Minimizing the inter-island transport using an optimization methodology for complex coastal regions and archipelagos ☆

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Abstract

Computation of geostrophic flow velocity, which is done by integrating the thermal wind equations, from shipboard measurements alone is a classical problem in physical oceanography. Geostrophic flows have a balance struck between the coriolis force and the pressure gradient. The computation of geostrophic flow velocity has been well established for open oceans, without any coastlines or islands. In this paper, we describe a novel optimization methodology for computing the transport streamfunction along island coastlines in complex coastal regions and archipelagos by minimizing the inter-island transport. The objective of our optimization methodology, which utilizes the Fast Marching Method (FMM) or Level Set Method (LSM) to compute the minimum vertical area between all pairs of islands, is to minimize the velocity hot-spots. The estimates of transport streamfunction from this methodology can potentially be very useful for the initialization of velocity fields in the ocean models, by utilizing the temperature (T) and salinity (S) field maps (Objective Analysis maps) for the computation of geostrophic flow velocity. Application of this methodology has been illustrated in the complex domain of Philippines Archipelago.

Key words:

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Field Mapping, Geostrophy, Fast Marching Method, Level Set Method, Objective Analysis, Archipelagos, Complex coastal regions.

1. Introduction

For ocean flows, which evolve over long spatial-time scales and away from the immediate vicinity of the sea-surface, the dominant terms in the horizontal momentum equations are the terms corresponding to the Coriolis force and the pressure gradient. Such a flow field, where a balance is struck between the Coriolis and the pressure forces, is called geostrophic. The thermal wind equations are obtained for geostrophic flow by assuming that the vertical momentum equation is approximately given by hydrostatic balance. The thermal wind relation (Equation 1) is a key theoretical relationship of the observational oceanography as it provides a method by which observations of temperature (T) and salinity (S) as a function of depth can be used to infer ocean currents (Wunsch, 1996; Marshall and Plumb, 2008).

$$-f\frac{\partial(\rho v)}{\partial z} = g\frac{\partial \rho}{\partial x} \quad and \quad f\frac{\partial(\rho u)}{\partial z} = g\frac{\partial \rho}{\partial y} \tag{1}$$

Here, ρ is the density, u and v are the horizontal fluid velocity in the zonal (x) and meridional (y) directions respectively, and $f = 2\Omega\ sin\phi$ is the Coriolis parameter for the spherical earth rotating at a rate of Ω at latitude ϕ. This relation has proved to be extremely useful for initializing the velocity from temperature and salinity Objective Analysis (OA) field maps in open oceans.

The OA methodology for the initialization of tracer fields (e.g. temperature, salinity and biology) has been well formulated for open oceans without any landforms (convex simply-connected domains), but the OA in complex coastal regions (multiply-connected domains) is one of the 'last' mapping problems which remains to be studied in detail (Agarwal, 2009). New methodology for Objective Analysis in complex coastal regions and archipelagos using Fast Marching Method (FMM) and Level Set Method (LSM) have been proposed by Agarwal and Lermusiaux (2009). These new OA methodologies will likely be very useful in improving the World Ocean Atlas (WOA) climatologies (Levitus, 1982; Locarnini et al., 2006; Antonov et al., 2006; Garcia et al., 2006a,b) in complex domains and archipelagos.

In this paper, we describe a novel optimization methodology for computing the transport streamfunction along island coastlines in complex coastal regions and archipelagos. These transport streamfunction estimates will be used for specifying the boundary conditions to compute velocity under geostrophic balance from the temperature and salinity field maps (Agarwal and Lermusiaux, 2009) alone. The objective function of this optimization methodology is to minimize velocity hot-spots. To set up the objective function, we require the minimum vertical area between all pairs of islands, which are computed using the Fast Marching Method (Sethian, 1996, 1999) or the Level Set Method (Osher and Sethian, 1988). This new methodology can potentially be very useful for the initialization of velocity in the ocean models.

Our research study for complex coastal regions and archipelagos has been motivated by the Philippines Straits Dynamics Experiment (PhilEx) sponsored by the Office of Naval Research. The goal of PhilEx is to enhance understanding of the oceanographic processes and features arising in and around straits, and to improve the capability to predict the inherent spatial and temporal variability of these regions using models and advanced data assimilation techniques.

Our research is carried out within the Multidisciplinary Simulation, Estimation and Assimilation System (MSEAS: http://mseas.mit.edu) group. MSEAS consists of a set of mathematical models and computational methods for ocean predictions and dynamical diagnostics, for optimization and control of autonomous ocean observation systems, and for data assimilation and data-model comparisons. It is used for basic and fundamental research and for realistic simulations and predictions in varied regions of the world's ocean, recently including monitoring (Lermusiaux, 2007), naval exercises including real-time acoustic-ocean predictions (Xu et al., 2008) and environmental management (Cossarini et al., 2009). Several different models are included in the MSEAS, including a new free-surface primitive-equation dynamical model which uses two-way nesting free-surface and open boundary condition schemes (Haley et al., 2008). This new free-surface code is based on the primitive-equation model of the Harvard Ocean Prediction System (HOPS). Additionally, barotropic tides are calculated from an inverse tidal model (Logoutov, 2008).

The paper is organized as follows: In Section 2, we introduce the new optimization methodology for computing the transport streamfunction and velocity under geostrophic balance by minimizing the inter-island transport. In Section 3, application of our new methodology, for the complex region of Philippines Archipelago is presented. Section 4 consists of a summary and conclusions.

2. New optimization methodology for estimating transport streamfunction along island coastlines and for computing velocity under geostrophic balance

Flow estimation based on thermal wind balance (Equation 1), is a classical problem in physical oceanography (Wunsch, 1996; Marshall and Plumb, 2008). Historically, the only significant routine measurements possible were the temperature, T, and salinity, S, of the water at various depths. The equation of state for seawater then permits the estimation of density at a given pressure from the temperature and salinity measurements. The thermal wind equations (Equation 1) when integrated in the vertical give:

$$\begin{aligned}\rho v(x,y,z,t) &= \frac{-g}{f}\int_{z_0}^{z}\frac{\partial \rho}{\partial x}dz + \rho v_0 \\ \rho u(x,y,z,t) &= \frac{g}{f}\int_{z_0}^{z}\frac{\partial \rho}{\partial y}dz + \rho u_0\end{aligned} \tag{2}$$

where, z_0 is the level of no motion for $v_0, u_0 = 0$ or a level of reference for $v_0, u_0 \neq 0$. The formulation has been well defined for the open oceans without any landforms. For complex coastal regions having landforms such as islands and peninsulas, estimation of the inter-island transport is first required before proceeding with the geostrophic formulation discussed above.

The optimization methodology for estimating the inter-island transport is described by the flowchart in Figure 1 and is discussed below. The objective of this methodology is to find a set of constant values for the transport streamfunction (Ψ) along the island coastlines that produce a suitably smooth initialization velocity field, e.g. with the fewest large velocity hot-spots, i.e. minimize the maximum absolute velocity in the initialized geostrophic flow field. The working assumptions for the methodology are listed below:

1. Coastlines in the given domain can be divided into two distinct subsets:

(a). Set A: N coastlines along which the transport streamfunction is unknown, $N \neq 0$.

(b). Set B: M coastlines along which the transport streamfunction is known.

2. The solution for the transport streamfunction Ψ_0 exists for the case which includes coasts in set B, but coasts in set A, along with the corresponding interiors, are replaced by open ocean (e.g. island sunk to 10m depth).

3. The difference between the initial solution Ψ_0 and the final solution Ψ is not extremely large. Otherwise, the information from Ψ_0 would not be accurate enough.

Ψ_0 contains useful information like the relative position of major currents to various coastlines and the effects of topography on the flow. Thus, the information in Ψ_0 can be utilized to estimate the constant value of the transport streamfunction along the island coastlines by constructing an optimization functional for minimizing the inter-island transport subject to weak constraints. The construction of the optimization functional is now discussed.

The problem is divided into three parts to construct the optimization functional. The optimization functional (E) in the general form, which is a summation of three terms, is given by:

$$E = E_1 + E_2 + E_3 \tag{3}$$

where, E_1 is the minimizing target for the transport between all pairs of the unknown (Set A) coasts, E_2 is the minimizing target for the transport between all pairs of unknown (Set A) and known (Set B) coasts and E_3 is the minimizing target for the transport between all pairs of the unknown (Set A) coasts and the open boundaries of the domain. These three terms in Equation 3 are:

1. Constructing the optimization functional for minimizing the transport between all pairs of island coastlines with unknown (Set A) transport streamfunction: Let C_n and C_m be two of the coasts (coast n and coast m) in Set A. Ψ_0 is not constrained to be a constant along these coasts. Find the grid point i^0 on the coastline n and the grid point j^0 on the coastline m such that $[i^0, j^0] = \arg\max\limits_{[i,j]} |\Psi_{0n}(i) - \Psi_{0m}(j)|$ and $\delta\Psi_{n,m} = \Psi_{0n}(i^0) - \Psi_{0m}(j^0)$. Here, we denote Ψ_0 at point b on coastline a by $\Psi_{0a}(b)$.

The optimization functional for minimizing the inter-island transport between islands n and m is given by $(\Psi_{C_n} - \Psi_{C_m} - \delta\Psi_{n,m})^2$, where, Ψ_{C_a} is the unknown optimized, constant value of the transport streamfunction along the coast a. This optimization function can be weighted by $w_{nm} = 1/d_{nm}^2$ where, d_{nm} is the minimum distance between C_n and C_m. However, since the objective is to smooth the resulting initialization velocity flow field, the above weighting will be appropriate if the ocean depth is uniform in between all pairs of islands. An alternative weighting along with its computational methodology for non-uniform ocean depths will be proposed later in this section.

2. Constructing the optimization functional for minimizing the transport between all pairs of island coastlines with known (Set B) and unknown (Set A) transport streamfunction: Let C'_k be one of the coasts along which the transport streamfunction $\Psi_{C'_k}$ is known (Set B) and C_n be the coast in Set A. Ψ_0 is not constrained to be a constant along C_n. Find the grid point i' on the coastline n such that $[i'] = \arg\max_{[i]} |\Psi_{0n}(i) - \Psi_{C'_k}|$ and $\delta\Psi'_{n,k} = \Psi_{0n}(i') - \Psi_{C'_k}$.

The optimization functional for minimizing the inter-island transport between islands n and k is given by $(\Psi_{C_n} - \Psi_{C'_k} - \delta\Psi'_{n,k})^2 = (\Psi_{C_n} - \Psi_{0n}(i'))^2$. As before, we propose to weight these optimization function by $w'_{nk} = 1/d'^2_{nk}$ where, d'^2_{nk} is the minimum distance between C_n and C'_k.

3. Constructing the optimization functional for minimizing the transport between all pairs of island coastlines with unknown (Set A) transport streamfunction and the open boundaries of the domain: Let (C''_b) be the open boundary, $\{b\}$ be the set of open boundary points and C_n be the coast in Set A. Ψ_0 is not constrained to be a constant along C_n. Find the grid point i'' on the coastline n and the grid point b'' on the open boundary such that $[i'', b''] = \arg\max_{[i,b]} |\Psi_{0n}(i) - \Psi_{C''_b}(b)|$ and $\delta\Psi''_{n,b} = \Psi_{0n}(i'') - \Psi_{C''_b}(b'')$. Here, we denote Ψ_0 at the point b on the open boundary by $\Psi_{C''_b}(b)$.

The optimization functional for minimizing the inter-island transport between the island n and the open boundary is given by $(\Psi_{C_n} - \Psi_{C''_b}(b'') - \delta\Psi''_{n,b})^2 = (\Psi_{C_n} - \Psi_{0n}(i''))^2$. As before, we propose to weight this optimization function by $w''_{nb} = 1/d''^2_{nb}$ where, d''^2_{nb} is the minimum distance between C_n and the open boundary C''_b.

The weighted average of the optimization functionals constructed from the above three parts is given by:

$$E = \frac{1}{2}\sum_{n=1}^{N}[\sum_{m=1,m\neq n}^{N} w_{nm}(\Psi_{C_n} - \Psi_{C_m} - \delta\Psi_{n,m})^2 +$$
$$\sum_{k=1}^{M} w'_{nk}(\Psi_{C_n} - \Psi_{0n}(i'))^2 + w''_{nb}(\Psi_{C_n} - \Psi_{0n}(i''))^2] \tag{4}$$

The minimum of E is computed by solving the standard least square problem i.e by setting gradients with respect to Ψ_{C_j}'s equal to zero. Therefore, the solution to the optimization problem in Equation 4 is given by:

$$\Psi_{C_j}[\sum_{m=1,m\neq j}^{N} 2w_{jm} + \sum_{k=1}^{M} w'_{jk} + w''_{jb}] - \sum_{m=1,m\neq j}^{N} 2w_{jm}\Psi_{C_m}$$
$$= \sum_{m=1,m\neq j}^{N} 2w_{jm}\delta\Psi_{j,m} + \sum_{k=1}^{M} w'_{jk}\Psi_{0j}(i') + w''_{jb}\Psi_{0j}(i'') \tag{5}$$

Equation 5 represents a system of N equations which can be solved to obtain the transport streamfunctions (Ψ_{C_j}) along coastlines in set A. These streamfunction values, which smooth the velocity field, will be used as Dirichlet boundary conditions while solving the geostrophic flow equations using the temperature and salinity field maps. The illustration of this methodology in the complex domain of Philippines Archipelago is discussed in Section 3.

We now discuss new and more suitable weights to be used in Equation 5. Consider the stream function (Ψ) for a two-dimensional horizontal flow. It is defined such that the flow velocity can be expressed as:

$$\vec{u} = (u, v) = \frac{1}{H}\nabla \times \Psi\hat{k} \Rightarrow u = \frac{1}{H}\frac{\partial \Psi}{\partial y}, v = -\frac{1}{H}\frac{\partial \Psi}{\partial x} \tag{6}$$

Here, H is the ocean depth. The transport between a pair of islands having streamfunction ψ_1 and ψ_2 is given by:

$$\psi_2 - \psi_1 = \int_A \vec{u}.\hat{n} dA \tag{7}$$

where, A is the vertical area between the two islands and $\hat{n}$ is the unit vector normal to the vertical area. Equation 6 and 7 suggests that the appropriate weight function to smooth the velocity field should be $w_{nm} = 1/A_{nm}^2$, where, A_{nm} is the minimum vertical area along any path between the two islands. The weight function ($w_{nm} = 1/d_{nm}^2$) will be appropriate when the ocean depth is uniform in between all pairs of islands. Since the ocean depth is not uniform, a new methodology is required to compute the minimum area along any path between a pair of islands. Using the Fast Marching Method (FMM), which is described in Appendix A, is a very convenient and efficient way to compute A_{nm}. Simulations have been performed with several other weight functions to confirm that the proposed weight function based on the minimum vertical area (A_{nm}) is the most appropriate for smoothing the velocity flow field.

Fortran-90 code have been written to utilize the weight function based on the minimum vertical area between islands, which can be computed using the FMM/LSM. For obtaining the minimum vertical area, the scalar speed function in the Eikonal Equation (Equation 8) is chosen to be F(x,y) = 1/H(x,y). This completes the description of our new optimization methodology for obtaining the transport streamfunction along island coastlines and the geostrophic flow velocities in complex coastal regions.

3. Estimation of the velocity under geostrophic balance in Philippines Archipelago

Estimation of velocity under geostrophic balance in the Philippines Archipelago is illustrated in this section. The methodology for minimizing the inter-island transport, which is described in Section 2, is utilized for computing a smooth geostrophic velocity flow field.

We have proposed to utilize weight functions based on the minimum vertical area along each pair of islands in the algorithm for minimizing the inter-island transport. The estimation of the minimum vertical area has been carried out using the FMM by specifying the scalar speed function in the Eikonal equation (Equation 8) as F(x,y) = 1/H(x,y), where H is the ocean depth. The temperature and salinity data are from the World Ocean Atlas 2005. They are mapped using FMM-based OA scheme (Agarwal and Lermusiaux, 2009) and the SPDE (stochastically forced partial differential equation) approach (Balgovind et al., 1983; Lynch and McGillicuddy, 2001), with the Helmholtz equation employed for the field. The streamfunction and velocity fields (at depths 0m, 100m) are shown in Figure 2. These streamfunction and velocity plots obtained using the temperature and salinity field maps from FMM-based OA scheme (Figure 2 (Left)) show a very good comparison with the streamfunction and velocity plots obtained using the temperature and salinity field maps based on the stochastically forced Helmholtz equation (Figure 2 (Right)). These maps suggest that the velocity is maximum in the Mindoro strait, near the Mindanao island and in the Balabac strait. The maximum absolute velocity, which is in the Balabac strait, is 79.7 cm/s. At lower depths, the velocity remain high in the Mindoro strait and near the Mindanao island. There is a large inter-island transport across the Mindoro strait since the vertical area between the Mindoro and Palawan island is very large.

Minimum inter-island distance can be obtained using the FMM by specifying the scalar speed function in the Eikonal equation (Equation 8) as 1 for the sea points and 0 for the land points. The velocity fields obtained using the weight functions based on the minimum inter-island distance has significantly large magnitudes, particularly in the Balabac strait. The maximum absolute velocity is 140.9 cm/s, which is significantly larger than the maximum absolute velocity obtained using weight functions based on the minimum vertical area (79.7 cm/s). Such high velocity magnitudes, which are obtained due to the inaccurate computation of inter-island transport, are clearly not acceptable. These results clearly show that the weight functions based on the minimum vertical area will produce smooth geostrophic flow field with minimum velocity hot spots.

4. Summary and Conclusions

We have discussed our new optimization methodology for the estimation of velocity under geostrophic balance in complex coastal regions and archipelagos. This methodology utilizes FMM for the computation of the minimum vertical area between all pairs of islands. The minimum area is required for specifying the weight functions in this new optimization methodology to obtain the transport streamfunction which minimizes the inter-island transport and produces a smooth velocity flow field. The transport streamfunction can then be utilized to estimate the geostrophic flow velocity from the temperature and salinity field maps alone. We have illustrated this method by applying it in a subdomain of the Philippines Archipelago. We believe that this new method along with FMM-based OA scheme (Agarwal and Lermusiaux, 2009; Agarwal, 2009) will be very useful for initialization of ocean models in complex coastal regions and archipelagos.

A. Fast Marching Algorithm

The Fast Marching Method (FMM) for monotonically advancing fronts, which has been proposed by Sethian (1996, 1999), is described. This method leads to an extremely fast scheme for solving the Eikonal equation (Equation 8). The Level set method (Osher and Sethian, 1988) relies on computing the evolution of all the level sets by solving an initial value partial differential equation using numerical techniques from hyperbolic conservation laws. As an alternative, an efficient modification is to perform the work only in the neighborhood of the zero level set, as this is known as the 'narrow band approach'. The basic idea of this approach is to tag the grid points as either "alive", "land mines" or "far away" depending on whether they are inside the band, near its boundary, or outside the band, respectively. The work is performed only on *alive* points, and the band is reconstructed once the land mine points are reached.

FMM, which allows boundary value problems to be solved without iterations, is now discussed in detail. The method is applicable to monotonically advancing fronts (i.e. the front speed ($F \geq 0$ or $F \leq 0$) which are governed by the level set equation. The steady state form of the level set equation is the Eikonal equation (Equation 8) which says that the gradient of the arrival time surface is inversely proportional to the speed of the front. For the two dimensional case, the stationary boundary value problem is given by:

$$|\nabla T|F(x,y) = 1 \quad s.t. \quad \Gamma = \{(x,y)|T(x,y) = 0\} \tag{8}$$

where Γ is the starting position of the interface. The first order finite difference discretization form of the Eikonal equation (Sethian, 1999) at the grid point (i,j) is given by:

$$\begin{gathered}[max(D_{ij}^{-x}T,0)^2 + min(D_{ij}^{+x}T,0)^2 + \\ max(D_{ij}^{-y}T,0)^2 + min(D_{ij}^{+y}T,0)^2]^{1/2} = \frac{1}{F_{ij}}\end{gathered}$$

or,

$$\begin{gathered}[max(max(D_{ij}^{-x}T,0), -min(D_{ij}^{+x}T,0))^2 + \\ max(max(D_{ij}^{-y}T,0), -min(D_{ij}^{+y}T,0))^2] = \frac{1}{F_{ij}^2}\end{gathered} \tag{9}$$

Equation 9 is essentially a quadratic equation for the value at each grid point (assuming that values at the neighboring nodes are known). An iterative algorithm for computing the solution to Equation 9 was introduced by Ruoy and Tourin (1992). FMM is based on the observation that the upwind difference structure of Equation 9 means that the information propagates "one way", i.e. from the smaller values of T to the larger values. Therefore, FMM rests on solving Equation 9 by building the solution outward from the smallest time value T. The front is swept ahead in an upwind manner by considering a set of points in a *narrow band* around the existing front and bringing new points into the *narrow band* structure.

The fast marching algorithm is:

1. Initialize
   (a) *Alive* points: Let A be the set of all grid points (i,j) on the starting position of the interface Γ; set $\mathrm{T_{ij}} = 0$ for all points in A.
   (b) *Narrow Band* points: Let the *Narrow Band* be the set of all grid points (i,j) in the immediate neighborhood of A; set $T_{ij} = \frac{d}{F_{ij}}$ for all points in the *Narrow Band* where, d is the grid separation distance.
   (c) *Far Away* points: Let the *Far Away* region be the set of all remaining grid points (i,j); set $T_{ij} = \infty$ for all points in the *Far Away* region.
2. Marching Forward
   (a) Begin Loop: Let (i_{min},j_{min}) be the point in the *Narrow Band* with the smallest value for T.
   (b) Add the point (i_{min},j_{min}) to A; remove it from the *Narrow Band.*
   (c) Tag as neighbors any points $(i_{min}-1,j_{min})$, $(i_{min}+1,j_{min})$, $(i_{min},j_{min}-1)$, $(i_{min},j_{min}+1)$ that are either in the *Narrow Band* or the *Far Away* region. If the neighbor is in the *Far Away* region, remove it from that list and add it to the *Narrow Band.*
   (d) Recompute values of T at all neighbors in accordance with Equation 9. Select the largest possible solution to the quadratic equation.
   (e) Return to the top of the loop.

Here are some properties of the fast marching algorithm. The smallest value in the *Narrow Band* is always correct. Other *Narrow Band* or *Far Away* points with larger values of T cannot affect the smallest value. Also, the process of recomputing T values at the neighboring points cannot give a value smaller than any of the accepted value at *Alive points*, since the correct solution is obtained by selecting the largest possible solution to the quadratic equation (Equation 9). Thus the algorithm marches forward by selecting the minimal T value in the *Narrow Band* and recomputing the values of T at all neighbors in accordance with Equation 9.

The key to an efficient version of the algorithm lies in finding a fast way to locate the grid point in the *Narrow Band* with the minimum value for

T. To do so, the heapsort algorithm (Williams, 1964; Sedgewick, 1988) with backpointers is often implemented and it is the algorithm we used here. This sorting algorithm generates a "complete binary tree" with the property that the value at any given parent node is less than or equal to the value at its child node. Heap is represented sequentially by storing a parent node at the location k and its child at locations $2k$ and $2k + 1$. The member having the smallest value is stored at the location $k = 1$.

All *Narrow Band* points are initially sorted in a heapsort. The fast marching algorithm works by first finding, and then removing, the member corresponding to the smallest T value from the *Narrow Band* which is followed by one sweep of DownHeap to ensure that the remaining elements satisfy the heap property. The DownHeap operation moves the element downwards in the heap till the new heap satisfies the heap properties. *Far Away* neighbors are added to the heap using the Insert operation which increases the heap size by one and brings the new element to its correct heap location using the UpHeap operation. The UpHeap operation moves the element upwards in the heap till the new heap satisfies the heap properties. The updated values at the neighbor points obtained from Equation 9 are also brought to the correct heap location by performing the UpHeap operation.

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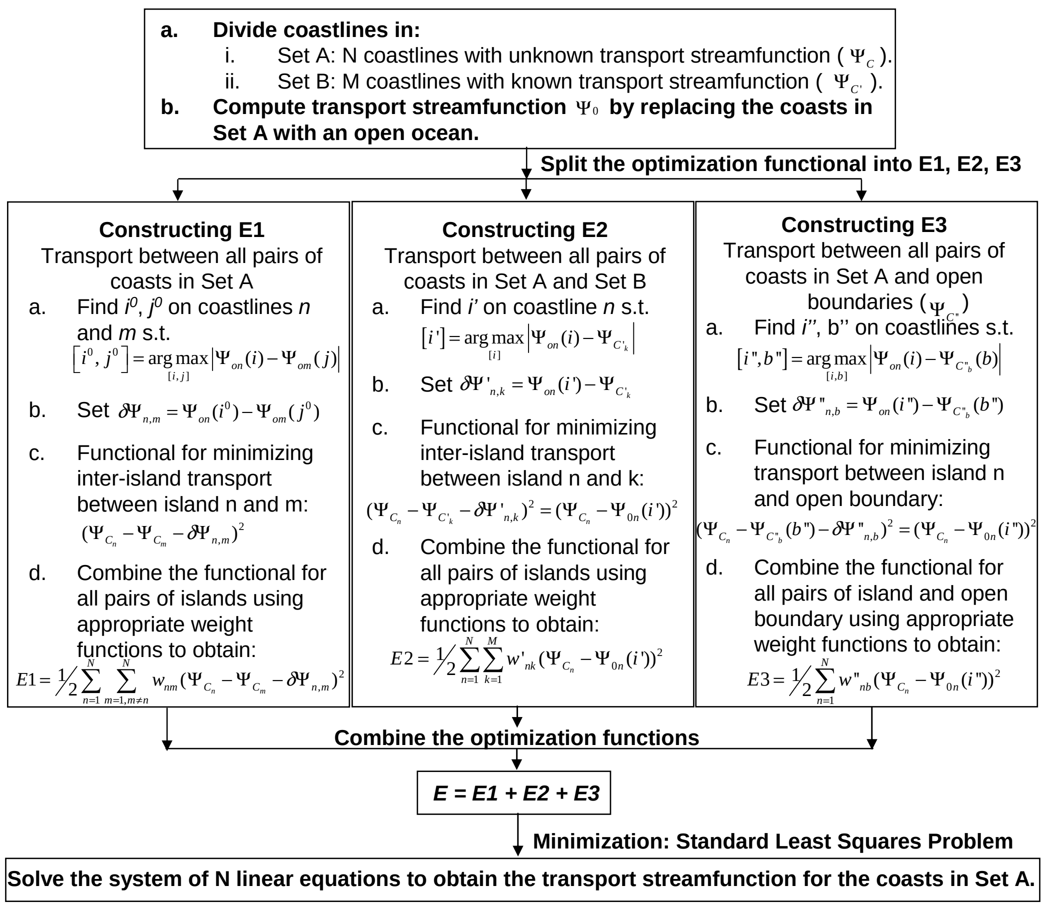

Figure 1: Flowchart for constructing Optimization Function and computing transport streamfunction.

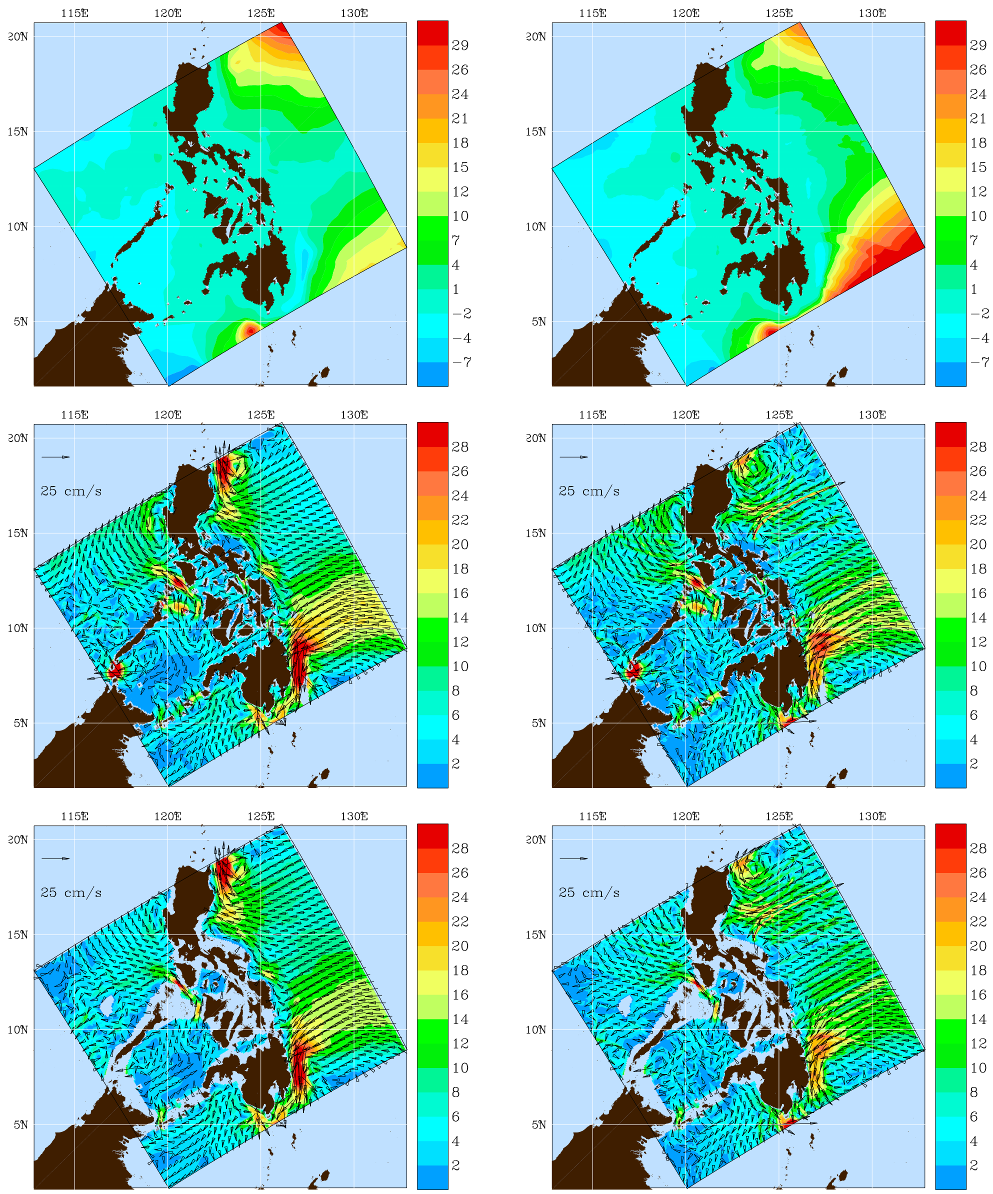

Figure 2: Velocity estimation under geostrophic balance (weight functions based on the minimum vertical area) from field maps (WOA05) obtained using the FMM (Left) and using the SPDE Approach (Right): (Top) Streamfunction, Velocity at depths: (Middle) 0m; (Bottom) 100m.