ON THE MAPPING OF MULTIVARIATE GEOPHYSICAL FIELDS: STUDIES OF THE SENSITIVITY TO ERROR SUBSPACE PARAMETERS

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Abstract

The influence of *a priori* parameters on the error and variability subspace estimation and corresponding three-dimensional, multivariate and multiscale mapping methodology described in (Lermusiaux *et al.* (2000) is investigated. The sensitivities of the subspace and of the *a posteriori* fields and errors to the size of the subspace, scales considered and nonlinearities in the dynamical adjustments are exemplified and studied. The applications focus on the mesoscale to subbasin-scale physical fields in the northwestern Levantine during February 10-March 15 and March 19-April 16, 1995. Different cases are illustrated and compared among each other as well as to a benchmark. For additional evaluation, forecasts generated from the various analyses are compared to *in situ* and satellite data. A back-of-the-envelope rule for estimating the size of subspaces is also outlined. Several properties of the multiscale subspace estimation and analysis scheme are discussed. The sensitivities to size confirm that the truncation to a subspace is efficient and that the use of quantitative criteria is important to determine adequate sizes. The sensitivities to scales show that, for a given region, smaller scales usually require larger subspaces, but that dynamics can alter this property. The sensitivities to the dynamical adjustment reveal that nonlinearities can modify the dominant variability decomposition, especially the dominant eigenvectors, and that changes are largest for the features and regions with high shears. Dynamical information is extracted from the estimated variability variance fields, eigenvalue spectra, multivariate eigenvectors and (cross)-covariance functions. Dominant dynamical balances and the spatial distribution of hydrographic and velocity characteristic scales are obtained for primary regional features. In particular, the Ierapetra is found to be close to gradient-wind balance and coastal-trapped waves are anticipated to occur along the northern escarpment of the basin.

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1. INTRODUCTION

In mapping geophysical fields and their uncertainties, it is important to assess the sensitivity to *a priori* parameters. The present work carries out such studies for the *a priori* error and variability subspace estimation and corresponding mapping methodology described in a companion manuscript (Lermusiaux *et al.*, 2000, henceforth, LAL00). With this three-dimensional, multivariate and multiscale approach (App. A), the dominant components of the *a priori* error probability density functions of the fields or state to be mapped, i.e. the *a priori* error subspace, are constructed in two parts. The observed portions of the subspace are specified based on measurements and multiscale statistical models. The non-observed portions are then built by dynamical adjustments to the observed portions. Once this *a priori* error subspace is computed, the data sets of interest can be mapped into gridded fields and error properties by Bayesian estimation in the subspace. Presently, the error statistics are assumed to be described by covariance matrices and the mapping (analysis) is reduced to a minimum error variance estimation problem. The subspace is then determined by the dominant eigendecomposition of the *a priori* error covariance.

Two main objectives of this manuscript are to investigate (i) the sensitivity of the state and error estimates to the properties of the error subspace (e.g. size, dynamics, adjustment) and (ii) the sensitivity of the subspace itself to its size and to the data and dynamics utilized in its construction. The types of questions considered thus include: How many, and why so many, eigenvectors should be employed to represent most of a covariance matrix or to carry out a mapping? What is the sensitivity of a variability or error subspace to the parameters of the model employed in the dynamical adjustments? How, and by how much, should such subspaces be dynamically adjusted? And finally, what is the performance of forecasts generated from the various analyses? The sensitivity studies are carried out by examining the field and error estimates obtained from different *a priori* parameters. For additional assessment, results are also qualitatively and quantitatively compared to those of the univariate horizontal (2D) scheme of the Harvard Ocean Prediction System (HOPS, e.g. Lozano *et al.*, 1996; Robinson, 1996; Lermusiaux, 1999). Since this benchmark (Sects. 3–4) is a multiscale analysis, the present three-dimensional, multivariate and multiscale mapping of all physical fields and their error covariances is referred to in short as the "multivariate 3D" scheme. Ultimately, forecasts generated from the various analyses are compared to *in situ* and satellite data, using root-mean-square differences and correlation coefficients for measures of skill. The third objective is to extract some dynamical knowledge from the variability subspaces and fields computed. This is carried out by studying the spatial distribution of the estimated hydrographic and velocity variances, shapes and amplitudes of dominant multivariate eigenvectors, and structures of (cross)-covariance functions.

The applications illustrated are in the Levantine sea and focus on the mesoscale to subbasin-scale physical errors and fields. The measurements were collected during the LIW95 campaign (Roether *et al.*, 1996; Malanotte-Rizzoli *et al.*, 1996), carried out from January to April 1995 in the Eastern Mediterranean by the Physical Oceanography of the Eastern Mediterranean (POEM) group. To introduce this region and its contemporary upper-thermocline features, the results of a multivariate 3D estimation of the potential density at 105 m and velocity at 5 m are plotted on Fig. 1. Note that several surface circulation features, because of their approximate geostrophic balance, are easily distinguished along the steepest slopes of isopycnals. To date, most investigations in the Levantine that combine data and dynamics have resolved the subbasin-scales and are usually based on this close-to-geostrophy hypothesis (e.g. Hecht *et al.*, 1988; Milliff and Robinson, 1992; Özsoy *et al.*, 1993; Robinson and Malanotte-Rizzoli, 1993; Malanotte-Rizzoli *et al.*, 1999).

There are various techniques for the analysis of geophysical fields, some of which also estimate uncertainties. A relatively comprehensive overview of such techniques and of issues involved in determining accurate *a priori* error statistics is presented in LAL00 and references cited therein. Determining (error) covariance or structure functions, and estimating their parameters and covariance matrix form, is also related to this work. Recent studies include: in atmospheric science, Rabier *et al.* (1998), Riishojgaard (1998), Dee and da Silva (1999), Gaspari and Cohn (1999), Gneiting (1999), Franke and Barker (2000); in physical oceanography, Cressie and Huang (1999), Miller and Cornuelle (1999), Menemenlis and Chechelnitsky (2000), Molinari and Festa (2000); and, in biological oceanography, Abbott and Letelier (1998). Today, error models, in particular the efficient representation of error covariance matrices, are essential for error predictions (Ehrendorfer, 1997; Lermusiaux, 1997, henceforth, LER97) and data assimilation (Robinson *et al.*, 1998; Chin *et al.*, 1999; Derber and Bouttier, 1999; Lermusiaux, 1999a,b; Lermusiaux and Robinson, 1999; Verron *et al.*, 1999; Mitchell and Houtekamer, 2000; Tippett *et al*, 2000).

In what follows, Sect. 2 outlines a simple rule for estimating the size of error subspaces. In Sect. 3, variations of the *a priori* error subspace with its size and with the scales in the data, and the changes in the *a posteriori* fields as a function of these variations are illustrated for multivariate 3D analyses of temperature and salinity. In Sect. 4, such sensitivity studies are carried out, but for the complete physical fields: effects of subspace size, scales considered and nonlinearities in the dynamical adjustments on the subspace itself and on the *a posteriori* fields and errors are investigated in detail. In doing so, the subspaces are also utilized to help derive some dynamical properties (e.g. characteristic scales, dominant balance, dynamical identity) of geophysical features and phenomena estimated. In Sect. 5, the performance of forecasts initialized based on the present field and error mapping scheme is evaluated. The summary and conclusions are in Sect. 6. In the Appendices, algorithms and notation are summarized, relevant scalings and non-dimensional numbers listed, and main parameters of the dynamical model overviewed.

2. THE SIZE OF ERROR SUBSPACES: BACK-OF-THE-ENVELOPE CALCULATION

A simple rule giving an approximate order of magnitude for the size of multivariate error subspaces is outlined. It is based on experience with more rigorous and quantitative calculations (LER97, Lermusiaux, 1999a,b). It is exemplified for the case where errors are the variability that occurs in a region over specific time and space scales.

Consider first one state variable, e.g. temperature or zonal velocity. For the vertical variability, 2 to 3 empirical orthogonal functions (EOFs) or dynamical modes are often found to explain most of the significant variance (e.g. De Mey and Robinson, 1987; Haney *et al*, 1995; De Mey, 1997; Wunsch, 1997; Pedder and Gomis, 1998; von Storch and Frankignoul, 1998). For the horizontal variability, most studies to date focus on surface fields. In that case, the number of dominant patterns or modes usually varies between 15 to about 50: for example, consider a surface field in the Gulf Stream or Atlantic region (e.g. Thacker and Lewandowicz, 1996, 1997; Everson *et al.*, 1997; Kaplan *et al.*, 1997). For the 3D variability, an approximate subspace size is simply the product of vertical and horizontal requirements. With the above values, this product yields a size of 30 to 150.

In the case of several state variables, a simple hypothesis is to assume that the size of the subspace necessary to describe most of a normalized form of the multivariate variability is proportional to the number of variables. For example, considering the primitive equation (PE) model[1] of HOPS, the approximate size of a mesoscale error subspace is then 135 to 650 for the above values, which is of the order of 10^2 to 10^3. In summary, the order of magnitude of the size p of an error subspace is often well approximated by,

$$\mathcal{O}(p) \simeq \sum_v n_h^v \times n_z^v \ , \tag{1}$$

where n_h^v (n_z^v) is an estimate of the horizontal (vertical) requirements for the state variable v. Similarly, a range of likely p is obtained from the products of the minimum and maximum horizontal and vertical requirements, i.e. $\sum_v \min(n_h^v \times n_z^v) \leq p \leq \sum_v \max(n_h^v \times n_z^v)$. Such back-of-the-envelope rules don't require extensive 3D computations: values can be quickly obtained for the specific region and processes considered. They provide quick estimates of the number of state samples, breed vectors or singular vectors (e.g. Molteni *et al.* 1996; Toth and Kalnay, 1997; Barkmeijer *et al.* 1998) that are needed to explain

[1] The PE model employed (App. C) governs five state variables, henceforth called the "PE fields": at any time, it represents four three-dimensional fields, the temperature T, salinity S and internal horizontal velocities $\widehat{u}$ and $\widehat{v}$ (zonal x and meridional y directions), and one two-dimensional field, the barotropic transport stream function ψ.

most of the variance in an analysis or ensemble forecast, and are thus very useful today.

The above sizes, even though large when compared to classic 1D requirements, are small when compared to the number of discrete state variables. To exemplify this property, consider a covariance matrix $\mathbf{B}$ for a horizontal, univariate tracer field. In its non-dimensional form (from here on denoted by *), the covariance function employed is a "Mexican hat" (e.g. Carter and Robinson, 1987; Louis *et al.*, 1997), i.e. the negative of the second derivative of a two-dimensional Gaussian function. Its scalable parameters are the zonal l_a^x and meridional l_a^y zero-crossings, zonal l_b^x and meridional l_b^y e-folding decay-scales and decorrelation time τ. The elements of $\mathbf{B}^*$ are then of the form $C^*(\mathbf{r}_1, \mathbf{r}_2) = (1 - a^2)\ e^{-b^2/2}$, where $a^2 = (\mathbf{r}_1 - \mathbf{r}_2)^T \mathbf{L}_a^{-2} (\mathbf{r}_1 - \mathbf{r}_2)$ and $b^2 = (\mathbf{r}_1 - \mathbf{r}_2)^T \mathbf{L}_b^{-2} (\mathbf{r}_1 - \mathbf{r}_2) + (\Delta t_i / \tau)^2$ are scalars, and $\mathbf{r}_1$ and $\mathbf{r}_2$ denote horizontal position vectors, with $\mathbf{r} = (x, y)$. The 2-by-2 anisotropic scaling matrices contain the local zero-crossings, $\mathbf{L}_a = \mathbf{diag}(l_a^x, l_a^y)$, and local length-scales, $\mathbf{L}_b = \mathbf{diag}(l_b^x, l_b^y)$. The scalar Δt_i is the interval between the data time t_i and estimation time t_0 (App. A). The discretized domain utilized covers the northwestern Levantine (500 km in the x and 520 km in the y direction) with a uniform resolution of 10 km (App. C). For one 2D tracer field, the size of $\mathbf{B}^*$ is thus $[2073 \times 2073]$.

The normalized cumulative eigenvalue spectrum of $\mathbf{B}^*$ corresponding to subbasin-scales ($l_a^x = l_a^y = 200$ km, $l_b^x = l_b^y = 100$ km and $\tau = \infty$) and mesoscales ($l_a^x = l_a^y = 60$ km, $l_b^x = l_b^y = 30$ km and $\tau = \infty$) are given on Fig. 2 (for convenience, scales are chosen homogeneous). Figure 3 shows a few horizontal, non-dimensional eigenvectors for the subbasin-scale case (mesoscale eigenvectors are in LER97 and LAL00). For the subbasin-scales, 15 eigenvectors represent 85% of the variance, 36 vectors 99%, and 50 vectors 99.8%. For the mesoscales, larger numbers are required: 15 eigenvectors represent 20% of the variance, 50 vectors 56%, and 100 vectors 83%. For at least 90% of the variance (see Fig. 2), 19 vectors are necessary for the subbasin-scales and 128 for the mesoscales, which is only 0.7% and 4.7% of the number of discrete state variables, 2703, respectively. Note that in the statistical curves of Fig. 2, all realizations of the 2D subbasin-scale/mesoscale fields are accounted for. However, in the real ocean, during a specified time scale, the dynamical evolution is a constraint. Gyres, meanders or eddies are not everywhere at the same time. Hence, for a given percentage of variance, purely statistical sizes (Fig. 2) are usually upper-bounds on the dynamical size needed at a given time.

3. MULTIVARIATE THREE-DIMENSIONAL ANALYSIS OF TRACER FIELDS: SUBSPACE SIZE

Hydrographic data (dynamical tracers T and S) are frequent observations in oceanography. The sensitivity of the *a posteriori* state $\mathbf{x}_{\mathrm{trc}}^{\mathrm{a}}$ to the size of the *a priori* error subspace is thus first exemplified for the case of a global mapping of temperature and salinity fields. The *a priori* error subspace is constructed assuming that the *a priori* error covariance of these tracers $\mathbf{B}_{\mathrm{trc}}$ is "observed" in the sense of LAL00, i.e. historical (synoptic) data are available to specify its dominant eigendecomposition $\mathbf{B}_{\mathrm{trc}}^p$. Variations of $\mathbf{B}_{\mathrm{trc}}^p$ with the scales considered in the data are illustrated. Some differences between the multivariate 3D scheme and Kalman update (see App. A.4) are also exemplified.

The data to be gridded are 146 T and S profiles. They originated from 49 CTD (Conductivity-Temperature-Depth) and 97 salted AXBT (Aircraft-Deployed Expendable Bathythermographs) observations gathered in the Levantine basin during February 10–18 and on March 15, 1995, respectively[2] (Fig. 4a). Their 5 m mesoscale coverage is estimated on Fig. 4b. The numerical domain (App. C) contains 2073 horizontal grid points and 20 vertical levels (leading to 108120 tracer state variables); its bottom topography is plotted on Fig. 4c. The 146 profiles lead to 5840 data-points (dimension of $\mathbf{y}^{\mathrm{o}}$, eq. A1b), i.e. the number of T, S scalar data residuals on vertical levels. In all examples to follow, the first *a priori* estimate $\mathbf{x}_{\mathrm{trc}}^{\mathrm{b}}$ ($w = 0$ in Apps. A.2–3) is the horizontal average of $\mathbf{y}^{\mathrm{o}}$. The scales to be mapped range from subbasin-scales to mesoscales.

[2] The data manipulations, which include a few horizontal and vertical extensions, and the estimation of the salinities of the AXBTs, are described in LER97.

The univariate 2D mapping scheme of HOPS, employed as a benchmark, assumes *a priori* that two independent scales are present and that errors are homogeneous horizontally. It uses a successive correction, minimum error variance technique (e.g. Daley, 1991; Lorenc, 1992): the largest scales are gridded first and the resulting fields constitute the background in the mapping of the smaller scales. Successively for each scale, global horizontal analyses of the scale-filtered T and S data are thus carried out at various levels.

For fair comparisons with this benchmark, the present multivariate 3D scheme also follows a two-scale successive correction approach ($w = 1, 2$ in Apps. A.2–3) and also assumes horizontally homogeneous *a priori* errors, scale by scale. Two multivariate 3D mappings (App. A.3) are thus carried out successively. For each scale, $\mathbf{B}^p_{\mathrm{trc}}$ is constructed based on a statistical model fit to data in the horizontal and on data EOFs in the vertical (App. A, Table A.a). In this Section, the data used to do so is $\mathbf{y}^{\mathrm{o}}$.

3.1 Subbasin-scale

Univariate 2D benchmark. Figure 5 shows four subbasin-scale ($w = 1$) univariate 2D analyses for T and S. The *a priori* horizontal covariances were specified using a "Mexican hat" (Sect. 2), of zero-crossings $l^x_a = l^y_a = 200$ km, decay-scales $l^x_b = l^y_b = 100$ km and decorrelation time-scale $\tau = 50$ days, all estimated based on the subbasin-scale data residuals. The data error covariance at data-points was assumed diagonal (at each level, $\mathbf{R} = r\mathbf{I}$). The corresponding horizontally uniform, non-dimensional (0 to 1) variance was calibrated to $r^* = 0.03$, mainly based on experience.

Multivariate 3D scheme. The subbasin-scale ($w = 1$) multivariate 3D results are summarized by Figs. 6–7 (construction of $\mathbf{B}_{\mathrm{trc}}$) and Figs. 8–10 (mappings). The dominant eigendecomposition of $\mathbf{C}^z_{\mathrm{trc}}$ (eq. A10) is illustrated (Fig. 6) by its normalized cumulative variance and four dominant T, S eigenvectors, i.e. four EOFs of the subbasin-scale data residuals (eqs. A7–A9). The first multivariate vector (64% of the variance) is surface-intensified, vanishing around 500–600 m, with a slightly larger amplitude in T than in S. For T, the maximum is around 150 m. For S, it is more pronounced and deeper, near 225 m. The dominant subbasin-scale S variability thus occurs at intermediate water depths, below that of T. On average, the vector best explains a vertical projection of the T, S subbasin-scale variability in the northwestern Levantine: e.g. a displacement, strengthening or weakening of the Rhodes Gyre (e.g. Milliff and Robinson, 1992). The second vector (25% of the variance) exhibits T, S values of opposite signs (adding effects in density), a larger amplitude in T than in S and a sustained decrease to zero around 225 m, depth of the S maximum on the first vector. It relates to upper-layer variability during February–March 1995 (Malanotte-Rizzoli *et al.* 1996; LER97). The third vector explains 5% of the variance, with T, S in phase from the surface down to 700–800 m depth (compensating effects in density). Its shape is such that it can for example account for a tilting or stretching/squeezing of the cyclonic gyres and anticyclonic cone-shaped vortices (Brenner, 1993). The fourth vector only explains 2% of the variance.

The horizontal covariance $\mathbf{C}^{\mathbf{r}^*}_{\mathrm{trc}}$ (eq. A11) is computed based on the subbasin-scale Mexican hat used in the univariate 2D analyses. From (eqs. A13–15), the subbasin-scale vertical (eq. A10) and horizontal (eq. A12) decompositions are then combined. The resulting normalized variance is explained by a few hundred 3D vectors, as shown on Fig. 7. Using (eq. A16) in (App. A.3), the subbasin-scale mapping ($w = 1$) is finally carried out.

Figure 8 plots parts of the state $\mathbf{x}^{\mathrm{a}}_{\mathrm{trc}}$ for the case where $p = 500$ in (eq. A16), i.e $\mathbf{B}_{\mathrm{trc}}$ in (eq. A13) is limited to its dominant 500 eigenvectors which explain 99.6% of the *a priori* error variance (Fig. 7). The size of $\mathbf{x}^{\mathrm{a}}_{\mathrm{trc}}$ being 108120, this analysis is 216 times cheaper than a full covariance formulation (App. A.4) and about 12 times cheaper than a Representer method (Bennett and Chua, 1994), the number of Representers being 5840. For example, the elapsed-time was 15 minutes on a Sun Ultra and the full formulation would have thus lasted about 2.25 days (see LAL00 for more on computational costs). Comparing results (Figs. 8 and 5), the T, S maps are alike. For the subbasin-scale, the assumptions of zero vertical covariance and of uncorrelated T and S error fields does not appear restrictive. This is mainly because there is here more than enough data coverage for the subbasin-scale. The vertical and T–S correlations are as expected stronger with the

3D scheme, but the differences between fields of Figs. 5 and 8 are within data error bounds.

Figure 9 shows the subbasin-scale $\mathbf{x}^{\mathrm{a}}_{\mathrm{trc}}$ for the case where p is only 250 in (eq. A16). The relative Root Mean Square (RMS) differences between the T, S maps of Fig. 8 (500 vectors) and Fig. 9 (250 vectors) are less than 1%. This is an interesting result. Once most (e.g. here $\geq 98\%$, see Fig. 7) of the error covariance has converged, the remaining eigenvectors can be ignored if their eigenvalues are negligible in comparison to data error eigenvalues (see App. A.4). Considering costs, the mapping with $p = 250$ would be 10^3 times cheaper than the full covariance mapping. Finally, using 10 vectors as on Fig. 10 is not sufficient. Indeed, for 99% of the variance, the quick product of the horizontal and vertical requirements (Sect. 2) indicates here a total of $36 \times 8 = 288$ vectors (see eq. 1 and use Fig. 2a and Fig. 6), i.e. an order of 10^2 multivariate 3D vectors. This back-of-the-envelope calculation agrees with the numerically computed spectra of Fig. 7.

3.2 Mesoscale

Univariate 2D benchmark. Figure 11 shows four T and S mesoscale ($w = 2$) univariate 2D analyses. Zero-crossings were estimated based on data at $l^x_a = l^y_a = 60$ km, decay-scales at $l^x_b = l^y_b = 30$ km and decorrelation time-scale at $\tau = 7$ days. The non-dimensional data error variance r^* for $\mathbf{R} = r\mathbf{I}$ was calibrated to 0.03, as in Sect. 3.1.

Multivariate 3D scheme. In the mesoscale 3D mapping, $\mathbf{x}^{\mathrm{b}}_{\mathrm{trc}}$ is the subbasin-scale state of Fig. 8. The construction of the dominant mesoscale covariance $\mathbf{B}_{\mathrm{trc}}$ is illustrated by Fig. 12. In the vertical (Fig. 12a), the variance of the mesoscale data residuals (eq. A10) decays less rapidly with the eigenvalue number than in the subbasin-scale case (Fig. 6). In the horizontal, $\mathbf{C}^{\mathbf{r}^*}_{\mathrm{trc}}$ (eq. A11) is computed based on the mesoscale Mexican hat used in the univariate 2D analyses. From (eqs. A13–15), the mesoscale vertical (eq. A10) and horizontal (eq. A12) decompositions are then combined. As shown on Fig. 12b, the resulting normalized mesoscale variance is mostly explained by a few 3D vectors, but compared to the subbasin-scale (Fig. 7), slightly more vectors are needed. Using (eq. A16) in (App. A.3), the mesoscale mapping ($w = 2$) is finally carried out.

Figures 13–14 show $\mathbf{x}^{\mathrm{a}}_{\mathrm{trc}}$ obtained using the dominant 500 and 750 mesoscale vectors, respectively. Differences with the fields of Fig. 11 are small, at several locations within data error bounds. However, the 3D scheme which uses the significant EOFs of the mesoscale residuals (eqs. A7–10) yields larger vertical correlations in $\mathbf{x}^{\mathrm{a}}_{\mathrm{trc}}$ than the 2D scheme does. For example, in the upper 0 to 50 m mixed-layer, the measurement noise and unresolved vertical scales are filtered more on Figs. 13–14. Small mid-depth features of significant vertical extension, e.g. see T, S around 225 m, also appear better represented. From our experience, for mesoscale mappings with data resolutions similar to those of Fig. 4a, a disadvantage of the univariate 2D analysis is in fact its sensitivity to environmental noise. Finally, considering mesoscale error subspace sizes, the relative RMS differences between maps of Figs. 13 and 14 are less than 3%. Even though 750 vectors explain 5% more *a priori* mesoscale variance than 500 vectors do (Fig. 12b), most of the additional 250 vectors have eigenvalues smaller than the local data error variance. They thus mainly yield mesoscale corrections within data error bounds.

For the 3D scheme, two main properties (see App. A.4) have been exemplified. First, once most of the *a priori* error covariance has converged (e.g. 99%), the remaining eigenvectors can be truncated in most cases. Second, the *a priori* error vectors of eigenvalues negligible with respect to data errors have negligible impacts on the *a posteriori* state. Even if the *a priori* error covariance has not converged, these vectors can usually be omitted.

4. MULTIVARIATE THREE-DIMENSIONAL ANALYSES OF PE FIELDS

Primitive equation fields and their errors are now considered. The examples shown are still for the northwestern Levantine basin, but the focus is on the period from March 19 to April 16, 1995 (Sect. 3 considered tracer fields and only used data collected before and on March 15). Four mapping cases are discussed (Table 1): one univariate 2D benchmark and three variations of the multivariate 3D scheme.

Table 1. The four analyses selected and their main differences

Case	Scheme Employed	Size of *a priori* Error Subspace	Velocity Covariances	Velocity Dynamics Utilized
1:	Univariate 2D (benchmark)	full covariances (all 2D vectors)	Not built: direct integration of thermal-wind equations	Geostrophy
2:	Multivariate 3D	$p = 322$	Built via an ensemble of adjustment PE integrations	Linear momentum PE model
3:	Multivariate 3D	$p = 322$	As above	Nonlinear momentum PE model
4:	Multivariate 3D	$p = 400$	As above	Nonlinear momentum PE model

The univariate 2D benchmark (Case 1) first carries out horizontal analyses of the T and S data. This is done in two successive scale corrections, as in Sect. 3. The total velocity is then computed by assuming geostrophic balance with the gridded tracers and integrating the thermal-wind equations up and down from a level of reference.

For fair comparisons, the present multivariate 3D scheme (Cases 2–4) also uses a two-scale successive correction approach ($w = 1, 2$ in App. A.2) to construct the *a priori* error subspace. For each scale, the tracer portion $\mathbf{B}_{\rm trc}$ of the *a priori* error covariance $\mathbf{B}$ is specified as in Sect. 3. The non-observed total velocity portions are then built in statistical and dynamical accord with $\mathbf{B}_{\rm trc}$, through an ensemble of adjustment momentum integrations (App. A, Table A.b). These integrations lead to an ensemble of PE adjusted fields. Variability samples, i.e. the differences between these fields and the *a priori* state $\mathbf{x}^{\rm b}$, are then computed, normalized and organized by singular value decomposition (SVD). Once a convergence criterion determines that the number of samples is large enough to explain most of the variance, an estimate $\mathbf{B}^p$ of the dominant components of $\mathbf{B}$ is obtained. In building this covariance, for the subbasin-scales, $\mathbf{x}^{\rm b}$ is the horizontal average of the tracer data, with null velocity; for the mesoscales, $\mathbf{x}^{\rm b}$ is the result of the subbasin-scale mapping.

The multivariate 3D cases of Table 1 were selected to illustrate the main tendencies in an extensive set of computations. They exemplify the sensitivities of the error subspace and state $\mathbf{x}^{\rm a}$ to: (i) the nonlinear terms of the momentum equations used in the adjustment of the non-observed velocity variability to the observed tracer variability (Cases 2 and 3), and (ii) the size of the subspace, i.e. the number p of *a priori* error eigenvectors utilized in the mapping (Cases 3 and 4). Case 4 is the mapping briefly exemplified in LAL00.

In the remainder of this Section, the PE field and error covariance estimates of the four cases are compared and studied. For conciseness, the subbasin-scale mappings are briefly overviewed in Sect. 4.1; the detailed presentation is restricted to the mesoscale stage $w = 2$. The estimates of the *a priori* error covariance for the mesoscale fields, $\mathbf{B}^p = \mathbf{E}\boldsymbol{\Pi}\mathbf{E}^T$, are compared in Sect. 4.2; the *a posteriori* fields, in Sect. 4.3; and the *a posteriori* mesoscale error covariances, $\mathbf{B}^{{\rm a}^p} = \mathbf{E}^{\rm a}\boldsymbol{\Pi}^{\rm a}{\mathbf{E}^{\rm a}}^T$, in Sect. 4.4. The dynamical possibilities raised by the study of error and variability subspaces are emphasized in Sects. 4.2 and 4.4.

4.1 Subbasin-scale

For each case (Table 1), the data $\mathbf{y}^{\rm o}$ employed for the subbasin-scale stage are 512 profiles of the MODB (Med5) winter climatology for the Levantine (Brankart, personal communication) and the T/S profiles obtained from the *in situ* CTD and AXBT data gathered during February 10–18 and March 15, 1995 (Fig. 4a). For coherence with the recent evolution of the deep Eastern Mediterranean waters (Roether *et al.*, 1996), the MODB data were cut at 1250 m depth: only *in situ* data are utilized below 1250 m. In all cases, $\mathbf{x}^{\rm b}$ is in hydrostatic equilibrium: its tracer fields are the horizontal averages of the data $\mathbf{y}^{\rm o}$ and its velocity is null.

For the univariate 2D scheme (Case 1), the tracer fields are estimated from $\mathbf{y}^{\rm o}$ as in Sect. 3.1, using the same *a priori* horizontal error covariances of T and S. The total velocity fields are assumed in geostrophic balance with these gridded tracers, from a level of no motion at 600 m (since the 2D scheme is univariate and geostrophy is linear, velocities are only computed after the mesoscale tracer analyses, Sect. 4.3). For the multivariate 3D Cases 2–4, the $\mathbf{B}^p_{\rm trc}$'s are also as in Sect. 3.1, except for the variations of the size p. Three estimates of the subbasin-scale $\mathbf{B}^p$ are then built as indicated in Table 1, using the PE model of App. C. With these $\mathbf{B}^p$'s, three mappings of the data $\mathbf{y}^{\rm o}$ for this first stage are finally carried out, following (App. A.3).

4.2 Mesoscale: data and *a priori* error covariance

For each case (Table 1), the data to be gridded for the mesoscale correction are 236 CTD profiles gathered from March 19 to April 16 (Fig. 15). They lead to 9440 data-points (dimension of $\mathbf{y}^{\rm o}$). The field estimation is centered on March 27.

The four $\mathbf{x}^{\rm b}$'s (Cases 1 to 4) are the four subbasin-scale fields of Sect. 4.1; they contain no mesoscale by construction. Their *a priori* error is thus mesoscale variability: i.e. for Cases 2 to 4, $\boldsymbol{\Gamma} = \mathbf{I}$ in (eq. A23). The observations used to estimate the corresponding covariances consist of the 289 CTD profiles gathered from Feb. 10 to April 16; they include the data $\mathbf{y}^{\rm o}$ and are denoted by $\mathbf{y}^{\rm o}_{\rm ctd}$. These four covariance estimations are now described.

Univariate 2D benchmark. The *a priori* horizontal error covariances of T and S are specified using a Mexican hat, as in Sect. 3.2. Its zero-crossings and decay-scales are calibrated on basin average to the mesoscale residuals of the 289 CTD profiles, which yields $l^x_a = l^y_a = 60$ km and $l^x_b = l^y_b = 30$ km, respectively. The non-dimensional data error variance r^* for $\mathbf{R} = r\mathbf{I}$ is adjusted to 0.03. Velocity errors are not computed; velocities are directly estimated from geostrophic balance with a level of no motion at 600 m.

Multivariate 3D scheme. In the construction of the mesoscale $\mathbf{B}^p = \mathbf{E}\boldsymbol{\Pi}\mathbf{E}^T$, the dominant decomposition of its tracer submatrix, $\mathbf{B}^p_{\rm trc} = \mathbf{E}_{\rm trc}\boldsymbol{\Pi}_{\rm trc}\mathbf{E}^T_{\rm trc}$, is computed first (App. A, Table A.a). This is done by combining the vertical EOFs of the mesoscale tracer residuals $(\mathbf{y}^{\rm o}_{{\rm ctd}_i} - \mathcal{H}_i(\mathbf{x}^{\rm b})$, $i = 1, \ldots, 289$, eq. A7) with the eigendecomposition of an analytical horizontal tracer covariance (of elements specified using the mesoscale Mexican hat of the benchmark, see above). The resulting cumulative error variance is a curve similar to that of Fig. 12b (it differs slightly because of the different $\mathbf{x}^{\rm b}$ and data $\mathbf{y}^{\rm o}_{\rm ctd}$ employed). For the sizes p utilized (Table 1), the dominant 322 tracer eigenvectors explain 67.2% of the total tracer variance, the dominant 400, 72%, and dominant 500, 76.5%. The complete $\mathbf{B}^p$'s are then obtained from $\mathbf{B}^p_{\rm trc}$, by adjustment momentum integrations (App. A, Table A.b). The $j = 1, \ldots, r$ dynamical adjustments lasted for 2 model-days, with r increasing based on the convergence criterion $\rho \geq \alpha$ (eq. A22), with the limit α set to 0.93. This coefficient ρ assesses the added value of new parallel batches of adjustment runs; some of its values are listed in Table 2.a.

For Case 2, $\rho \geq \alpha$ was achieved after 322 runs, for $p = 322$ and $\tilde{p} = 288$. For Case 4, 400 runs were necessary, based on $p = 400$ and $\tilde{p} = 360$. This larger dimension is due to the nonlinear momentum terms which create flow variability not estimated by the linear adjustments of Case 2. Since Case 3 is simply the nonlinear extension of Case 2, convergence is not an objective. Overall, the rate of increase of ρ diminishes as $\tilde{p} \leq p$ grows: this is partly because the singular values of $\mathbf{B}$ decay rapidly.

Table 2.b estimates the overall similarity among the three $\mathbf{B}^p$'s, using (eq. A22). Comparing Cases 2 and 3, averaged on volume and state variables, nonlinear momentum terms only change about 2.7% of the global, non-dimensional PE mesoscale variability. In the present examples, since $\mathbf{u}^j = \sqrt{q}\mathbf{e}^j$ in (eq. A17), the tracer covariance in a $\mathbf{B}^p$ is exactly $\mathbf{B}^p_{\rm trc}$. Therefore, the covariances involving total velocities in the $\mathbf{B}^p$'s of Cases 2 and 3 differ by about $(1 - \rho)n/n_{\rm vel} = 5.4\%$, where n is the state vector size (218,943) and $n_{\rm vel}$ the number of velocity variables (110,823). This 5.4% difference is an indicator of the volume-averaged importance of the nonlinear momentum terms in estimating the total velocity covariance. Its local variations will be exemplified in Sect. 4.2.a. Comparing Cases 3 and 4, they are about 13.2% apart. The tracer eigenvectors of high number (small eigenvalues) here still affect the error subspace estimate[3]. Finally, comparing Cases 2 and 4, their differences combine the nonlinear and size effects (2.7% and 13.2%).

[3] This 13.2% rise is partly due to the 4.8% increase in normalized 3D tracer variance explained (from 67.2% at $p = 322$ to 72% at $p = 400$), see above.

Table 2. Convergence of and similarity between error subspaces

a. Convergence coefficient			
for Case 2:	$\rho_{254-288} = 0.919$	$\rho_{288-322} = 0.930$	
for Case 3:	same values as for Case 4.		
for Case 4:	$\rho_{288-322} = 0.928$	$\rho_{322-360} = 0.929$	$\rho_{360-400} = 0.935$
b. Similarity coefficient			
between Cases 2–3:	$\rho = 0.973$		
between Cases 2–4:	$\rho = 0.845$		
between Cases 3–4:	$\rho = 0.868$		

In what follows, the three $\mathbf{B}^p = \mathbf{E}\Pi\mathbf{E}^T$'s (Cases 2–4) are further intercompared by discussing and studying their error variances (Sect. 4.2.a), covariance eigenvectors (Sect. 4.2.b) and covariance functions (Sect. 4.2.c). These estimates give an important advantage over the univariate 2D scheme (Case 1) which only provides 2D error variance fields.

4.2.a A priori mesoscale error variance

Univariate 2D benchmark. The *a priori* tracer error variances are set to the horizontally-averaged variance of the residuals of $\mathbf{y}^{\mathrm{o}}_{\mathrm{ctd}}$: they are depth-dependent. The corresponding velocity error variances are set to these of velocities in geostrophic balance with the tracers.

Multivariate 3D scheme. The error variances form the diagonal of the $\mathbf{B}^p$'s. For fair comparisons with the benchmark, tracer variances are horizontally uniform in the data domain (Fig. 15). For Cases 2 and 3 ($p = 322$), the T (S) error standard deviation at 5 m is 0.40°C (0.080 PSU), while at 500 m, it is 0.17°C (0.039 PSU). For Case 4 (see LAL00), standard deviations are logically slightly larger: to first order[4], using the variance numbers given above, by 3.3% to 3.6% ($100\times(72-67.2)/2$, divided by 72 and 67.2). The total velocity variances are however non-uniform in 3D. By construction, large *a priori* errors (Table 3) correspond to the locations and features (Fig. 1) estimated to have large mesoscale flow variability.

For Case 2 (linear momentum), at the surface, the largest standard deviations are as expected above the Rhodes basin (flanked by steep topography) within the deep, subbasin-scale Rhodes Gyre. Other features of above average flow variability are, in decreasing order, the Ierapetra, Mersa Matruh Gyre (one of its lobes is in the domain) and central branch of the Mid-Mediterranean Jet. At 500 m depth, the largest $\widehat{u}, \widehat{v}$ deviations are near the inflow/outflow of the Kasos and Karpathos Straits, and again above the Rhodes basin.

For Case 3 (nonlinear momentum, $p = 322$), at the surface, it is not the Rhodes basin, but the high velocity, smaller size Ierapetra which is estimated to dominate. With respect to Case 2, the nonlinear correction in standard deviation for the Ierapetra is 50% for the surface internal velocity and 10% for the barotropic transport (Table 3). For the larger Mersa Matruh Gyre, the correction is smaller, especially in ψ (30% more for $\widehat{u}, \widehat{v}$, but only 5% more for ψ). For the Mid-Mediterranean Jet and subbasin-scale Rhodes Gyre, the nonlinear terms barely increase the barotropic and surface flow variabilities. At 500 m depth, as for Case 2, the two Straits and Rhodes basin dominate the internal deviations, with a nonlinear correction of 3% and 7.7%, respectively. However, Case 3 also estimates that the Ierapetra and an anticyclone of the Mersa Matruh-Shikmona Gyre (MM-S-G) complex have above average internal deviations at 500 m. For these deep mesoscale vortices, nonlinear terms are thus important at depth. Overall, the average increase in standard deviation is 3% for ψ, 8% for the surface $\widehat{u}, \widehat{v}$, and 4.6% for the 500 m $\widehat{u}, \widehat{v}$ (percentages relative to Case 2). These upper-ocean computations agree with the volume-averaged velocity variance increase of 5.4% (estimated above from Table 2.b). For Case 4 (nonlinear momentum, $p = 400$), results are similar to these of Case 3, except that standard deviations are a bit larger (LAL00), to first order and overall, by about 6.6% to 7.6% (13.2/2 to 13.2/2/0.868, see footnote 4).

[4] Assuming $x_2 > x_1 > 0$, to link ratios of standard deviations to ratios of variances, and inversely, one can use: $\frac{x_2^2 - x_1^2}{2x_2^2} \leq \frac{x_2 - x_1}{x_2} \leq \frac{x_2^2 - x_1^2}{2x_1^2}$.

Table 3. *A priori* mesoscale error standard deviations (St.Dv.) for velocity: estimates of Cases 2 and 3/4

Case	Depth	Features (and horizontal averages)	ψ error St.Dv.	Internal velocity error St.Dv.
2.	Surface (5 m)	Above the Rhodes basin:	0.85 Sv	15.5 $\mathrm{cm\,s^{-1}}$
		Ierapetra:	0.8 Sv	14.5 $\mathrm{cm\,s^{-1}}$
		Mersa Matruh Gyre:	0.8 Sv	13 $\mathrm{cm\,s^{-1}}$
		Mid-Mediterranean Jet:	0.68 Sv	13.5 $\mathrm{cm\,s^{-1}}$
		Surface average:	0.65 Sv	12.5 $\mathrm{cm\,s^{-1}}$
	500 m	Kasos and Karpathos Straits:		1.7 $\mathrm{cm\,s^{-1}}$
		Above the Rhodes basin:		1.3 $\mathrm{cm\,s^{-1}}$
		500 m average:		0.65 $\mathrm{cm\,s^{-1}}$
3./4.	Surface (5 m)	Ierapetra:	0.87/0.9 Sv	22/23 $\mathrm{cm\,s^{-1}}$
		Above the Rhodes basin:	0.87/0.9 Sv	16/16 $\mathrm{cm\,s^{-1}}$
		Mersa Matruh Gyre:	0.84/0.86 Sv	17/18 $\mathrm{cm\,s^{-1}}$
		Mid-Mediterranean Jet:	0.69/0.72 Sv	14/15 $\mathrm{cm\,s^{-1}}$
		Surface average:	0.67/0.7 Sv	13.5/14 $\mathrm{cm\,s^{-1}}$
	500 m	Kasos and Karpathos Straits:		1.75/1.8 $\mathrm{cm\,s^{-1}}$
		Above the Rhodes basin:		1.4/1.5 $\mathrm{cm\,s^{-1}}$
		Ierapetra:		0.95/1 $\mathrm{cm\,s^{-1}}$
		Anticyclone in MM-S-G complex:		1/1.05 $\mathrm{cm\,s^{-1}}$
		500 m average:		0.68/0.7 $\mathrm{cm\,s^{-1}}$

Two general remarks can be made: (i) for accurate flow variability variances, not only the background density field, but also the regional topography, nonlinear and diffusion terms (App. C) should be considered; (ii) even though subspaces are used, the present global estimates of mesoscale variances are in accord with and complement previous local estimates (e.g. Özsoy *et al.*, 1993; Robinson and Golnaraghi, 1993).

4.2.b A priori mesoscale error eigenvectors and spectrum

Case 2. Figure 16 illustrates the first and second non-dimensional singular vectors, columns of $\mathbf{E}^*$ in (eqs. A21–23). As expected from the variance results (Sect. 4.2.a), for linear momentum dynamics, the dominant mesoscale variability during early Spring correspond to large-mesoscale variations of the Rhodes Gyre. In the past few decades, the Rhodes Gyre has indeed been observed to be the main subbasin-scale feature of the region (e.g. Milliff and Robinson, 1992; Nittis and Lascaratos, 1998). The two vectors shown explain 1.08% and 1.05% of the variance explained by the 322 vectors, respectively.

The first vector (Panels 1a–b) covers the principal core of the Gyre, above the Rhodes basin. Horizontally (Panel 1a), the main T and S structures are almost axisymmetric, with horizontal decay-scales of about 40 km (radius of 60 km). Their shapes are close to large-mesoscale Mexican hats. On the open-ocean side, the main ψ structure is also close to a Mexican hat, indicating a predominant geostrophic equilibrium (from the isocurve $\psi = 0$, the ratio of minimum to maximum $\Delta\psi$ is about 4, as for the tracers). Close to the coastline, the $\Delta\psi$'s vanish, mainly because of decreasing depth and increasing coastal friction. Vertically, the T structure is surface intensified (Panel 1b), with an uniform extremum in the upper 20 to 200 m depth and a significant extension down to about 900–1000 m above the Rhodes basin. The S structure has a subsurface extremum within 250 m to 320 m and one significant zero-crossing near 100 m. The signs of T and S structures reverse at about 1500 m and 1300 m, respectively. Below, the largest T and S amplitudes are each near 2300 m, but these local extrema are much smaller than the global upper-thermocline extrema (about 100 and 20 times smaller, respectively). Overall, where T and S are both relatively large, they are in phase, compensating each other in density (at 5 m, they are in opposition, but T strongly dominates). The surface $\widehat{u}$ (Panel 1a) and cross-sections in $\widehat{u}$ and $\widehat{v}$ (Panel 1b) confirm the predominant geostrophic equilibrium. In addition to the two main lobes, a weak topographic wave pattern is visible in the $\widehat{u}$ map and sub-surface tracer cross-sections. Its average amplitudes are about 20% of those of the two main lobes. Vertically, the $\widehat{u}$ and $\widehat{v}$ structures reverse sign near 700 m. The extrema below this reversal are at about 1500 m, but their amplitudes are 10 times smaller than the surface ones.

Using ψ (Panel 1a), a minimum in total velocity is expected near 900 m.

The second vector (Panels 2a–b) mainly relates to displacements of the core of the Gyre, along the north-south axis of the Rhodes basin. Except for the main double lobes in T, S and ψ, and corresponding triple lobes in $\widehat{u}$ and $\widehat{v}$, most properties are as those of the first vector: e.g. ψ, T and S horizontally in phase where they are all important; depths of zero-crossings for T, S, $\widehat{u}$ and $\widehat{v}$ overall unchanged; predominant balance for ψ, T, S, $\widehat{u}$ and $\widehat{v}$ remains geostrophic equilibrium, etc. Nonetheless, such a displacement of the Gyre's core is logically suggested to interact more with topographic wave patterns (e.g. Smith, 1983) than in the first vector. Along the northern escarpment, the dominant wavelength of the pattern (Panels 2a–b) is about 160 km (scale of 25 km). Possible wave types and time scales are further discussed below.

The other dominant vectors (LER97) also relate to the main features (Fig. 1 and Table 3). This is because to explain a specific variation of, or interaction among, features, a group of vectors is usually required. Considering for example the vectors 3 to 16, their largest amplitudes assign the vectors: (5–9,11–16) to the Ierapetra; (3–7,9,12–16) to the Rhodes Gyre; (3–6,13–14) to wave patterns along the northern escarpment and Mid-Mediterranean Ridge; (12–14) to the Mersa Matruh Gyre; (8–16) to the Mid-Mediterranean Jet; (3–4,6,9–16) to the West Cyprus Gyre; (4,6,9,12–16) to the Asia Minor Current; and (9–10,12–16) to the Mersa Matruh-Shikmona Gyre complex. Since each vector is constructed to be orthogonal to all the more dominant ones and 3D dynamical couplings can be complex, a detailed subspace analysis requires additional computations (rotations, projections, wavelets, etc) beyond the present scope. Considering finally the rate of decay of the error spectrum, the error variance explained by the 322 vectors is distributed such that the dominant 50, 100 and 200 vectors explain 41.4%, 66.7% and 89.4% of the total variance, respectively.

Case 3. Figure 17 is as Fig. 16, but for nonlinear momentum adjustments (Table 1). In accord with the variance results (Table 3), the feature of dominant mesoscale variability is then the Ierapetra. The two vectors shown explain 1.59% and 1.24% of the variance explained by the 322 vectors, respectively. These percentages are larger than in Case 2 because nonlinear terms steepen the start and flatten the upper-middle of the eigenvalue spectrum. The first vector corresponds to the vortex itself, the second to eastward-westward displacements (for the sign plotted, it is a westward motion).

In comparison with Case 2, the horizontal T and S structures of the first vector (Panel 1a) are, when significant, closer to Mexican hats (almost no wave patterns) and of lesser width (radius near 50 km, decay-scale of 30 km). The lobes far from the vortex in the surface S are mainly due to the truncation and have a maximum amplitude that is insignificant based on $\rho = 0.928$ (Table 2.a). They are excited by radiating nonlinear effects (absent in Case 2) and can be seen where amplitudes are relatively small. Vertically (Panel 1b), T and S are more stretched than in Case 2 (Fig. 16), as one would expect for a ring-like vortex. The T is still surface intensified, but its uniform extremum is deeper, within 50 to 250 m. This agrees with previous results: e.g. the shallow surface mixed-layer of the Rhodes Gyre (e.g. Malanotte-Rizzoli *et al.*, 1996) and the structure of Levantine warm core eddies (e.g. Brenner, 1993). The dominant feature in T extends down to about 600 m, main depth of the present Ierapetra, instead of about 1000 m for the Rhodes Gyre (Fig. 16). The main structure in S has a zero-crossing near 80 m and a subsurface extremum within 250 m to 340 m. The sign in T/S reverses at about 1300 m/1100 m, which is shallower than for the Rhodes Gyre. The extrema below are also shallower, at about 1700 m for both T and S, but they remain smaller (50 and 20 times, respectively) than the upper-thermocline extremum. Overall, T and S are still in phase when they are both important. The ψ component (Panel 1a) is less close to geostrophic equilibrium than on Fig. 16, as shown for example by the asymmetries of its pattern. The $\widehat{u}$ and $\widehat{v}$ components (Panels 1a and 1b) confirm this fact. The nonlinear, inertial and viscous terms of the momentum equations as well as topography modify the classic geostrophic antisymmetries of $\widehat{u}$ and $\widehat{v}$ with respect to the x and y axes. The horizontal lobes (Panel 1a) present sharp gradients and are tilted, with their heads wider than their tails. The maximum velocity is reached near a radius of 30 km. Vertically (Panel 1b), $\widehat{u}$ and $\widehat{v}$ reverse sign near 400 m, indicating a minimum in total velocity near 600 m based on ψ (Panel 1a). The $\widehat{u}$ and $\widehat{v}$ extrema below this reversal are near 1600 m and deeper, but their amplitudes are about 10 percent of the surface ones. The variations of the deep counter-flow and corresponding density feature are thus weak. However, they have a large vertical extent (here 1500 m) and can influence the stability of the Ierapetra, as found by forecast sensitivity studies in LER97 and argued in general by Dewar and Killworth (1995) and Killworth, *et al.* (1997).

Considering the second vector on Fig. 17, displacements of the Ierapetra can excite radiative momentum patterns (Panels 2a–b). Within the fixed mesoscale tracer variability, these adjustment patterns logically correlate most to density variations in quasi-geostrophic balance, hence the T and S radiation patterns on Panels 2a–2b. The horizontal $\widehat{u}$ is close to a "45 degree shamrock" which indicates a thermal-wind balance (Daley, 1991). However, at all depths, $\widehat{u}$ and $\widehat{v}$ have clear asymmetries and amplitude variations which result from local peculiarities of the topography, density, nonlinearity and bottom friction (see dynamics below). In the vertical, most properties are as those of the first vector.

Since the two dominant vectors are modified, the others also differ, even though most of subspace itself remains unchanged (see Table 2.b). Considering again the vectors 3 to 16, their largest amplitudes assign the vectors: (3–6,8,14–16) to the Ierapetra; (6–16) to the Rhodes Gyre; (4,5,8) to wave patterns along the northern escarpment and Mid-Mediterranean Ridge; (3–4,7–10,12) to the Mersa Matruh Gyre; (8–12,14–16) to the Mid-Mediterranean Jet; (10–14,16) to the West Cyprus Gyre; (10,12,16) to the Asia Minor Current; and (8–16) to the Mersa Matruh-Shikmona Gyre complex. Overall, the dominant 50, 100 and 200 vectors explain 41.1%, 66.2% and 89.4% of the error variance explained by the 322 vectors, respectively. These percentages are close to these of Case 2: the result is that here nonlinear terms impact the dominant vectors of the subspace but don't change much of the overall spectrum.

Case 4. The error subspaces of Cases 3 and 4 were estimated to be similar at 86.8% (Table 2.b). Up to an arbitrary sign, their dominant vectors 1 to 15 are almost identical (the vectors for Case 4 are only slightly less noisy and are thus not shown). In particular, the two dominant vectors (LAL00) confirm that the Ierapetra dominated the nonlinear mesoscale variability during the Spring of 1995. The vectors of Cases 3 and 4 only start to differ somewhat by vector 16, mainly in the small mesoscales. The additional vectors 323 to 400 also include 3D patterns of relatively large horizontal scales and usually "wiggly" vertical structures, or vice-versa. This agrees with the characteristics of the tracer vectors 323 to 400 used in the adjustment integrations and indicates that the ordering of scales in the Levantine sea is complex. The decay of the error spectrum is similar to that of Case 3: the 50, 100 and 200 dominant vectors explain 38.4%, 62.7% and 85.2% of the error variance explained by the 400 vectors, respectively.

Dynamical properties. From the dominant mesoscale vectors and fields obtained, the types and scales of possible linear wave patterns along the northern escarpment and Mid-Mediterranean Ridge (e.g. Case 2), and the dynamical balance of the Ierapetra (e.g. Cases 3–4) can be briefly investigated. To do so, some scaling estimates are necessary. Note that the values obtained from Cases 3 and 4 are similar, indicating a robustness of the results.

Using the definitions of App. B and Fig. 16, locally for a wave packet along the escarpment, $\rho_0 = 1028$ kg/m^3, $L = 25$ km, D is about 1000 m, H is on Fig. 4c, $f_0 = 8.45\ 10^{-5}\,\mathrm{s}^{-1}$ and $\beta_0 = 1.86\ 10^{-11}\,\mathrm{m}^{-1}\,\mathrm{s}^{-1}$ (for $\theta_0 = 35.5$). Computing a reference $\rho_s(z)$ by horizontal average of the density field, this leads an average N_s within D of about $2.5\ 10^{-3}\,\mathrm{s}^{-1}$, a local $R_D = N_s D/f_0 \simeq 30\,\mathrm{km}$ and $R_{\mathrm{ext}} \simeq 1200\,\mathrm{km}$. For U, based on the second vector (Fig. 16, Panels 2a–b), the sum of the surface internal and external components yields: $U \simeq (3.9\times 1.5\ 10^{-2} + 5.6\ 10^{7}\times 0.5\ 10^{-2}/H/L)\sqrt{1893} \simeq 3\,\mathrm{cm\,s}^{-1}$ ($1.5\ 10^{-2}$ and $0.5\ 10^{-2}$ are the non-dimensional amplitudes, 3.9 and $5.6\ 10^{7}$ the normalization factors, and 1893 the eigenvalue). With contributions from other relevant vectors, the characteristic velocity U for a wave packet is about $5\,\mathrm{cm\,s}^{-1}$. Hence, the local values of the non-dimensional numbers are (see App. B): $Ro = 0.015$, $Bu_{\mathrm{ext}} = 2\ 10^3$, $Bu = 1.4$, $\beta = 0.23$, $\alpha \simeq 25$ and $s \simeq 0.6$ (excepted just east of the island of Rhodes where α and s reach 50 and 1, respectively). The small Ro and large Bu_{ext} indicate that the rigid-lid linear PE regime is an acceptable approximation. From β, the planetary vorticity gradient is relatively small and from $\alpha \gg 1$, the topographic vorticity gradient is large: β_0 can be neglected compared to $|\nabla_h H|\frac{f_0}{D}$. From $Bu \sim 1$, the length-scales of the horizontal flow and internal stretching (stratification's strength) match. Since s is close to one, quasi-geostrophy may not apply. Finally, for the model topography (Fig. 4c), s and α are relatively uniform along the escarpment waveguide (about 60 km wide). Based on these numbers and interpretations, the topographic wave packets are of the coastal-trapped type (Huthnance, 1992; Pedlosky, 1987; Brink, 1991). Note that here, wave patterns arise during rigid-lid momentum adjustments (fixed tracer disturbances): for the barotropic mode along the escarpment, to first-order in s, the frequency is then of the order $\frac{|\nabla_h H| f_0/D}{k}$, where $k = 1/L$ is the wave number. This leads a period of about 1.45 days, i.e. of the order of 1 to 2 days.

In the ocean, wave packets would be generated by complete dynamical adjustments, and also by winds, atmospheric pressure and coastal flows. A complex wave spectrum around the above barotropic values can be expected. First, the natural topography is more jagged and at several locations steeper than the model topography, which we estimate could shift the period by about 50%. Second, several wave types are possible along the escarpment and/or Mid-Mediterranean Ridge: in addition to a suite of coastal-trapped waves (e.g. Leblond and Mysak, 1971) like Kelvin waves (internal, external or double) and continental shelf waves (barotropic or baroclinic), inertial and Rossby waves can be generated. For complete statements on scales, shapes and forcings, additional observational, analytical and numerical studies are necessary (e.g. Huthnance, 1978; Holland and Webster, 1994). The dominance of such wave patterns and associated mesoscale variations in the vectors related to the Rhodes Gyre (see Cases 2–4) suggests that they could be in a large part responsible for the observed multiple centers of the Gyre (Milliff and Robinson, 1992). In particular, the main core of the Gyre is not stationary, but undergoes translations and precessions of synoptic time-scales (seen in our simulations).

Using Fig. 17 and local scale analysis, an approximate dynamical balance for the Ierapetra can be obtained. One has $\rho_0 = 1028$ kg/m^3, $L = 20$ km, $D = 400$ m, $H \simeq 2500$ m (Fig. 4c), $f_0 = 8.15\ 10^{-5}\,\mathrm{s}^{-1}$ and $\beta_0 = 1.89\ 10^{-11}\,\mathrm{m}^{-1}\,\mathrm{s}^{-1}$ (for $\theta_0 = 34.1$). The vertical averages within D of N_s and of the horizontal $\Delta\rho$ are $2.5\ 10^{-3}\,\mathrm{s}^{-1}$ and 0.15 kg/m^3, respectively. Hence, $R_D = 12$ km and $R_{\mathrm{ext}} = 770$ km. From Table 3, the mesocale component of the total surface U is 25 $\mathrm{cm\,s^{-1}}$. Adding the subbasin-scale component (Sect. 4.1) yields $U \simeq 45\,\mathrm{cm\,s^{-1}}$. Hence, for the Ierapetra, the relevant non-dimensional numbers are (see App. B): $Ro = 0.28$, $Bu_{\mathrm{ext}} = 1500$, $Bu = 0.38$, $\beta = 0.017$, $\alpha = 1.8$ and $s = 0.5$. The size of Ro attests that nonlinearities are locally important while β and Bu_{ext} confirm that the planetary vorticity and stretching of the free-surface do not matter. In fact, the above numbers suggest that the dominant horizontal momentum balance within the vertical extent D is the gradient-wind balance: solving for U in, $\frac{U^2}{L} - f_0 U + \frac{\Delta\rho g D}{\rho_0\, L} \simeq 0$, gives a root $U \simeq 50\ \mathrm{cm\,s^{-1}}$ (accounting for the uncertainty in the characteristic scales, this root lies between 40 and 70 $\mathrm{cm\,s^{-1}}$). In the vertical, since Ro and Bu are of the same order, W is large enough for the vertical convergence and horizontal divergence to balance ($W \simeq UD/L$) and the horizontal changes in density to match the vertical ones ($\Delta\rho \simeq \Delta\rho_s$). The values of α and s indicate that the local topographic valley (Fig. 4c) is a factor in maintaining the location of the vortex. Finally, the viscous terms are only important in the bottom layers and at the sub-mesoscales: the horizontal momentum Ekman number $Ek_h = A_h/f_0 L^2$ is 0.92 for scales under 20 km but decays to $5\ 10^{-4}$ at 50 km (see App. C).

4.2.c A priori mesoscale error covariance functions

The mesoscale covariance function estimates of Cases 2–4 are now compared. The covariance between the surface (5 m) temperature at (33.80N,27.85E) and the other state variables is considered as an example. The plots correspond to elements of a row of $\mathbf{B}^p$ (size $218943 \times p$). The position (33.80N,27.85E) is about 80 km east-southeast of the Ierapetra.

Case 2. Figure 18 shows that the horizontal T–T and T–S Mexican hat covariances are well represented. Vertically (Panel b), the characteristics of the tracer covariances are: T–T and T–S

uniform within a 30 m surface mixed-layer; subsurface extremum in T–S near 300 m; T–T and T–S positive down to about 1200 m where they change sign; etc. The T–ψ covariance (Panel a) indicates that, locally, quasi-geostrophic balance is a good approximation. However, the horizontal T–$\widehat{u}$ and T–$\widehat{v}$ covariances are not exactly classic geostrophic double-lobe structures, antisymmetric with respect to the x and y axis, respectively. At the surface (Panel a), the main reason is that inertial terms matter at (33.80N,27.85E). In fact, if only the inertial and geostrophic terms were important, after 2 days of adjustment integration, with $2\pi/f \simeq 0.89$ days locally, the T–$\widehat{u}$ pattern at 5 m would be tilted clockwise by almost 45 degrees. With the horizontal, vertical and bottom diffusions, this tilt at 5 m is here reduced to about 30 degrees (Panel a). Note that even though inertial oscillations occur (e.g. in numerical simulations, the center of the Ierapetra at the surface rotates in small circles at frequencies close to f), their effects on covariances are not always desired[5]. In the vertical (Panel b), T–$\widehat{u}$ and T–$\widehat{v}$ have a zero-crossing within 500 to 600 m. From the T–ψ pattern, the total velocity covariances near (33.80N,27.85E) have a vertical minimum near 700 m. Below the zero-crossings, T–$\widehat{u}$ and T–$\widehat{v}$ are opposite to their surface structures, but have substantially smaller amplitudes, with maxima near 10% of the surface values.

Cases 3 and 4. By construction, the tracer auto-covariance functions for Cases 3 and 2 (Fig. 18) are equal. Those of Case 4 are very similar (see LAL00). The main difference is a relative volume-averaged increase in variance of $100 \times (72 - 67.2)/67.2 = 7.1\%$ (Sect. 4.2), which is distributed in the 3D T–T and T–S fields according to the shape of the tracer vectors 323 to 400. Figure 19 thus only illustrates cross-covariances with velocity fields. Comparing Cases 2 (Fig. 18) and 3 (Fig. 19a), overall, the nonlinear adjustment of velocities modifies the structures of the tracer-velocity cross-covariances, but amplitudes remain similar. Focusing on differences, the horizontal T–$\widehat{u}$ structure has a smaller tilt, of about 15-to-20 degrees clockwise: the nonlinear terms dissipate some inertial oscillations. Cross-covariances now extend to the Ierapetra and their gradients are tighter due to nonlinearities. The T–ψ structure has become asymmetric, with cyclonic T–ψ correlations at the Ierapetra. In the vertical, structures in the upper-layers extend deeper than in the linear adjustment case. The other vertical changes are mainly due to the lesser 15-to-20 degree rotation in the horizontal. Considering the velocity cross-covariances of Cases 3 (Fig. 19a) and 4 (Fig. 19b), shape and amplitude differences are small. This agrees with the corresponding global similarity coefficient (Table 2.b).

4.3 Mesoscale: *a posteriori* PE fields

At the mesoscale stage ($w = 2$), for all four cases (Table 1), the mapping procedures are as overviewed in Sect. 4.1. The differences lie in the *a priori* states (the subbasin-scale estimates), data set and mesoscale covariance parameters (Sect. 4.2). To exemplify the sensitivity to subspace parameters, the focus is on Cases 2 and 3 for the effects of nonlinear momentum terms, and on Cases 3 and 4 for the effects of the size of the subspace.

Figures 20–22 illustrate the results, variable by variable. It is useful to first summarize some physical reasons for the differences between the fields before (Figs. 11 and 13–14) and after March 15 (Fig. 1 and Figs. 20–22). For the region of common data (Figs. 4 and 15), variations are mainly due to: the flattening of the temperature and salinity front along the northern coastline which provides part of the support for the Asia Minor Current; the contraction of the cyclonic Rhodes Gyre towards a rounder shape which is especially seen at 105 and 225 m depths; the weakening of the cyclonic West Cyprus Gyre; the stretching with some surface weakening of the Ierapetra eddy; the mesoscale meanderings of the sampled portions of the Mersa Matruh Gyre; the deep anticyclonic convections in the Mersa Matruh-Shikmona Gyre complex; and the formation of a surface pycnocline and thus surface mixed-layer. By late March, the depth of this overall warming

[5] Inertial oscillations arising in adjustments can have undesired effects on covariances They can be made insignificant by allowing the duration of the adjustment integrations to be random within an inertial period. A location where this would be useful is inside the Rhodes Gyre where nonlinear terms are weak: the tilt in the 5 m T–$\widehat{u}$ and T–$\widehat{v}$ covariances there mainly depends on the duration of adjustment, due to inertial rotations. However, at the Ierapetra, where nonlinear terms dominate, the duration of adjustment did not influence the tilt in the velocity covariances (the tilt is desired).

surface mixed-layer varies from 5 to 10 m at the top of the two subbasin-scale cyclonic gyres, to about 50 to 100 m around the anticyclonic gyres and rings. For more on the physical processes during the spring of 1995, we refer to LER97.

Tracer fields. Figure 20 illustrates the T, S components of $\mathbf{x}^{\mathrm{a}}$ obtained by multivariate 3D analysis (App. A.3) in the Case 2. By construction, the tracer fields for Case 3 (not shown) are identical to these of Case 2 (up to machine precision). Comparing Cases 2 (Fig. 20) and 4 (Fig. 10 in LAL00), their two tracer fields have small differences, in accord with the 4.8% increase in the 3D tracer variance explained (from 67.2% to 72%). Differences are largest mainly in the surface layers and in the small mesoscales, which is logically the main variability spanned by the additional vectors 323 to 400 used in Case 4 (Sect. 4.2.b). The results of the comparison with the benchmark Case 1 (see Figs. 9–10 in LAL00) are analogous to these of Sect. 3. The large mesoscales to the subbasin-scales have small differences, within data error bounds. The submesoscales to small mesoscales present larger differences, especially in the surface layers. This is because many of these small scales are environmental noise for the available data resolution (Fig. 15a) and because the 400 eigenvectors only explain 72% of the total tracer variance (Sect. 4.2).

Transport fields. Figure 21 compares the barotropic transport stream functions. Panel (a) is the result of Case 1 which computes ψ by vertically averaging the total velocities in thermal-wind balance with the horizontal T and S analyses (see Fig. 9 in LAL00), assuming a level of no motion at 600 m. Panel (b) is the ψ component of $\mathbf{x}^{\mathrm{a}}$ for Case 2: the transport is then globally estimated from the T and S data (Fig. 15) using App. A.3. Panel (c) is ψ for Case 3, Panel (d) ψ for Case 4. The advantages of Cases 2–4 (Fig. 21b–d) are that their ψ estimates, and corresponding uncertainties (Sect. 4.4), are in accord with the dominant SVD of the PE adjusted perturbations. As described in LAL00, most of the regional dynamical features can be distinguished in these ψ fields. However, in Case 1, some features likely do not have an acceptable ψ estimate. Comparing Panels (b) and (c), the effects of nonlinear terms in the momentum adjustments agree with the local mesoscale standard deviations of Table 3. Differences are largest for the Ierapetra and Mersa Matruh Gyre. Comparing Panels (c) and (d), increasing the size of the subspace leads to small 2 to 10% variations in ψ, in accord with the global covariance similarity of 86.8% (Table 2.b). The largest changes are at the Ierapetra, above the Rhodes basin and along the Asia Minor Current and Mid-Mediterranean Jet.

Internal velocity fields. Figure 22 compares the surface internal velocities. Panel (a) is the result of Case 1, which computes $\widehat{u}, \widehat{v}$ in thermal-wind balance with the horizontal T and S analyses. Panel (b) are the surface $\widehat{u}, \widehat{v}$ components of $\mathbf{x}^{\mathrm{a}}$ for Case 2: $\widehat{u}, \widehat{v}$ are then globally estimated from the T and S data (Fig. 15) using App. A.3. Panel (c) is the same, but for Case 3; Panel (d) is for Case 4. The differences between Cases 1 and 2 are small and mainly due to differences in their respective tracer fields. Introducing the nonlinear terms in the momentum equations (Cases 2 to 3) mainly affects the Ierapetra and Mersa Matruh Gyre. The other features in the surface circulation remain relatively unchanged. Increasing the size of the subspace (Cases 3 to 4) leads to variations in the small mesoscales, in accord with the variability spanned by the additional vectors considered.

Overall, the regional increases in velocity amplitudes from Cases 2 to 3, and from Cases 3 to 4, logically agree with the *a priori* standard deviations of Table 3 and global values obtained from Table 2.b. The comparison of Cases 1 and 4 reveals that their $\widehat{u}, \widehat{v}$ fields differ mainly because of the different tracer fields (see above) and because of ageostrophic motions in several regions and features (see LAL00 for details). The variations between Panel a and Panels b–d show that nonlinear terms (e.g. centrifugal force) are necessary to explain velocities at the Ierapetra: as concluded in Sect. 4.2.b, the gradient-wind balance is locally a good approximation of the momentum equations but geostrophy is not.

4.4 Mesoscale: *a posteriori* error covariance

4.4.a A posteriori mesoscale error variance

Univariate 2D benchmark. The *a posteriori* tracer error variances result from the 2D analyses, e.g. the 5 m errors on Fig. 15b. These error fields give an estimate of the horizontal mesoscale coverage of the data $\mathbf{y}^o$ (236 CTDs). The velocity error variances are set to these of velocities in geostrophic balance.

Multivariate 3D scheme. The *a posteriori* error standard deviations for Cases 2 and 3 are shown on Figs. 23 and 24 (for Case 4, see Fig. 13 of LAL00). An important result is that error properties are overall as one would expect dynamically even though only 322 or 400 vectors are employed. The surface tracer errors of Cases 2–4 (Figs. 23a–24a) are similar to these of Case 1 (Fig. 15b). The differences are due to the use in Cases 2–4 of *a priori* T and S covariances with non-zero multivariate and vertical correlations. By construction, the tracer errors for Cases 2 and 3 are identical (up to machine precision).

In general, the structures of error standard deviations of total velocities (Figs. 23–24) differ from these of tracers which mimic the patterns of hydrographic ship-tracks. Differences in shape are much larger for $\widehat{u}, \widehat{v}$ errors (under geostrophic influence) than for ψ errors (under hydrostatic influence). The relationships among the velocity error patterns, regional dynamical features, and hydrographic horizontal and vertical resolutions (un-sampled regions, distances between ship-tracks, spacings between profiles along ship-tracks, non-uniform profile lengths, etc) are discussed in detail in Sect. 5.b.ii of LAL00. These discussions apply here. The velocity errors for Cases 2–4 differ from these of Case 1 (not shown) because, in Cases 2–4, *a priori* error covariances involving velocities are non-homogeneous and anisotropic in 3D and because the dominant momentum balance is not everywhere geostrophy (diffusion and nonlinearities matter locally). In particular, comparing Cases 2 and 3 (Figs. 23–24), the nonlinear momentum terms impact the *a posteriori* ψ errors and $\widehat{u}, \widehat{v}$ errors at various depths and locations. Differences are largest near the Ierapetra, Mersa Matruh Gyre and Mersa Matruh-Shikmona Gyre complex which are the features found to be the most nonlinear *a priori* (Sect. 4.2). Error patterns in Case 3 are broader and more uniform (less depend on data location) than in the linear Case 2 because of nonlinear mixing. The transport errors (Figs. 23b–24b) have similar shapes but amplitudes in Case 3 are on average larger by about 4 to 6% (in accord with the *a priori* variance increase of 5.4%, Sect. 4.2.a), with peak increases at about 10 to 20% for the Ierapetra. The effects of nonlinearities are larger on internal velocity errors. At 5 m (Figs. 23a–24a), the $\widehat{u}, \widehat{v}$ errors are about 30 to 50% greater at the Ierapetra, and about 10% greater in the Kasos Strait, around the Rhodes Gyre and in regions of low data resolution east of 28E. At 410 m and 1500 m, $\widehat{u}, \widehat{v}$ errors remain of broader extent and larger (e.g. see the Ierapetra, anticyclone in the Mersa Matruh-Shikmona Gyre complex and around the Rhodes Gyre), but the increase (around 5–10%) is usually smaller than at the surface. Comparing Cases 3 (Fig. 24) and 4 (Fig. 13 in LAL00), the error amplitudes of Case 3 are slightly smaller, by an amount that logically agrees with the global similarity coefficient (Table 2.b) and smaller regional *a priori* error estimates (Table 3). Patterns for the two cases are also very close, excepted at the small mesoscales because of the additional space spanned by the vectors 323-to-400.

4.4.b A posteriori mesoscale error eigenvectors and spectrum

The first and second non-dimensional *a posteriori* error singular vectors for Cases 2 and 3 are illustrated on Figs. 25 and 26, respectively (for Case 4, see Fig. 14 of LAL00). In the linear Case 2, the first vector accounts for 5.7% of the normalized *a posteriori* error variance; the second for 5.1%. For Case 3, these numbers are 5.2% and 4.8% respectively: they are smaller because additional kinetic energy due to the nonlinear terms remains un-measured in the *a posteriori* eigenvalue spectrum. For Case 4, values are 4.25% and 4.05%, which are smaller than for Case 3 due to the larger subspace size. Overall, the eigenvalue spectra differ for the same reasons, but by much smaller amplitudes. In Case 2, the dominant 50 vectors explain 75.8% of the variance explained by all 322 vectors; the dominant 100 explain 87.2%. In Case 3, these numbers remain nearly unchanged, 75.5% and 87.2%. In Case 4, they are logically slightly smaller: 71.5% for 50 vectors and 84.5% for 100 vectors out of all 400 vectors. For all Cases, in comparison to *a priori* values (Sect. 4.2.b), the analysis has here steepened the top and flattened the middle and end of the error spectrum.

Comparing *a posteriori* vectors one by one, the three Cases have almost the same first vector (Panels 1 on Figs. 25–26). However, due to the nonlinear adjustments, other vectors are reordered in Cases 3–4: e.g. up to the effects of the orthogonality constraint and arbitrary sign, the second vector of Case 2 is as the third of Case 3 (not shown) and inversely (Fig. 26). Finally, the dominant vectors of Cases 3 and 4 (Fig. 14 of LAL00) are almost identical, indicating a convergence of the subspace.

For each of Cases 2–4, the data $\mathbf{y}^o$ have redistributed the variance in the subspace. Globally, the dominant *a posteriori* vectors have little in common with the *a priori* ones. In Case 2 (Fig. 25), the two dominant vectors are associated with the low data resolution along 30.75E (Fig. 15a) and lack of data in the northeast corner (north of 35.5N, east of 29.5E). In Cases 3 (Fig. 26) and 4 (Fig. 14 of LAL00), this holds for the first vector, but not for the second. Because of the nonlinear adjustments, the second vector corresponds to the low data resolution patches centered on 29E, which is across the Mersa Matruh-Shikmona Gyre complex where nonlinearities matter (Sect. 4.2). Locally, the properties of the dynamics and data coverage also affect these vectors. A study of regional dynamical influences is given for Case 4 in LAL00; similar statements apply to the Cases shown here. Focusing on the effects of data coverage, considering for example the first vector of Cases 2–4 (e.g. Figs. 25–26, Panels 1a–b), for each variable, the amplitude of the pattern south of 35.5N (northern ship-transect) is at least twice as small as it is north of 35.5N (no data in this corner). The zero-crossings (maxima) of the T and S patterns of this first vector are also aligned with the (lack of) meridional ship-transects (Fig. 15): from 34N to 35.5N included, zero-crossings are every 0.5 degrees of latitude, while north of 35.5N, the last lobe covers 0.8 degrees of latitude, across the no-data corner. The profile depths also influence error patterns. For example, east of 30.25E, all profiles are about 1000 m deep, except for two at (31E,35.5N) and (31E,34.5N) which are 2000 m deep. Logically, these two deep profiles have reduced the error on $\widehat{u}$ at 35N but not on $\widehat{v}$ (see Panels 1b from 1000 m to 2000 m). Analogous facts hold for other vectors. Other lessons that can be learned from the analysis of *a posteriori* error covariances, including guidance on the design of sampling strategies, are discussed in LAL00.

4.4.c A posteriori mesoscale error covariance functions

The comparison of Figs. 18 and 19 (Sect. 4.2.c) with the *a posteriori* error covariance functions plotted on Figs. 27 and 28 clearly shows that error covariance functions are also modified by data. All amplitudes are reduced in accord with the data coverage and measurement error model (App. A and LER97). At the position (33.80N,27.85E), both the horizontal and vertical correlation scales have been reduced to small mesoscale values.

Case 2. Keeping data coverage (Fig. 15a) in mind, the horizontal structures (Fig. 27a) clearly show the influences of the low data resolution south of 34N within 28–29E and of the three ship-tracks along 26E, 26.5E and 27E. In the vertical, the T–T and T–S structures have stronger negative lobes and are more surface intensified because *a priori* T and S data errors are modeled to decay with depths (LER97 and Fig. 31 in Sect. 5.2). The T–$\widehat{u}$ and T–$\widehat{v}$ structures have however remained similar and surface intensified, except that the negative lobe of the T–$\widehat{u}$ structure is now to the east, in accord with thermal-wind effects and the lower data resolution in that direction.

Cases 3 and 4. Only Case 4 is shown (Fig. 28). The covariances for Case 3 are very similar, except that amplitudes are smaller by an amount in accord with the global similarity coefficient of Table 2.b (these amplitude changes can also be seen by comparing Figs. 27 and 28). In the horizontal (Fig. 28a), the structures of Case 4 are similar to these of Case 2 (Fig. 27a), except that gradients are tighter due to nonlinear and larger subspace size effects. In the cross-covariances with velocity fields, differences are largest where features have been shown to be most nonlinear (e.g. Table 3). In the vertical, gradients are also tighter for all covariances. The T–$\widehat{u}$ and T–$\widehat{v}$ structures have remained similar, but the T–T and T–S structures are more complex, in accord with the additional small mesoscales explained by the vectors 323–400.

5. QUANTITATIVE EVALUATION OF FIELDS AND IMPACT ON FORECAST PERFORMANCE

To complement the sensitivity studies (Sects. 3–4), the two mapping schemes (Cases 1–4) and error subspaces of Cases 2–4 are now evaluated by measuring the quality of forecasts initialized from the four different analyses. Starting from each gridded state (e.g. Figs. 20–22), the numerical PE model (App. C) is first integrated forward in time. The forecasts are then compared to *in situ* hydrographic data and satellite sea surface temperature (SST) data. Due to the interactions between tracers and velocities, the effects of the initial flow fields and velocity error subspaces are also assessed.

Even though the *in situ* data are local and intermittent, during March 26–April 8, samplings are relatively intensive (see Fig. 31a below). The corresponding CTD profiles are used for the forecasts' evaluations. Presently, daily satellite images are often cloudy and can contain unphysical, non-uniform variations, sometimes of more than 1°C. Filtering and time-averaging can smooth out this noise, but can also smooth out some oceanic features or patch together sparse daily snapshots (which can occur when a storm passes over the domain in a few days). The high-resolution images provided by GISIS (1995) are here interpolated onto the numerical model grid, masking cloudy areas. During the week of March 27–April 3, on three days, different halves of the domain are clear and contain limited noise (e.g. Fig. 29a). This leads to a relatively good coverage for the week-composite (Fig. 30a), even though one should beware of non-synoptic patching. These interpolated day and week images are used for the forecasts' evaluations.

An issue is that the interval during which forecasts can be compared is limited by the predictive capability of the prediction system. Beyond this limit, nonlinear integrations of data and model errors could reverse conclusions (i.e. indicate that the worse mapping is best). For the area, period, regime of interest and data available, the present limit is near 10 days (LER97). Forecasts are thus not compared beyond that. Remaining issues are to define the error and choose its measure. Since comparing forecasts to analyses of data can be misleading, data and forecasts are here directly compared at data points. These data-forecast residuals can be a good estimate of the forecast error, in so far as they are larger than the estimate of the errors in the data. They are measured by root-mean-squares (RMS), correlation coefficients and pattern correlation coefficients (PCC).

5.1 Satellite SST data

Considering first daily images (e.g. Fig. 29), it is challenging to determine qualitatively which of the forecasts is closest to the satellite SST, even though forecasts differ. Quantitatively, their global scores are indeed similar (see Table 4 below). Focusing on differences, overall, gradients are more accurate in the multivariate 3D scheme (Cases 2–4) than in the benchmark (Case 1). The branch of the Mid-Mediterranean Jet near 33N (on Fig. 29a, seen starting at 29E) is also better represented in Cases 2–4 (Figs. 29c,d,e). The lobes of the Mersa Matruh Gyre are visible in Cases 1 and 4 (Figs. 29b,e), but not so much in the Cases with the smaller subspace size (Figs. 29c,d). For the Ierapetra and its surrounding cold vortices, Cases 1 and 4 have similar large mesoscales. At the small mesoscales, Cases 1 and 4 are better than Cases 2 and 3; overall, the Ierapetra gradients of Case 4 are the most accurate. Comparable comments hold for the other daily images (not shown).

Considering the week composite (Fig. 30), similar statements can again be made, including for the Rhodes Gyre which is now visible on the SST. Of course, time-scales longer than a week are smoothed out in the forecast averages (Figs. 30b–e). The sub-mesoscales and localized steep gradients (due to patching) that remain on the week SST image are noise, which should not be considered in this week-average comparison.

In Table 4, the skill of forecasts generated from the different analyses is quantitatively evaluated. In all computations, only cloud-free points are considered. For the three relatively clear days (Table 4a), the correlation coefficient between the satellite SST day composite and the estimated temperature field at 5 m for that day is given. To obtain these coefficients, the horizontal mean of each field is removed first. The week composite

Table 4. Day and week measures of skill for temperature: satellite SST vs. estimated T fields at 5 m

a. *Day Cor. Coef.* Day composite vs.	Case	March 27 Nowcast	March 28 1-day forecast	April 3 7-day forecast
	1	0.57	0.76	0.82
	2	0.67	0.77	0.81
	3	0.67	0.76	0.81
	4	0.66	0.76	0.81
b. *Week measures*	Case	Cor. Coef.	PCC	RMS Diff.
(March 27–April 3	1	0.76	0.18	0.3912
week composite vs.	2	0.75	0.20	0.3894
average of eight	3	0.75	0.21	0.3870
estimated T fields)	4	0.76	0.20	0.3895

(Table 4b) is then compared to the week average of the day-to-day temperature forecasts at 5 m. To take advantage of the larger coverage (Fig. 30a), more measures are evaluated. In addition to the correlation coefficient, a PCC and the RMS of the residuals between the clear satellite data and averaged forecasts are also calculated. The PCC is defined by, PCC $\doteq \frac{(\mathbf{T}^S-\mathbf{T}^b)^T\,(\widehat{\mathbf{T}}-\mathbf{T}^b)}{||\mathbf{T}^S-\mathbf{T}^b||_2\;||\widehat{\mathbf{T}}-\mathbf{T}^b||_2}$, where the vector $\mathbf{T}^S$ is the satellite week composite, $\widehat{\mathbf{T}}$ the estimated week average and $\mathbf{T}^b$ the background set to the satellite month composite for March 1995 (only common clear points are considered).

For the day snapshots (Table 4a), the main difference is for the nowcast on March 27. The estimates of Cases 2–4 are about 15% better than that of the univariate 2D Case 1. On other days, skill measures are very similar. A surprising result is that there is very little difference among Cases 2–4. This is mainly due to the uncertainty in the satellite data, even though it also reflects the subspace convergence. Another effect of data uncertainties is the increasing correlation with time. For the week composite (Table 4b), correlation coefficients are very close for all cases. The PCC numbers indicate that Case 1 is the least effective, while Case 3 is best (this shows that adding more vectors can at times decrease performance). These PCC numbers are relatively low. This is because they focus on the small mesoscale (the satellite month composite is removed) for which the skill is smaller than for the large mesoscale and subbasin-scale due to the initial data resolution, and also because skin effects are in the week and month composites but not in the average $\widehat{\mathbf{T}}$ fields. The RMS numbers indicate a RMS error slightly larger for Case 1 than for Cases 2–4, but differences are not very significant (the estimated error in the *in situ* data at the surface is around 0.3°C, see Fig. 31 below). For the day images, the RMS and pattern correlation coefficient (not given) lead to the same conclusions. Overall, the satellite data indicate that the skills of Cases 2–4 are similar and only a few times slightly larger than the skill of Case 1.

5.2 *In situ* data

Considering the *in situ* T and S data (Fig. 31a), time is divided in periods of three days. For the data collected in each of these periods, data-forecast residuals are computed at the intersections of the CTDs with the model grid, using the forecast for the center of the period (details on the measurement models, see App. A, are in LER97 and LAL00). These residuals are measured by RMS, averaging level by level. On Fig. 31b, the resulting RMS-error (RMSE) estimates are plotted for the temperature and salinity of Cases 1 and 4, on 12 horizontal model levels (from 0 to 500 m depth). Overall, forecast errors increase with forecast durations and decrease with depths in proportion to the amplitude of the variability. For March 26–28 (Rhodes Gyre area) and March 29–April 1 (near 30E), most RMSEs are below the estimated error standard deviations of the data. Both schemes likely issue good forecasts. At certain depths, Case 1 is better than Case 4, and vice versa. Focusing where RMSEs are significant, Case 4 is better than Case 1 at the surface, while the opposite holds near 500 m. For April 3–5 (data along the Cretan Arc), RMSEs are much larger, in accord with the forecast duration but also with the lesser quality of the numerical model in this region. For temperature, Case 1 is better than Case 4 at all depths. For salinity, Case 4 is substantially better than Case 1 in the surface layers, but is worse below. For April 6–8 (period with most data), in the surface layers, Case 4 is better for temperature, but not for salinity. At mid-depths, the situation reverses. Near 500 m, Case 1 is best overall but errors become smaller than the estimated data error.

These results first confirm that the two schemes have similar skill. They also restate that the univariate 2D scheme can be too sensitive to small scale noise (e.g. Sect. 3.2), hence the usually lesser skill in the surface layers. They show that the present multivariate 3D scheme can sometimes be less effective at depth. To improve this, either the subspace can be enlarged or the statistical specification of the initial covariance decomposition (Apps. A.1–2) refined. For example, in coastal regions, the lack of better performance at depth is likely due to the tracer covariances which were simplified (e.g. horizontally homogeneous, vertical structures dominated by open-ocean data instead of coastal ones) for fair comparisons with the benchmark (see Sect. 4.2).

6. SUMMARY AND CONCLUSIONS

This foregoing study investigated the influence of *a priori* parameters on the error and variability subspace estimation and corresponding mapping methodology described in (Lermusiaux *et al.*, 2000, henceforth, LAL00). The sensitivities of the subspace and of the *a posteriori* gridded fields and errors to the size of the subspace, scales considered and nonlinearities in the dynamical adjustments were exemplified and studied for mesoscale to subbasin-scale physical fields in the northwestern Levantine during February 10-March 15 and March 19-April 16, 1995. The *a posteriori* temperature, salinity and velocity fields and errors obtained were evaluated by comparison among each other and by using the univariate 2D analysis scheme of HOPS as a benchmark. Forecasts generated from the various analyses were qualitatively and quantitatively compared to *in situ* and satellite data. A simple back-of-the-envelope rule for the required size of error subspaces was also outlined and utilized.

Several three-dimensional, multivariate and multiscale properties of the subspace estimation algorithm and analysis scheme were illustrated and discussed. The global mapping was shown to: estimate simultaneously all physical fields and errors from the tracer data; usefully decompose multivariate error covariances; lead to velocity fields that are close to being PE adjusted and *a posteriori* velocity errors that are PE consistent; improve vertical correlations and the filtering of the environmental noise; and, yield more accurate field gradients. Overall, in comparison to the geostrophic benchmark, the largest effects were found where topography, nonlinear and diffusion terms mattered. In particular, *a posteriori* errors were found to be complex, reflecting the properties of both the available data sets and regional dynamics (not always close to geostrophic). Based on the forecasts from the various analyses, the multivariate 3D scheme performed as well as the benchmark. Similar conclusions have been drawn in meteorology (e.g. Andersson *et al.*, 1998). Presently, this is likely in part because of the limited data and because the 3D scheme was simplified for fair comparisons.

From the studies of the sensitivity to scales, it was confirmed that for a fixed domain size, smaller scales usually require larger subspaces. This is valid especially when subspaces are specified based only on common statistical models. When a dynamical model is used or when the subspace itself is dynamical, the ordering of scales becomes more complex and the size requirements more variable. From the studies of the sensitivity to the size of the subspace, it was exemplified that truncating the *a priori* error covariance to a converged subspace is efficient and that *a priori* error eigenvectors of eigenvalues negligible with respect to data errors can be neglected (App. A.4). In the Levantine example, increasing the subspace size usually improved the analysis at depths and the small mesoscales. Overall, the variations of *a posteriori* fields with the parameters of the error subspace were as indicated by a global similarity coefficient (Table 2): if *a priori* error covariance matrices differed by about 5% (in the standard deviation sense), so did the fields and *a posteriori* errors. From the studies of the sensitivity to the linear-nonlinear dynamics of the adjustments, it was found however that even when subspaces are globally similar, some of their dominant vectors can be quite different. The nonlinear momentum correction modified the dominant vectors from large-mesoscale variations of the Rhodes Gyre to mesoscale variations of the Ierapetra eddy. It also modified the shape of error spectra, the structures of covariance functions and some properties of the *a posteriori* fields and errors. Relative linear-nonlinear differences were computed and as anticipated found to be largest for the features and regions with high shears, around deep eddies or gyres (e.g. Ierapetra, Mersa Matruh Gyre), along fronts (e.g. Mid-Mediterranean Jet) and in Straits.

Several dynamical properties were obtained by studying the *a priori* estimates of the tracer and velocity variances, spectra of eigenvalues, shapes and amplitudes of dominant multivariate eigenvectors, and structures of (cross)-covariance functions. The spatial distribution of the characteristic hydrographic and velocity scales and dominant dynamical balances were quantitatively obtained for most features of the northwestern Levantine. In particular, the Ierapetra eddy was found to be close to gradient-wind balance in the Spring of 1995. Coastal-trapped waves were also inferred to be likely along the northern escarpment of the basin and to account for a significant portion of the global variations of momentum and density fields in the region. Some interactions among features were also discussed.

Many physical and computational properties of the present methodology are discussed in LAL00, including general directions for improvements, several algorithm variations and problem-dependent specifics. Focusing on research directions associated with the present sensitivity analyses, a first issue relates to data availability and accurate multiscale statistical models. Mainly for fair comparisons with the univariate 2D scheme but also for the lack of sufficient data, the multivariate 3D scheme was simplified: a successive correction approach was used instead of a general one for multiple interacting scales and *a priori* tracer covariance functions were assumed homogeneous and isotropic in the horizontal instead of being non-homogeneous and anisotropic in 3D as velocity covariances were. Both of these assumptions can be removed in principle (LAL00). In regions with more data than here (to allow accurate fits), the performance of the 3D scheme may then be improved. Another direction concerns algorithms for efficient dynamical adjustment of the error subspace estimate. For example, specific models or processes (e.g. free surface, coupled biochemical irreversibilities, stochastic internal waves) may require specific adjustment procedures. Additional convergence criteria (LER97) should also be investigated. For example, our experience in other regions suggest that taking into account each variable separately as well as all variables as a whole (eqs. A22) is useful. In these criteria, ideal normalizations for specific purposes should also be further researched.

It is important to note that most results obtained here apply to the analysis step of data assimilation schemes combining data and forecasts based on their respective error subspaces. Hence, considering time evolutions, the research opportunities include: subspaces for multiresolution interpolation and nested grids, efficient error initializations for real-time error forecasting and adaptive sampling (e.g. Lermusiaux, 1999b; Arango *et al.*, 2001), model improvements based on accurate error estimates (LER97), dynamical studies based on the quantitative organization of multivariate variability subspaces (e.g. Lermusiaux, 2001; Lermusiaux and Robinson, 2001), and complex interdisciplinary applications (e.g. Robinson and Lermusiaux, 2001).

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APPENDIX A
A priori error subspace estimation and mapping methodology

This Appendix summarizes algorithms introduced in LAL00 for the estimation (initialization) of 3D multivariate and multiscale geophysical fields and their dominant errors. An important step is the construction of the dominant *a priori* error covariance. Details and properties of these algorithms are in LAL00. Wherever possible, the notation of Ide *et al.* (1997) is adopted. The translation into other standards (e.g. Jazwinski, 1970; Gelb, 1974; Daley, 1991; Bennett, 1992; Wunsch, 1996; Robinson *et al.*, 1998) is easily carried out.

The framework is that of continuous-discrete estimation. The true (superscript ^{t}) state vector $\mathbf{x}^{\mathrm{t}} \in \mathrm{R}^n$ is assumed subject to the stochastic dynamical and measurement models, respectively,

$$d\mathbf{x}^{\mathrm{t}} = \mathcal{M}(\mathbf{x}^{\mathrm{t}}, t)\, dt + d\boldsymbol{\eta}^{\mathrm{t}}(t)\ , \qquad \text{(A1a)}$$

$$\mathbf{y}^{\mathrm{o}} = \mathcal{H}\big(\mathbf{x}^{\mathrm{t}}(t_0)\big) + \boldsymbol{\epsilon}\ , \qquad \text{(A1b)}$$

where $\mathcal{M}$ is the dynamics operator, $\boldsymbol{\eta}^{\mathrm{t}}$ a random process of zero mean and covariance matrix $\mathbf{Q}$, $\mathbf{y}^{\mathrm{o}} \in \mathrm{R}^m$ the data vector, $\mathcal{H}$ the measurement operator and $\boldsymbol{\epsilon}$ a random process of zero mean and covariance matrix $\mathbf{R}$. Specifics of (A.1.a–b) are in LER97 and LAL00. The time t_0 for the estimation is fixed. An unbiased estimate of $\mathbf{x}^{\mathrm{t}}(t_0)$ is denoted by $\mathbf{x} \in \mathrm{R}^n$. The state error covariance is defined by $\mathbf{P} \doteq \mathcal{E}\big\{\big(\mathbf{x} - \mathbf{x}^{\mathrm{t}}(t_0)\big)\big(\mathbf{x} - \mathbf{x}^{\mathrm{t}}(t_0)\big)^T\big\} \in \mathrm{R}^{n\times n}$, where $\mathcal{E}\{\cdot\}$ is the statistical mean operator. For the present minimum error variance criterion (Sect. 1), an error subspace is the portion of R^n of dimension p which is spanned by the eigenvectors corresponding to the dominant p eigenvalues of a (normalized) error covariance. With this rational reduction, the optimal estimate becomes:

$$\left\{\mathbf{x}^{\mathrm{a}} \;\middle|\; \min_{\mathbf{x}}\ \mathrm{tr}\,[\mathbf{P}^{\mathrm{a}^p}],\ \text{knowing } [\mathbf{y}^{\mathrm{o}}, \mathbf{R}] \text{ and } \left[\mathbf{x}^{\mathrm{b}}, \mathbf{P}^{\mathrm{b}^p}\right]\right\}, \qquad \text{(A1c)}$$

in which $\mathbf{x}^{\mathrm{t}}(t_0)$ is subject to (A1a–b). The superscripts ($^{\mathrm{b}}$) and ($^{\mathrm{a}}$) refer to estimates before mapping (*a priori*) and after mapping (*a posteriori*). The superscript (^{p}) indicates a rank-p approximation: e.g. the *a priori*/*a posteriori* principal error covariance $\mathbf{P}^{\mathrm{b}^p}/\mathbf{P}^{\mathrm{a}^p}$ is the rank-p eigendecomposition of $\mathbf{P}^{\mathrm{b}}/\mathbf{P}^{\mathrm{a}}$, respectively. What follows summarizes the: approach and algorithms for specifying the observed portions of error covariances; present algorithm; mapping scheme; and, relations with the Kalman update. Quantities marked with asterisks are normalized.

A.1 Observed portions of the *a priori* error covariance: general specification

Following LAL00, the value of a field ϕ at location $(\mathbf{r}, z)$, where $\mathbf{r} = (x, y)$ and z are horizontal and vertical positions in a suitable coordinate system, is denoted by $\phi(\mathbf{r}, z)$. The *a priori* error covariance function for the fields $\phi(\mathbf{r}_1, z_1)$ and $\varphi(\mathbf{r}_2, z_2)$ is then,

$$C_{\phi\varphi}(\mathbf{r}_1, \mathbf{r}_2, z_1, z_2) \doteq \mathcal{E}\Big\{\big(\phi^{\mathrm{b}}(\mathbf{r}_1, z_1) - \phi^{\mathrm{t}}(\mathbf{r}_1, z_1)\big) \big(\varphi^{\mathrm{b}}(\mathbf{r}_2, z_2) - \varphi^{\mathrm{t}}(\mathbf{r}_2, z_2)\big)\Big\}. \qquad \text{(A2)}$$

An often efficient representation of (A2) is obtained by expanding the *a priori* errors into,

$$\phi^{\mathrm{b}}(\mathbf{r}, z) - \phi^{\mathrm{t}}(\mathbf{r}, z) = \sum_{i=0}^{\infty} \phi_i(\mathbf{r})\ Z^*_{\phi_i}(z), \qquad \text{(A3a)}$$

$$\varphi^{\mathrm{b}}(\mathbf{r}, z) - \varphi^{\mathrm{t}}(\mathbf{r}, z) = \sum_{i=0}^{\infty} \varphi_i(\mathbf{r})\ Z^*_{\varphi_i}(z), \qquad \text{(A3b)}$$

where the $Z^*_{\phi_i}(z)$'s and $Z^*_{\varphi_i}(z)$'s are vertical functions, normalized in some suitable sense. Substituting (A3a–b) into (A2) yields,

$$C_{\phi\varphi}(\mathbf{r}_1, \mathbf{r}_2, z_1, z_2) = \sum_{i,j=0}^{\infty,\infty} A_{\phi_i,\varphi_j}\ C^*_{\phi_i,\varphi_j}(\mathbf{r}_1, \mathbf{r}_2)\ Z^*_{\phi_i}(z_1)\ Z^*_{\varphi_j}(z_2), \qquad \text{(A4a)}$$

where the factor $A_{\phi_i,\varphi_j} = A_{\varphi_j,\phi_i}$ is the horizontal average of $\mathcal{E}\{\phi_i(\mathbf{r})\varphi_j(\mathbf{r})\}$ and $C^*_{\phi_i,\varphi_j}(\mathbf{r}_1, \mathbf{r}_2) \doteq \mathcal{E}\{\phi_i(\mathbf{r}_1)\varphi_j(\mathbf{r}_2)\}^*$ the non-dimensional horizontal cross-covariance function associated with ϕ_i and φ_j. The representation (A4a) is efficient if,

$$C_{\phi\varphi}(\mathbf{r}_1, \mathbf{r}_2, z_1, z_2) \simeq \sum_{i,j=0}^{I,J} A_{\phi_i,\varphi_j}\ C^*_{\phi_i,\varphi_j}(\mathbf{r}_1, \mathbf{r}_2)\ Z^*_{\phi_i}(z_1)\ Z^*_{\varphi_j}(z_2), \qquad \text{(A4b)}$$

with small indices I and J (i.e. A_{ϕ_i,φ_j}'s decay rapidly with increasing i and j).

Table A. *A priori* error subspace construction for one scale (w is omitted)

a. Observed Portions		
Tracer Vertical EOFs		
Compute scale-restricted *a priori* tracer residuals:	$\mathbf{d}_i = \mathbf{y}_i^{\mathrm{o}} - \mathcal{H}_i(\mathbf{x}^{\mathrm{b}})\,, \quad i = 1, \ldots, s\,.$	(A7)
Remove horizontal average, normalize:	$\mathbf{d}_i^* = \mathbf{N}_d^{-1}(\mathbf{d}_i - \overline{\mathbf{d}_i})\,.$	(A8)
Compute SVD of normalized tracer residuals:	$\mathrm{SVD}\left(\mathbf{S}_d^* \doteq \left[\mathbf{d}_1^*, \ldots, \mathbf{d}_s^*\right]\right) = \mathbf{E}_d^* \boldsymbol{\Sigma}_d^* \mathbf{V}_d^{*T}\,.$	(A9)
Leads to tracer vertical covariance decomposition:	$\mathbf{C}_{\mathrm{trc}}^z = \mathbf{E}_{\mathrm{trc}}^z \boldsymbol{\Pi}_{\mathrm{trc}}^z \mathbf{E}_{\mathrm{trc}}^{z\,T}\,,$ where $\mathbf{E}_{\mathrm{trc}}^z = \mathbf{N}_d \mathbf{E}_d^*$ and $\boldsymbol{\Pi}_{\mathrm{trc}}^z = \boldsymbol{\Sigma}_d^{*\,2}/s\,.$	(A10)
Tracer Horizontal Covariance Eigendecomposition		
Specify normalized covariance from analytical fit:	$\mathbf{C}_{\mathrm{trc}}^{\mathbf{r}^*}$ filled from $R_{\phi,\varphi}^*(\mathbf{r}_1, \mathbf{r}_2)$ in (A5).	(A11)
Compute SVD or eigendecompose:	$\mathbf{C}_{\mathrm{trc}}^{\mathbf{r}^*} = \mathbf{E}_{\mathrm{trc}}^{\mathbf{r}^*}\, \boldsymbol{\Pi}_{\mathrm{trc}}^{\mathbf{r}}\, \mathbf{E}_{\mathrm{trc}}^{\mathbf{r}^*\,T}\,.$	(A12)
Tracer 3D Covariance Eigendecomposition		
Based on Kronecker product of (A10) and (A12):	$\mathbf{B}_{\mathrm{trc}} = \mathbf{C}_{\mathrm{trc}}^z \otimes \mathbf{C}_{\mathrm{trc}}^{\mathbf{r}^*}\,.$	(A13)
Sort eigenproducts $\lambda_z\lambda_r$ and truncate to subspace:	p dominant $\lambda_z\lambda_r \Rightarrow$ diagonal $\boldsymbol{\Pi}_{\mathrm{trc}}\,.$	(A14)
Construct corresponding 3D eigenvectors:	p dominant $\mathbf{e}_z \otimes \mathbf{e}_r \Rightarrow$ rank-p orthogonal $\mathbf{E}_{\mathrm{trc}}\,.$	(A15)
Result is rank-p decomposition for considered scale:	$\mathbf{B}_{\mathrm{trc}}^p = \mathbf{E}_{\mathrm{trc}} \boldsymbol{\Pi}_{\mathrm{trc}} \mathbf{E}_{\mathrm{trc}}^T\,.$	(A16)
b. Complete Primitive Equation Based Formulation		
Form ensemble of perturbed *a priori* tracer fields:	$\mathbf{x}_{\mathrm{trc}}^{\mathrm{b}^j} = \mathbf{x}_{\mathrm{trc}}^{\mathrm{b}} + \mathbf{E}_{\mathrm{trc}} \boldsymbol{\Pi}_{\mathrm{trc}}^{\frac{1}{2}} \mathbf{u}^j\,, \quad j = 1, ..., r\,,$ where here $\mathbf{u}^j = \sqrt{q}\mathbf{e}^j\,.$	(A17)
Form ensemble of unbalanced *a priori* state vectors:	$\widetilde{\mathbf{x}}^{\mathrm{b}^j} \doteq \left(\widehat{\mathbf{u}}^{\mathrm{b}}, \widehat{\mathbf{v}}^{\mathrm{b}}, \mathbf{x}_{\mathrm{trc}}^{\mathrm{b}^j}, \mathbf{p}^{\mathrm{b}}\right),$	(A18)
Run r dynamical momentum adjustments:	Integrate in (A1a) with $\mathbf{x}_0 = \widetilde{\mathbf{x}}^{\mathrm{b}^j}$ to yield adjusted $\mathbf{x}^{\mathrm{b}^j}$.	(A19)
Form differences of PE adjusted fields, normalize:	$\mathbf{S}^{\mathrm{b}} \doteq \left[\mathbf{x}^{\mathrm{b}^1} - \mathbf{x}^{\mathrm{b}}\,;\ \ldots\,;\ \mathbf{x}^{\mathrm{b}^q} - \mathbf{x}^{\mathrm{b}}\right];\ \ \mathbf{S}^* = \mathbf{N}^{-1}\mathbf{S}^{\mathrm{b}}.$	(A20)
Compute SVD of rank $p \leq r$:	$\mathrm{SVD}_p(\mathbf{S}^{\mathrm{b}}) \doteq \mathbf{N}\ \mathrm{SVD}_p(\mathbf{S}^*) = \mathbf{E}\boldsymbol{\Sigma}\mathbf{V}^T;\ \ \mathbf{E} = \mathbf{N}\mathbf{E}^*.$	(A21)
Restart (A17–21) and evaluate convergence criterion:	$\rho = \dfrac{\sum_{i=1}^{\tilde{p}} \sigma_i\left(\widetilde{\boldsymbol{\Pi}}^{\frac{1}{2}} \widetilde{\mathbf{E}}^{*T} \mathbf{E}^* \boldsymbol{\Pi}^{\frac{1}{2}}\right)}{\sum_{i=1}^{p} \sigma_i(\boldsymbol{\Pi})} \geq \alpha\,,$ where α is a chosen limit $(1 - \epsilon \leq \alpha \leq 1)$ and $\sigma_i(\cdot)$ selects the singular value number i.	(A22)
Result is principal error covariance estimate:	$\mathbf{B}^p = \boldsymbol{\Gamma}\,\mathbf{E}\boldsymbol{\Pi}\,\mathbf{E}^T \boldsymbol{\Gamma}^T\,.$	(A23)

In Sect. 3.c of LAL00, algorithms for estimating the dominant components of error covariance matrices based on the functionals (A4b) are outlined for three types of assumptions involving: a single scale, multiple but independent scales and general multiple scales. In each of these assumptions, for each scale (and scale interactions in the third assumption), four situations arise (e.g. Figs. 2–3 in LAL00) depending on how the vertical and horizontal decompositions are obtained, either directly from data (EOFs of scale-restricted data residuals) or from an analytical model fit to data (eigendecomposition of a specified covariance matrix). For more details we refer to LAL00. The application dictates the choice among options; the algorithm chosen here is summarized below and in Table A.

A.2 Present construction of the *a priori* error subspace

The discrete state vector $\mathbf{x} \doteq (\widehat{\mathbf{u}}, \widehat{\mathbf{v}}, \mathbf{T}, \mathbf{S}, \mathbf{p}) \in \mathrm{R}^n$ contains gridded values of the $\widehat{u}$, $\widehat{v}$, T, S and ψ fields (see footnote 1). The corresponding *a priori* error covariance estimate, $\mathbf{B}$ (superscript $^{\mathrm{b}}$ omitted), is constructed as follows.

Observed portions. For the observed T and S, the assumption of multiple but independent scales is made and (A4b) becomes (LAL00),

$$C_{\phi\varphi}(\mathbf{r}_1, \mathbf{r}_2, z_1, z_2) \simeq \sum_{w=0}^{2} R_{\phi w,\varphi w}^*(\mathbf{r}_1, \mathbf{r}_2)\, Z_{\phi w,\varphi w}(z_1, z_2)\,, \quad \text{(A5)}$$

for ϕ and φ any four combinations of T and S. For fair comparisons with the univariate 2D scheme, for each scale w in (A5), $R^*_{T_wT_w}$, $R^*_{S_wS_w}$ and $R^*_{T_wS_w}$ are also assumed equal. The tracer submatrix $\mathbf{B}_{\rm trc}$ of the error covariance is then the sum of Kronecker products of vertical $\mathbf{C}^z_{\rm trcw}$ and horizontal $\mathbf{C}^{\mathbf{r}^*}_{\rm trcw}$ tracer covariances,

$$\mathbf{B}_{\rm trc} = \sum_w \mathbf{B}_{\rm trcw} = \sum_w \mathbf{C}^z_{\rm trcw} \otimes \mathbf{C}^{\mathbf{r}^*}_{\rm trcw} \ . \quad \text{(A6)}$$

The algorithm used here to construct a $\mathbf{B}^p_{\rm trcw}$ is outlined in Table A.a (A7–16): for each scale w, $\mathbf{C}^z_{\rm trcw}$ is obtained from data (A7–10) and $\mathbf{C}^{\mathbf{r}^*}_{\rm trcw}$ from an analytical model fit to data (A11–12). Normalization matrices are denoted by $\mathbf{N}$. They are block-diagonal: for each field, the corresponding element of $\mathbf{N}$ is the sample and spatial averaged variance. Further details are in Sect. 3 and App. B of LAL00.

Complete multivariate formulation. The velocity portions of $\mathbf{B}$ are computed through an ensemble of adjustment momentum integrations. The algorithm for a given scale w is outlined in Table A.b (specifics are in Sect. 4 of LAL00). An ensemble of perturbed tracers $\mathbf{x}^{\mathrm{b}^j}_{\rm trc} \doteq (\mathbf{T}^{\mathrm{b}^j}, \mathbf{S}^{\mathrm{b}^j})$ is created (A17). The corresponding PE adjusted velocities are obtained by momentum integrations (A19), starting from unbalanced, e.g. geostrophic, initial conditions (A18). Adjusted variability samples can then be formed (A20), normalized and organized by SVD (A21). New adjustments (A17–21) are carried out to increase p until the criterion (A22) determines that the number of samples is large enough. Once this occurs, $\mathbf{B}^p$ is obtained (A23). In (A22), the pairs $(\mathbf{E}^*, \boldsymbol{\Pi} = \frac{1}{q}\boldsymbol{\Sigma}^2)$ of rank p and $(\widetilde{\mathbf{E}}^*, \widetilde{\boldsymbol{\Pi}} = \frac{1}{\tilde{q}}\widetilde{\boldsymbol{\Sigma}}^2)$ of rank $\tilde{p} \leq p$ correspond to "new" and "previous" estimates of $\mathbf{B}^p$ (other criteria are in LER97). In (A23), $\boldsymbol{\Gamma}$ is a scaling, block diagonal matrix.

A.3 Error subspace mapping scheme

With $\mathbf{x}^{\rm b}$, $\mathbf{B}^p$ (A23) and (A1a–b), the extremum of (A1c) yields the *a posteriori* estimates $\mathbf{x}^{\rm a}$, $\mathbf{E}^{\rm a}$ and $\boldsymbol{\Pi}^{\rm a}$, hence $\mathbf{B}^{\mathrm{a}^p} \doteq \mathbf{E}^{\rm a}\boldsymbol{\Pi}^{\rm a}\mathbf{E}^{\mathrm{a}^T}$. For a direct inversion, (A1b) is linearized in the vicinity of $\mathbf{x}^{\rm b}$ and $\mathbf{x}^{\rm a}$ is hypothesized to be a linear function of $\mathbf{x}^{\rm b}$ and $\mathbf{y}^{\rm o} - \mathcal{H}(\mathbf{x}^{\rm b})$, each of which is assumed unbiased. The extremum is then (LER97),

$$\begin{aligned}\mathbf{x}^{\rm a} = \mathbf{x}^{\rm b} + \mathbf{E}\boldsymbol{\Pi}\mathbf{H}^{p^T}(\mathbf{H}^p\boldsymbol{\Pi}\mathbf{H}^{p^T} + \mathbf{R})^{-1} \\ (\mathbf{y}^{\rm o} - \mathcal{H}(\mathbf{x}^{\rm b})), \qquad \text{(A24a)}\end{aligned}$$

$$\begin{aligned}\mathbf{U}\boldsymbol{\Pi}^{\rm a}\mathbf{U}^T = \widetilde{\boldsymbol{\Pi}}^{\rm a} &\doteq \boldsymbol{\Pi} - \boldsymbol{\Pi}\mathbf{H}^{p^T} \\ &(\mathbf{H}^p\boldsymbol{\Pi}\mathbf{H}^{p^T} + \mathbf{R})^{-1}\mathbf{H}^p\boldsymbol{\Pi}, \quad \text{(A24b)} \\ \mathbf{E}^{\rm a} &= \mathbf{E}\,\mathbf{U} \ , \quad \text{(A24c)}\end{aligned}$$

where $\mathbf{H}$ is the linearization of $\mathcal{H}$ in the vicinity of $\mathbf{x}^{\rm b}$, $\mathbf{H}^p \doteq \mathbf{HE}$, the columns of $\mathbf{U}$ are ordered orthonormal eigenvectors of $\widetilde{\boldsymbol{\Pi}}^{\rm a}$ and $\boldsymbol{\Pi}^{\rm a}$ is diagonal. Details are in LAL00. The equations (A24a–c) can also be obtained by formally replacing covariances in the Kalman update step (e.g. Gelb, 1974) by their dominant eigendecompositions $\mathbf{B}^p$ and $\mathbf{B}^{\mathrm{a}^p}$. With the successive correction approach, (A24a–c) are repetitively employed for each scale w.

A.4 A few relations with the classic Kalman update

The difference between the Kalman update and (A24a) arises because of the different gains: $\mathbf{K} \doteq \mathbf{P}^{\rm b}\mathbf{H}^T\,[\mathbf{H}\mathbf{P}^{\rm b}\mathbf{H}^T + \mathbf{R}]^{-1}$ for the Kalman gain and $\mathbf{K}^p \doteq \mathbf{P}^{\mathrm{b}^p}\mathbf{H}^T\,[\mathbf{H}\mathbf{P}^{\mathrm{b}^p}\mathbf{H}^T + \mathbf{R}]^{-1}$ for the subspace gain. Three cases are considered next as a function of the measurement properties $\mathbf{H}$ and $\mathbf{R}$. The concept of Data Optimal (LER97) is used. The rank-p eigendecomposition $\mathbf{P}^{\mathrm{b}^p} = \mathbf{E}\boldsymbol{\Pi}\mathbf{E}^T$ and its complement, $\mathbf{P}^{\mathrm{b}^c} \doteq \mathbf{P}^{\rm b} - \mathbf{P}^{\mathrm{b}^p} = \mathbf{E}^c\boldsymbol{\Pi}^c\mathbf{E}^{c^T}$, are assumed known.

Measurements only sample the error subspace. The fixed-time Data Optimal subspace of rank-p is then observed. Keeping the distinction between normalized and non-normalized quantities, this situation can be defined by $\mathbf{H} = \boldsymbol{\Gamma}\mathbf{E}^{*^T}\mathbf{N}^{-1}$, where $\boldsymbol{\Gamma} \in \mathrm{R}^{m\times m}$ is a matrix of coefficients and $\mathbf{N}$ the normalization matrix (A21). Since $\mathbf{E}^{*^T}\mathbf{E}^{c^*} = 0$,

$$\begin{aligned}\mathbf{K} &= \mathbf{P}^{\rm b}\mathbf{H}^T\,(\mathbf{H}\mathbf{P}^{\rm b}\mathbf{H}^T + \mathbf{R})^{-1} \\ &= \mathbf{E}\boldsymbol{\Pi}\boldsymbol{\Gamma}^T\,(\boldsymbol{\Gamma}\boldsymbol{\Pi}\boldsymbol{\Gamma}^T + \mathbf{R})^{-1} \qquad \text{(A25a)} \\ \mathbf{K}^p &= \mathbf{P}^{\mathrm{b}^p}\mathbf{H}^T\,(\mathbf{H}\mathbf{P}^{\mathrm{b}^p}\mathbf{H}^T + \mathbf{R})^{-1} \\ &= \mathbf{E}\boldsymbol{\Pi}\boldsymbol{\Gamma}^T\,(\boldsymbol{\Gamma}\boldsymbol{\Pi}\boldsymbol{\Gamma}^T + \mathbf{R})^{-1} = \mathbf{K}\,. \quad \text{(A25b)}\end{aligned}$$

Hence, $\mathbf{K} - \mathbf{K}^p = 0$, and from (A24b–c), the error variance reduction achieved,

$$\begin{aligned}\mathbf{P}^{\mathrm{a}^p} &= \mathbf{E}(\boldsymbol{\Pi} - \boldsymbol{\Pi}\boldsymbol{\Gamma}^T\,(\boldsymbol{\Gamma}\boldsymbol{\Pi}\boldsymbol{\Gamma}^T + \mathbf{R})^{-1}\boldsymbol{\Gamma}\boldsymbol{\Pi})\mathbf{E}^T \\ &\quad + \mathbf{E}^c\boldsymbol{\Pi}^c\mathbf{E}^{c^T} = \mathbf{P}^{\rm a} \ , \qquad \text{(A25c)}\end{aligned}$$

is optimal in the full Kalman update sense. For such ideal measurements, the *a posteriori* states and errors of the minimum error variance and present estimates (A24a–c) are equal.

Measurements only sample the complement of the error subspace. The complement of the fixed-time, rank-p Data Optimal subspace is then observed. This situation can be defined by $\mathbf{H} = \boldsymbol{\Gamma}\mathbf{E}^{c^{*T}}\mathbf{N}^{-1}$, where $\boldsymbol{\Gamma} \in \mathrm{R}^{n-m \times n-m}$ is a matrix of coefficients,

$$\begin{aligned}\mathbf{K} =& \mathbf{P}^{\mathrm{b}}\mathbf{H}^T\left(\mathbf{H}\mathbf{P}^{\mathrm{b}}\mathbf{H}^T + \mathbf{R}\right)^{-1} \\ =& \mathbf{E}^c\boldsymbol{\Pi}^c\boldsymbol{\Gamma}^T\left(\boldsymbol{\Gamma}\boldsymbol{\Pi}^c\boldsymbol{\Gamma}^T + \mathbf{R}\right)^{-1} \qquad \text{(A26a)}\\ \mathbf{K}^p =& \mathbf{P}^{\mathrm{b}^p}\mathbf{H}^T\left(\mathbf{H}\mathbf{P}^{\mathrm{b}^p}\mathbf{H}^T + \mathbf{R}\right)^{-1} = 0\,. \qquad \text{(A26b)}\end{aligned}$$

Hence, $\mathbf{K} - \mathbf{K}^p = \mathbf{K}$ and from (A24b–c), $\mathbf{P}^{\mathrm{a}^p} = \mathbf{P}^{\mathrm{b}^p}$. If the domain of $\mathbf{H}$ spans the complement of the error subspace, the data has no influence on (A24a–c). Nonetheless, by construction, these data are associated with negligible components $\mathbf{P}^{\mathrm{b}^c}$ of the *a priori* error. They would thus improve $\mathbf{x}^{\mathrm{b}}$ by negligible amounts and can be neglected.

Measurements sample the full error space. This situation is exemplified by a complete measurement array $\mathbf{H} = \mathbf{I}$, which yields

$$\begin{aligned}\mathbf{K} =& \mathbf{P}^{\mathrm{b}}\left(\mathbf{P}^{\mathrm{b}} + \mathbf{R}\right)^{-1} = \left(\mathbf{E}\boldsymbol{\Pi}\mathbf{E}^T + \mathbf{E}^c\boldsymbol{\Pi}^c\mathbf{E}^{c^T}\right) \\ & \left(\mathbf{E}\boldsymbol{\Pi}\mathbf{E}^T + \mathbf{E}^c\boldsymbol{\Pi}^c\mathbf{E}^{c^T} + \mathbf{R}\right)^{-1} \qquad \text{(A27a)}\\ \mathbf{K}^p =& \mathbf{P}^{\mathrm{b}^p}\left(\mathbf{P}^{\mathrm{b}^p} + \mathbf{R}\right)^{-1} \\ =& \mathbf{E}\boldsymbol{\Pi}\mathbf{E}^T\left(\mathbf{E}\boldsymbol{\Pi}\mathbf{E}^T + \mathbf{R}\right)^{-1}\,. \qquad \text{(A27b)}\end{aligned}$$

The Frobenius norm of $\mathbf{K} - \mathbf{K}^p$ becomes negligible in comparison to that of $\mathbf{K}$ when the dominant eigenvalue of $\boldsymbol{\Pi}^c$ becomes negligible in comparison to the norm of $\mathbf{R}$. For example, for $\mathbf{R} = r\mathbf{I}$, the weight of the *a priori* error vector i in the Kalman gain (A27a) is proportional to $\frac{\lambda_i}{(\lambda_i + r)}$. For given accuracy, the eigenvectors corresponding to λ_i negligible in comparison to r are unnecessary. On the other hand, if the largest eigenvalue of $\boldsymbol{\Pi}^c$ is not negligible in comparison to r, $\mathbf{K}^p$ may not account for most of $\mathbf{K}$. Today, this situation is very uncommon for lack of good data. Should it occur, to avoid spurious effects of the truncation, one must increase p until all non-negligible data components are considered.

APPENDIX B
Relevant scalings and non-dimensional numbers

Table B defines the main non-dimensional numbers that are relevant for the present mesoscale to subbasin-scale study. The notation is standard (Gill, 1982; Pedlosky, 1987; Cushman-Roisin, 1994): g is the acceleration due to gravity; L and D the horizontal and vertical length-scales of motion; U and W the horizontal and vertical characteristic velocity scales; f_0 and β_0 the Coriolis parameter and its northward gradient at latitude θ_0; $\rho_s(z)$ the background density profile; and, ρ_0 a constant reference density. In addition, $N_s = \sqrt{-\frac{g}{\rho_0}\frac{\delta\rho_s}{\delta z}}$ is the Brunt-Väisälä frequency, $R_D = N_s D/f_0$ the internal Rossby radius of deformation, $R_{\mathrm{ext}} = \sqrt{gD}/f_0$ the external Rossby radius of deformation and $\nabla_h H$ the horizontal gradient of the bottom topography H. Except for g, all of these characteristic scales are assumed regional: they vary with the location and process considered.

For the synoptic and horizontally isotropic quasi-geostrophic scales in a stratified ocean, Ro, Bu_{ext}^{-1} and $s \ll 1$, but the other numbers may take a range of values: in particular, Bu and α increase with the strength of the stratification and topographic slope, respectively (e.g. Pedlosky, 1987). At these scales, using the internal energy (density) equation to estimate W, one can also show that $\frac{Ro}{Bu}$ is the ratio of vertical convergence to horizontal divergence $\frac{WL}{UD}$, and that the characteristic horizontal variations in density $\Delta\rho$ and vertical variations in background density $\Delta\rho_s$ are related via, $\frac{\Delta\rho}{\rho_0} = \frac{Ro}{Bu}\frac{\Delta\rho_s}{\rho_0}$.

Table B. Main non-dimensional numbers/parameters

Number/Parameter	Definition	Ratio
Rossby:	$Ro = \frac{U}{f_0 L}$	Relative vorticity to planetary vorticity (also nonlinear advection to Coriolis force)
External Burger:	$Bu_{\mathrm{ext}} = \frac{gD}{f_0^2 L^2} = \frac{R_{\mathrm{ext}}^2}{L^2}$	Relative vorticity to external vertical vortex-tube stretching (of free-surface)
Internal Burger:	$Bu = \frac{N_s^2 D^2}{f_0^2 L^2} = \frac{R_D^2}{L^2}$	Relative vorticity to internal vertical vortex-tube stretching (of isopycnal surfaces)
Planetary-β:	$\beta = \beta_0 \cdot \frac{L^2}{U}$	Planetary (ambient) vorticity gradient to relative vorticity gradient
Topographic-β:	$\alpha = \lvert\nabla_h H\rvert \frac{f_0}{D} \cdot \frac{L^2}{U}$	Topographic (ambient) vorticity gradient to relative vorticity gradient
Slope or topographic:	$s = \lvert\nabla_h H\rvert \cdot \frac{L}{D} = \alpha \, Ro$	Topographic gradient to motion aspect ratio

APPENDIX C
Dynamical model and parameters

The adjustment momentum integrations and forecasts are carried out using the numerical primitive equation model (see footnote 1) of HOPS (e.g. Robinson, 1996, 1999). The present values of the main parameters are listed in Table C.

Table C. Main parameters of the primitive equation model

Numerical Parameters	
Centroid lat. and long.	28.5N, 34.61W
Domain extension	500 km (x), 520 km (y)
Grid resolution	10 km
Grid size	51 (x), 53 (y), 20 (z,vertical hybrid levels:12 flat from 0 to 500 m and 8 sigma below)
Time step	450 sec.
State vector size	218,943
Physical Parameters	
Shapiro filter	F_u, F_v : 8 1 1 ; F_T, F_S : 4 1 3 ; $F_{\overline{\omega}_t}$: 2 1 1
Open boundary condition	$\hat{u}, \hat{v}$: ORI; T, S : ORI; ψ : ORE$_{\frac{1}{2}}$; $\overline{\omega}_t$: ORE$_{\frac{1}{2}}$
Vertical mixing	$A_v = 5$ cm^2 sec^{-1}, $A_v^{\mathrm{cvct}} = 100$ cm^2 sec^{-1} $K_v = 5$ cm^2 sec^{-1}, $K_v^{\mathrm{cvct}} = 100$ cm^2 sec^{-1}
Drag coefficient	$C_d = 0.02$
Rayleigh coastal friction	$\tau_c = 3600$ sec, $L_c = 10$ km
Rayleigh bottom friction	$\tau_b = 1800$ sec, $H_b = 2$ bottom levels, with $H_b \leq 200$ m

Horizontally, the numerical filtering and parameterization of subgrid-scale processes is based on a Shapiro filter (e.g. Shapiro, 1970): the symbols, F_u, F_v, F_T, F_S and $F_{\overline{\omega}_t}$ in Table C give the parameters of the filter for each of the five prognostic equations ($\overline{\omega}_t$ is the rate of change of barotropic vorticity) . The corresponding effective Laplacian horizontal eddy viscosity A_h and diffusivity K_h (LER97) decay with space scales. Under 20 km, they are both 3 10^8 cm^2 s^{-1}. At 50 km scales, $A_h = 1.6\ 10^5$ cm^2 s^{-1} and $K_h = 3.8\ 10^6$ cm^2 s^{-1}. Beyond 100 km scales, they are both almost null. The open-boundary conditions are of the Orlanski radiation type (Orlanski, 1976), either implicit or partially explicit (ORI/ORE$_{\frac{1}{2}}$). The vertical mixing is a Laplacian mixing (eddy viscosity A_v and diffusivity K_v). Convective adjustment is utilized when the water column is statically unstable (eddy viscosity A_v^{cvct} and diffusivity K_v^{cvct}). Across coastlines, the normal flow and tracer flux are set to zero. Along coastlines, the tangential flow is dissipated at the coast, using a Rayleigh friction of relaxation time τ_c and Gaussian decay L_c. At the bottom, for the momentum equations, a dynamic stress balance (drag coefficient C_d) and a Rayleigh friction (relaxation time τ_b and Gaussian decay H_b) are applied. In the cases illustrated in this work, stochastic forcings are not utilized. As discussed in LER97, for the present Levantine application, such forcings increase the sub-mesoscale variability but do not modify the main mesoscale and subbasin-scale results.