ϕ_0^t → computed using backward advection from t to 0.

ϕ_t^0 → computed using forward advection from 0 to t.

Composition Equations:

Forward flow map: $\phi_0^n = \phi_{n-1}^n(\phi_{n-2}^{n-1}(\cdots \phi_0^1)\cdots)$

Backward flow map: $\phi_n^0 = \phi_1^0(\phi_2^1(\cdots \phi_n^{n-1})\cdots)$

We see tire-like pattern in the forward flow map.

↳ Sign mistake in u, v.

- Need to have negative sign, but did not.

⇒ we were doing forward advection instead of backward.

⇒ we were computing backward flowmap instead of forward.

∴ What we were computing was:

$\bar{\phi}_0^n = \phi_n^{n-1}(\phi_{n-1}^{n-2}(\cdots \phi_1^0)\cdots)$.

Compare with the correct expression:

$\phi_0^n = \phi_{n-1}^n(\phi_{n-2}^{n-1}(\cdots \phi_0^1)\cdots)$.

$\bar{\phi}_0^n$ is not even a flow map!

↳ The sign change in u, v also made the divergence worse.