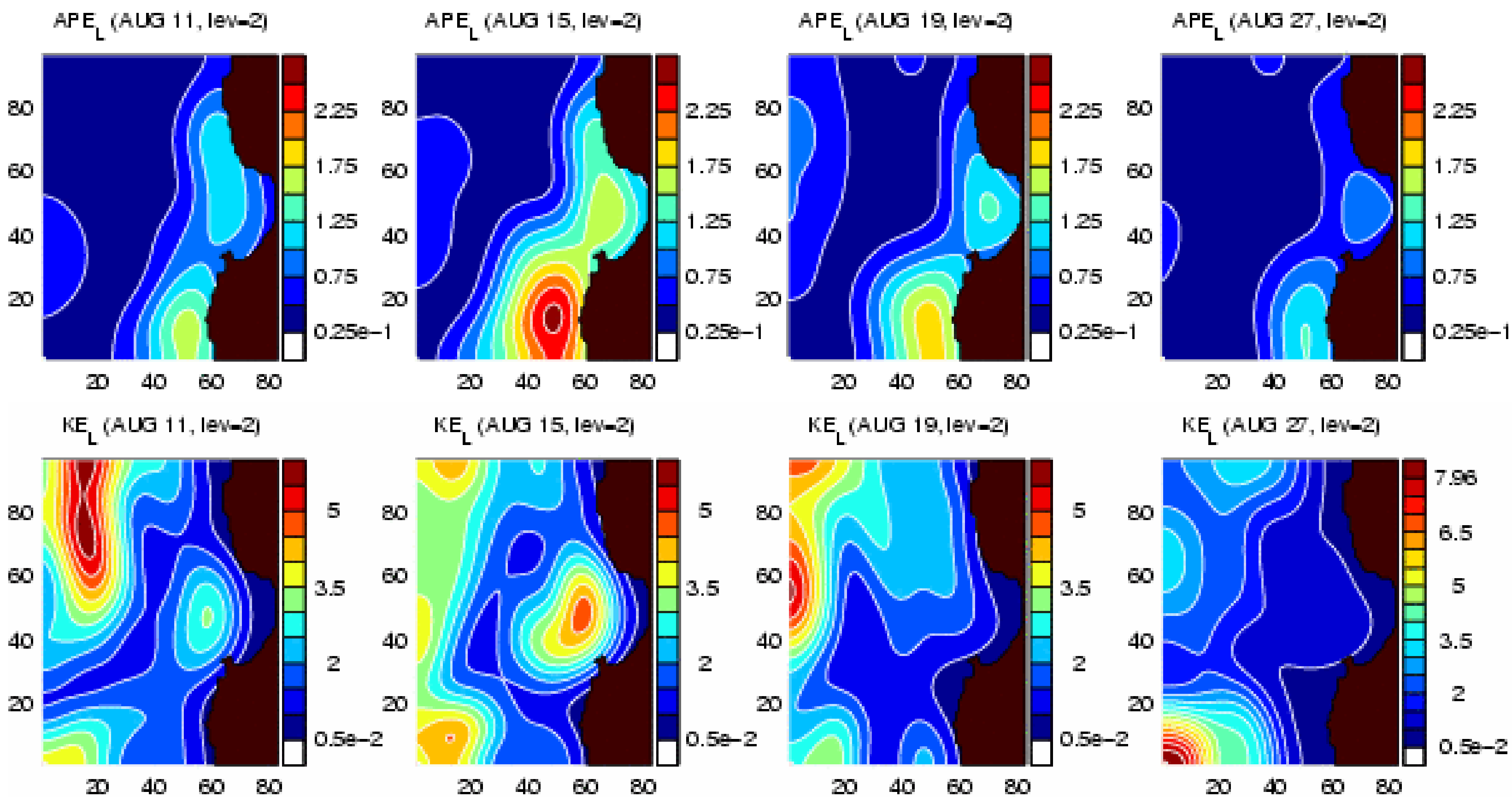


Question: How does the large-scale flow lose stability to generate the meso-scale structures?

Multi-Scale Energy and Vorticity Analysis

- Decomposition in space and time (wavelet-based) of energy/vorticity eqns.

Large-scale Available Potential Energy (APE)



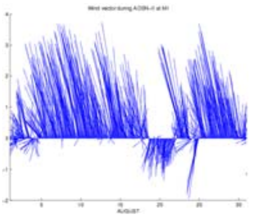
Large-scale Kinetic Energy (KE)

- Both APE and KE decrease during the relaxation period
- Transfer from large-scale window to mesoscale window occurs to account for decrease in large-scale energies (as confirmed by transfer and mesoscale terms)

Windows: Large-scale (>= 8days; > 30km), mesoscale (0.5-8 days), and sub-mesoscale (< 0.5 days)

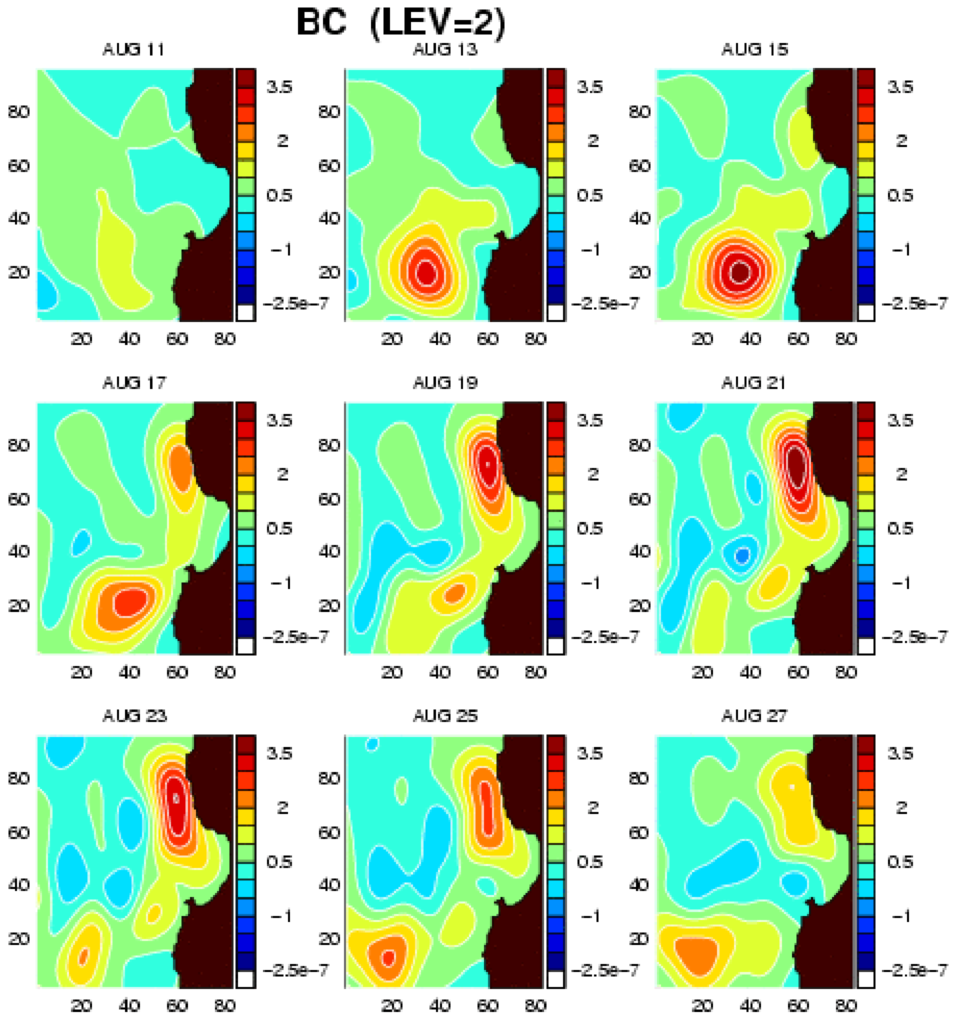
Dr. X. San Liang

Multi-Scale Energy and Vorticity Analysis

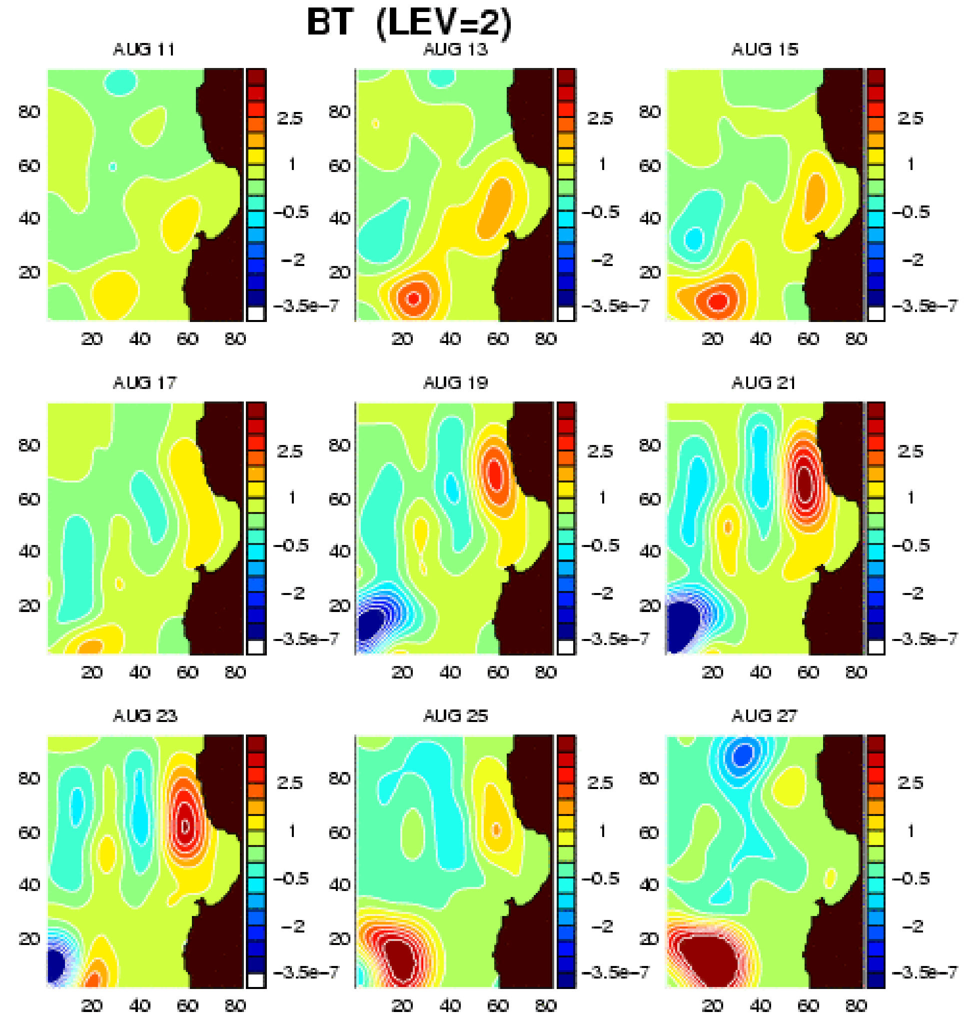


MS-EVA Analysis: 11-27 August 2003

Transfer of APE from large-scale to meso-scale



Transfer of KE from large-scale to meso-scale

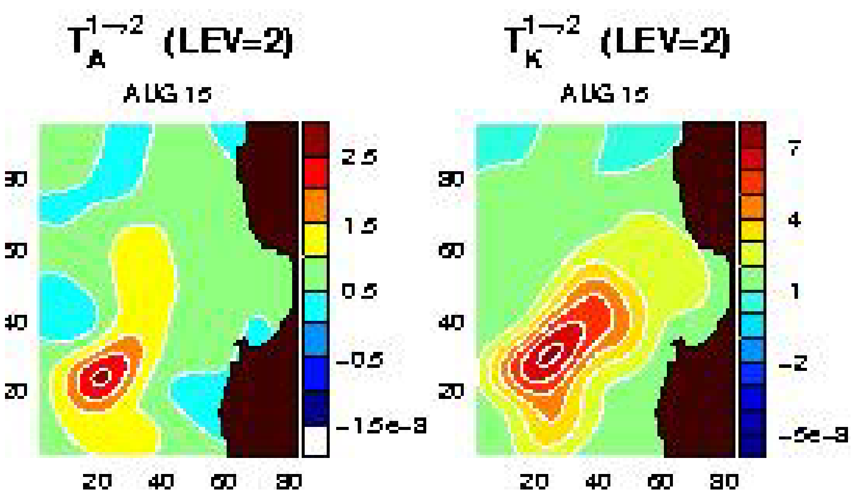


Multi-Scale Energy and Vorticity Analysis

Multi-Scale Dynamics

- Two distinct centers of instability: both of mixed type but different in cause.
- Center west of Pt. Sur: winds destabilize the ocean directly during upwelling.
- Center near the Bay: winds enter the balance on the large-scale window and release energy to the mesoscale window during relaxation.
- Monterey Bay is source region of perturbation and when the wind is relaxed, the generated mesoscale structures propagate northward along the coastline in a surface-intensified free mode of coastal trapped waves.

- Sub-mesoscale processes and their role in the overall large, mesoscale, sub-mesoscale dynamics are under study.



Energy transfer from meso-scale window to sub-mesoscale window.

Error Analyses and Optimal (Multi) Model Estimates

Strategies For Multi-Model Adaptive Forecasting

- <u>Error Analyses</u>: *Learn individual model forecast errors in an on-line fashion from model-data misfits based on Maximum-Likelihood*
- <u>Model Fusion</u>: *Combine models via Maximum-Likelihood based on the current estimates of their forecast errors*

3-steps strategy, using model-data misfits and error parameter estimation

1. Select forecast error covariance $\mathbf{B}$ and bias $\boldsymbol{\mu}$ parameterization $\boldsymbol{\alpha}$, $\boldsymbol{\beta}$

$$\mathrm{B} \approx \tilde{\mathrm{B}}(\boldsymbol{\alpha}); \qquad \boldsymbol{\mu} \approx \tilde{\boldsymbol{\mu}}(\boldsymbol{\beta}); \qquad \boldsymbol{\Theta} = \{\boldsymbol{\alpha}, \boldsymbol{\beta}\}$$

2. Adaptively determine forecast error parameters from **model-data misfits** based on the Maximum-Likelihood principle:

$\boldsymbol{\Theta}^* = \arg\max_{\boldsymbol{\Theta}} p(\boldsymbol{\mathcal{Y}}|\boldsymbol{\Theta})$ Where $\boldsymbol{\mathcal{Y}} = \{\mathbf{y}_1^o, \mathbf{y}_2^o, \ldots, \mathbf{y}_T^o\}$ is the observational data

3. Combine model forecasts via Maximum-Likelihood based on the current estimates of error parameters

O. Logoutov

Error Analyses and Optimal (Multi) Model Estimates

Forecast Error Parameterization

Limited validation data motivates use of few free parameters

- Approximate forecast error covariances and biases as some parametric family, e.g. homogeneous covariance model:

$$\mathbf{B}_m(i,j) = \sigma(\mathbf{x}_i)\sigma(\mathbf{x}_j)\rho\left(||\mathbf{x}_i - \mathbf{x}_j||\right); \quad \rho(r) = \exp\left(\frac{-r^2}{2L^2}\right)$$

  – Choice of covariance and bias models $\tilde{\mathbf{B}}$ and $\tilde{\boldsymbol{\mu}}$ should be sensible and efficient in terms of $\tilde{\mathbf{B}}\mathbf{v}, \tilde{\mathbf{B}}^{-1}\mathbf{v}$ and storage
    * functional forms (positive semi-definite), e.g. isotropic
      - facilitates use of Recursive Filters and Toeplitz inversion
    * feature model based
      - sensible with few parameters. Needs more research.
    * based on dominant error subspaces
      - needs ensemble suite, complex implementation-wise

Error Analyses and Optimal (Multi) Model Estimates

Error Parameter Tuning

Learn error parameters in an on-line fashion from model-data misfits based on Maximum-Likelihood

- We estimate error parameters via Maximum-Likelihood by solving the problem:

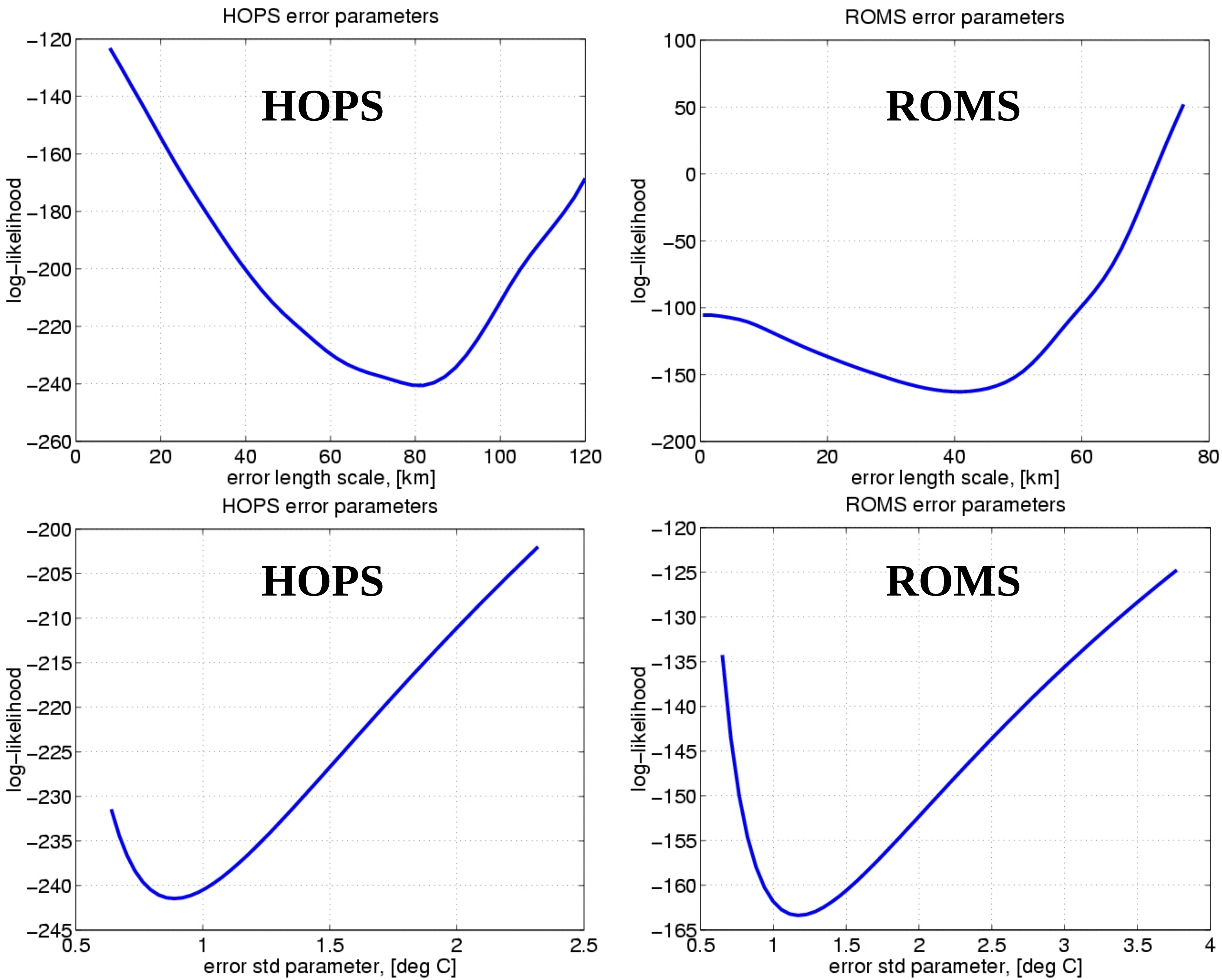
$$\boldsymbol{\Theta}^* = \arg\max_{\boldsymbol{\Theta}} p(\boldsymbol{\mathcal{Y}}|\boldsymbol{\Theta}) \qquad (1)$$

Where $\boldsymbol{\mathcal{Y}} = \{\mathbf{y}_1^o, \mathbf{y}_2^o, \ldots, \mathbf{y}_T^o\}$ is the observational data, $\boldsymbol{\Theta} = \{\boldsymbol{\theta}_1, \boldsymbol{\theta}_2, \ldots, \boldsymbol{\theta}_M\}$ the forecast error covariance parameters of the M models

- (1) implies finding parameter values that maximize the probability of observing the data that was, in fact, observed
- By employing a randomized algorithm, we solve (1) relatively efficiently

Error Analyses and Optimal (Multi) Model Estimates

Log-Likelihood functions for error parameters



Length Scale

Variance

Error Analyses and Optimal (Multi) Model Estimates

Model Fusion

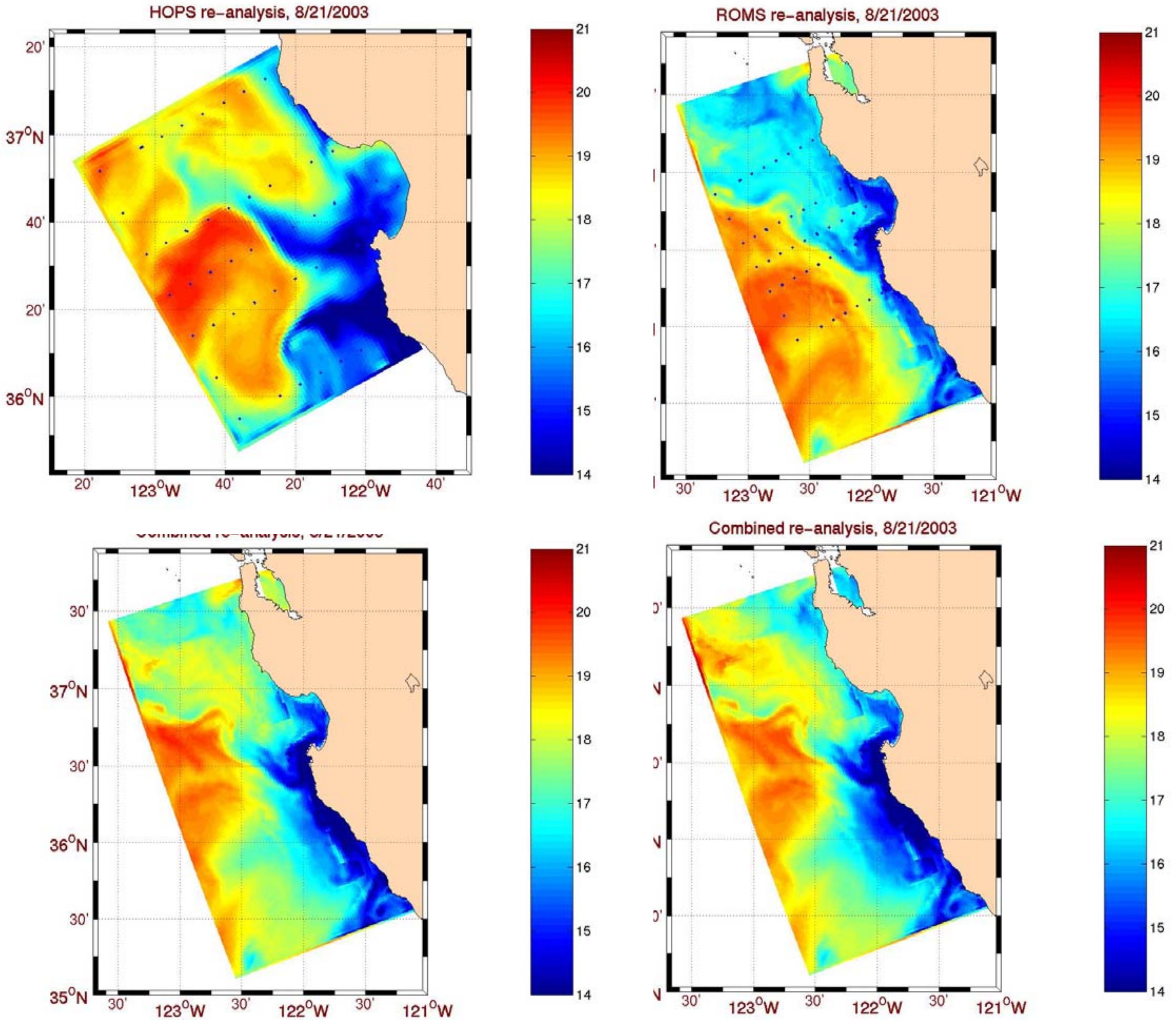
combine based on relative model uncertainties

- **Model Fusion**: once error parameters $\mathbf{\Theta}^*$ are available, combine forecasts $\mathbf{x}_m$ based on their relative uncertainties as:

$$\mathbf{x}^* = \arg\min_{\mathbf{x}} \sum_{m=1}^{M} \left(\mathbf{x} - \mathbf{H}_m \mathbf{x}_m\right)^T \mathcal{B}^{-1}_{(\mathbf{\Theta}_m)} \left(\mathbf{x} - \mathbf{H}_m \mathbf{x}_m\right)$$

Error Analyses and Optimal (Multi) Model Estimates

Two-Model Forecasting Example



HOPS and ROMS SST forecast

Left – HOPS (re-analysis)

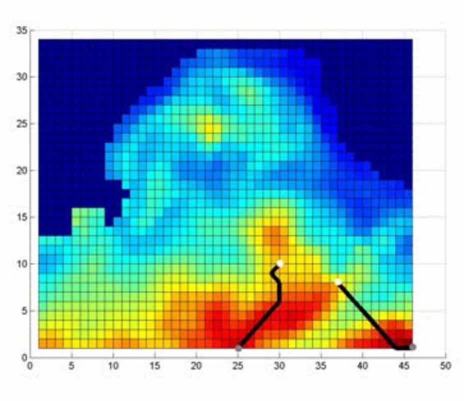
Right – ROMS (re-analysis)

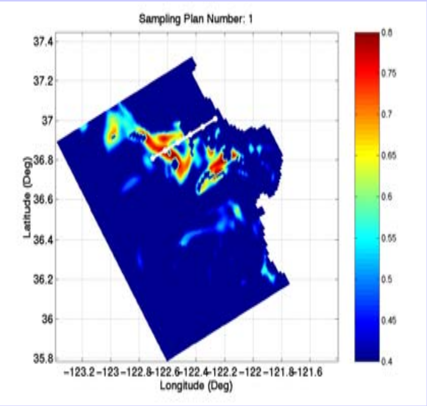
Combined SST forecast

Left – with *a priori* error parameters

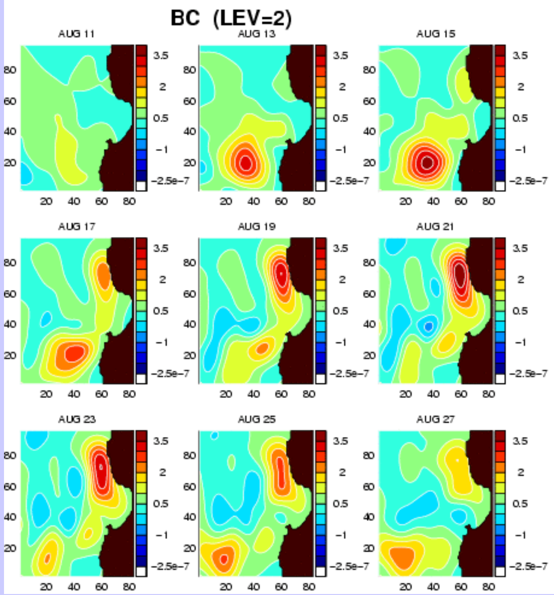
Right – with Maximum-Likelihood error parameters

CONCLUSIONS





- ESSE and MIP for fixed and fully variable adaptive sampling
- Model-data comparisons at near inertial scales, for improved smaller scale deterministic/ stochastic models
- Volume Term-by-Term and Flux balances computed for upwelling and relaxation periods (averaged and snapshots/time evolution). Shows complexity of 3D upwelling regimes, with strong eddying and meandering of coastal current



Ms Eva:

- Center west of Pt. Sur: winds destabilize the ocean directly during upwelling.
- Center near the Bay: winds enter the balance on the large-scale window and release energy to the mesoscale window during relaxation.

- Error model parameter parameterization via Bayesian Maximum likelihood