Mean multipath intensity relation for sound propagation through a random ocean front

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By considering the stochastic nature of the phase fluctuations in the ocean, the conventional ray theory intensity relaxation was extended in an earlier paper [J. A. Neubert, J. Acoust. Soc. Am. **51**, 310–322 (1972)] to permit consideration of partial coherence in multipath problems. Although this relation worked well in the open ocean [J. A. Neubert, J. Acoust. Soc. Am. **62**, 326–334 (1977)], it proves to be incomplete for sound propagation through a random ocean front. By also considering the amplitude fluctuations, a mean multipath intensity relation (as well as its standard deviation σ_I) is found that takes into consideration the strong horizontal sound-speed gradients that occur in certain important ocean frontal regions.

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INTRODUCTION

The Lagrangian (i.e., pathwise) relation for the sound pressure wave after having traveled an arc length s from ξ through the refractive field $n(\mathbf{x})$ is given by[1]

$$p[\mathbf{X}(s,\xi)] = A(s)\exp[ik_0 S(s)] \tag{1}$$

$$= p_0(\xi)\exp\left[-\frac{1}{2}\int_0^s \frac{ds'}{n(s')}\left(n\frac{dX_i}{ds'}\right)_{,i} + ik_0\int_0^s ds'\, n(s')\right], \tag{2}$$

where A is the amplitude, k_0 is the wavenumber,

$$S \equiv \int_0^s ds'\, n[\mathbf{X}(s')] \tag{3}$$

is the stochastic phase factor, and

$$p_0(\xi) = A(\xi)\exp[ik_0 S(\xi)]. \tag{4}$$

As in Ref. 1, the notation is as follows. The refractive index n is represented by

$$n = n[\mathbf{X}(s)] = n_0[\mathbf{X}(s)]\,[1+\alpha\mu(s)] = c_0/c[X(s)], \tag{5}$$

where c_0 is some convenient reference sound speed, $c[\mathbf{X}(s)]$ is the sound speed at path arc length s at point $\mathbf{X}$, $\mathbf{X}(s)$ varies from its initial point $\mathbf{X}(s=0)=\xi$ to its terminal point $\mathbf{x}$, $\alpha\mu$ is the fluctuation of n about its mean $n_0 = \langle n\rangle$, and $\alpha(0<\alpha\ll 1)$ is the normalized rms variance of the refractive index fluctuations [see Eqs. (3)–(7) of Ref. 1].

Equation (2) yields the multipath intensity relation I, which is discussed in detail in Sec. I. Taking the ensemble expectation of I yields the mean multipath intensity relation for $E\{I\}$ in Sec. II. In a similar manner, the standard deviation σ_I about $E\{I\}$ is evaluated. Both $E\{I\}$ and σ_I are dependent on the new stochastic factors $F_0(s)$ and $F_i(s)$. In Sec. III and Ref. 2, it is shown that these two stochastic factors are important (i.e., differ from unity) only in pronounced sound-speed frontal regions. Therefore the previous relations for $E\{I\}$ and σ_I are valid outside of these frontal regions but the complete relations derived below are necessary to describe sound propagation through a pronounced sound-speed frontal region.

I. GENERAL EXPRESSION FOR INTENSITY

In Ref. 1 the effect of the phase fluctuation on the mean multipath ray theory intensity in a random ocean was investigated. This resulted in the introduction of an important stochastic factor, which is called the "partial coherence factor." In this development the amplitude fluctuation behavior is retained as well. The result is two new stochastic factors that become important in sound propagation through an ocean front.[2] These stochastic factors occur when the approximation of Eq. (46) of Ref. 1 is not made. Furthermore, Eq. (42) of Ref. 1 left out one important term that does not effect the results of Ref. 1. The corrected relation is

$$\left(\frac{dX_i}{ds}\right)_{,i} = \left(\frac{dX_i}{ds}\right)_{0,i} - \alpha\left[\mu(s)\left(\frac{dX_i}{ds}\right)_0\right]_{,i} + \alpha\int_0^s ds'\left(\frac{[\mu(s')n_0(s')]_{,i}}{n_0(s)}\right)_{,i} + O(\alpha^2), \tag{6}$$

where

$$\left(\frac{dX_i}{ds}(s)\right)_0 = \frac{n_0(\xi)}{n_0(\mathbf{x})}\frac{dX_i}{ds}(0) + \frac{1}{n_0(\mathbf{x})}\int_0^s ds'\, n_{0,i}(s'). \tag{7}$$

Using the stochastic integral theory of Refs. 1, 3–6, the integral of the second two terms of Eq. (6) can be accurately approximated as follows using Eq. (7):

$$\int_0^s ds'\left[\mu(s')\left(\frac{dX_i}{ds'}\right)_0\right]_{,i} + \int_0^s ds'\int_0^{s'} ds''\left(\frac{[\mu(s'')n_0(s'')]_{,i}}{n_0(s')}\right)_{,i}$$

$$= -\int_0^s ds'\left(\mu(s')\frac{n_0(\xi)}{n_0(s')}\frac{dX_i}{ds'}(0)\right)_{,i} + \int_0^s ds'\int_0^{s'} ds''\left(\frac{[\mu(s'')n_0(s'')]_{,i} - \mu(s)n_{0,i}(s'')}{n_0(s')}\right)_{,i}, \tag{8}$$

$$= -\int_0^s ds'\, n_0(\xi)\frac{dX_i}{ds'}(0)\left(\frac{\mu(s')}{n_0(s')}\right)_{,i} + \int_0^s ds'\int_0^{s'} ds''\left[\left(\mu_{,i}(s'')\frac{n_0(s'')}{n_0(s')}\right)_{,i} + g(s',s'')\right], \tag{9}$$

$$\simeq -\int_0^s ds'\, n_0(\xi)\frac{dX_i}{ds'}(0)\mu'_{,i}(s') + \int_0^s ds' \int_0^{s'} ds''\, \mu'_{,ii}(s'',s'), \tag{10}$$

where

$$\mu'_{,i}(s') \equiv \left(\frac{\mu(s')}{n_0(s')}\right)_{,i} \tag{11}$$

$$\xrightarrow[n_0(s')\to 1]{} \mu_{,i}(s'), \tag{11a}$$

$$\mu'_{,ii}(s'',s') \equiv \left(\mu_{,i}(s'')\,\frac{n_0(s'')}{n_0(s')}\right)_{,i} \tag{12}$$

$$\xrightarrow[n_0(s'),n_0(s'')\to 1]{} \mu_{,ii}(s'') \tag{12a}$$

and

$$g(s',s'') \equiv \left([\mu(s'')-\mu(s')]\,\frac{n_{0,i}(s'')}{n_0(s')}\right)_{,i} \simeq 0. \tag{13}$$

The symbol $\simeq$ means that the quantity $g(s',s'')$ will give a negligible contribution compared to the other factors appearing in the following ensemble expectations.[4–6] Taking $n_0 \to 1$ gives the homogeneous limit.

Assume Eq. (34) of Ref. 1 so that

$$I(s) = I(\xi)\exp\left(-\int_0^s ds'\,\frac{dX_i}{ds'}\right)_{,i} \tag{14}$$

$$\simeq I_0(s)H_\alpha(s)F_\alpha(s), \tag{15}$$

where

$$I_0(s) \equiv I(\xi)\exp\left[-\int_s^0 ds'\left(\frac{dX_i}{ds'}\right)_{0,i}\right] \tag{16}$$

is the deterministic ray theory intensity as computed by conventional ray theory software,

$$H_\alpha(s) \equiv \exp\left(-\alpha n_0(\xi)\,\frac{dX_i}{ds}(\xi)\int_0^s ds'\,\mu'_{,i}(s')\right) \tag{17}$$

is the stochastic intensity path divergence factor, and

$$F_\alpha(s) \equiv \exp\left(\alpha\int_s^0 ds'\int_0^{s'} ds''\,\mu'_{,ii}(s'',s')\right) \tag{18}$$

is the stochastic intensity fading factor which results from pathwise phase fluctuations.

Therefore Eqs. (38)–(40) of Ref. 1 can be replaced by

$$I(\mathbf{x}) \simeq \left|\sum_{m=1}^{M} I_0^{1/2}(s_m)H_\alpha^{1/2}(s_m)F_\alpha^{1/2}(s_m)G_\alpha(s_m)e^{i\omega t}\right|^2 \tag{19}$$

$$= \sum_{m=1}^{M} I_0(s_m)H_\alpha(s_m)F_\alpha(s_m) + 2\sum_{m=1}^{M-1}\sum_{l=m+1}^{M} I_0^{1/2}(s_m)I_0^{1/2}(s_l)H_\alpha^{1/2}(s_m)H_\alpha^{1/2}(s_l) \times F_\alpha^{1/2}(s_m)F_\alpha^{1/2}(s_l)\cos[k_0(S_m - S_l)], \tag{20}$$

where M is the total number of multipaths, ω is the angular frequency, t is the time, and

$$G_\alpha(s_m) \equiv \exp(-k_0 S_m) \tag{21}$$

is the stochastic pathwise phase fluctuation factor. The first term in Eq. (20) represents the incoherent summation of intensities while the second term represents the coherent contribution to the multipath intensity.

II. THE MEAN MULTIPATH INTENSITY RELATION

The following developments parallel those of Appendix B of Ref. 1 (with a few notational changes) except the approximation of Eq. (46) of Ref. 1 is avoided. The ensemble expectation of Eq. (20) gives the mean multipath intensity

$$E\{I(\mathbf{x})\} \simeq \sum_{m=1}^{M} I_0(s_m)E\{H_\alpha(s_m)F_\alpha(s_m)\} + 2\sum_{m=1}^{M-1}\sum_{l=m+1}^{M} I_0^{1/2}(s_m)I_0^{1/2}(s_l)E\{H_\alpha^{1/2}(s_m) \times H_\alpha^{1/2}(s_l)F_\alpha^{1/2}(s_m)F_\alpha^{1/2}(s_l)\cos[k_0(S_m - S_l)]\}. \tag{22}$$

The standard deviation σ_I about $E\{I\}$ is defined by

$$\sigma_I \equiv E\left\{\left|\frac{p(\mathbf{x})}{[c(\mathbf{x})]^{1/2}} - E\left\{\frac{p(\mathbf{x})}{[c(\mathbf{x})]^{1/2}}\right\}\right|^2\right\} \tag{23}$$

$$\doteq E\{I(\mathbf{x})\} - \left|E\left\{\frac{p(\mathbf{x})}{[c(\mathbf{x})]^{1/2}}\right\}\right|^2, \tag{24}$$

which is Eq. (57) of Ref. 1.

Equations (22) and (24) are evaluated as follows via the conservation of energy for stochastic processes.[4–6] The first ensemble expectation in Eq. (22) is

$$E\{H_\alpha(s)F_\alpha(s)\} \doteq F_0(s)F_i(s), \tag{25}$$

where

$$F_0(s) \equiv \exp(-2\alpha^2 R_{a3}Q_3 s^2) = \exp(-s^2/d_0^2) \tag{26}$$

is the stochastic scatter loss factor [which is the ensemble expectation of the interaction of the stochastic path divergence factor and the stochastic fading factor; it represents the scattering away of acoustic energy and is dependent on the sound-speed gradients as shown in Eq. (29)] and

$$F_i(s) \equiv \exp(2\alpha^2 R_{a1}Q_1 s) = \exp(s/d_i) \tag{27}$$

is the stochastic scatter gain factor [which is the ensemble expectation of the stochastic divergence factor; it represents the scattering in of acoustic energy and is dependent on the sound-speed gradients as shown in Eq. (31)]. The decorrelation length (d_0) for $F_0(s)$ is

$$d_0 \equiv [\alpha(2R_{a3}Q_3)^{1/2}]^{-1}, \tag{28}$$

where

$$4s^2R_{a3}Q_3 \sim \int_0^s ds'\int_0^s ds''\int_0^{s''} ds'''\, n_0(\xi)\frac{dX_i}{ds'}(0)E\{\mu'_{,i}(s') \times [\mu'_{,jj}(s''',s'') - E\{\mu'_{,jj}(s''',s'')\}]\}. \tag{29}$$

The correlation length (d_1) for F_i is

$$d_i \equiv (2\alpha^2 R_{a1}Q_1)^{-1}, \tag{30}$$

where

$$4sR_{a1}Q_1 \sim \int_0^s ds'\int_0^s ds''\, n_0^2(\xi)E\{\mu'_{,i}(s')\mu'_{,i}(s'')\}. \tag{31}$$

The second ensemble expectation in Eq. (22) is

$$E\{H_\alpha^{1/2}(s_m)H_\alpha^{1/2}(s_l)F_\alpha^{1/2}(s_m)F_\alpha^{1/2}(s_l)\cos[k_0(S_m - S_l)]\} \doteq F_0^{1/2}(s_m)F_0^{1/2}(s_l)F_i^{1/2}(s_m) \times F_i^{1/2}(s_l)F_p(s_m)F_p(s_l)\cos[k_0(S_{0m} - S_{0l})], \tag{32}$$

where

$$S_0 \equiv \int_0^s ds'\, n_0[\mathbf{X}(s')] \tag{33}$$

is the deterministic phase factor and

$$F_p(s) = \exp(-\alpha^2 k_0^2 L s) \quad (34)$$

is the partial coherence factor [which is the ensemble expectation of the pathwise phase fluctuation factor; it gives the decrease in the coherent crosspath term of Eq. (36)], where

$$4sL \sim \int_0^s ds'' \int_0^s ds'' \, n_0(s') n_0(s'') \times E\{[\mu(s') - E\{\mu(s')\}][\mu(s'') - E\{\mu(s'')\}]\}. \quad (35)$$

Equations (25)–(35) are evaluated by direct application of the techniques given in Refs. 1, 3–6. The empirical method for determining the stochastic quantities $R_{a3}Q_3, R_{a1}Q_1$ and L from actual ocean data is given in Ref. 7.

Equations (22), (25), and (32) give the mean multipath intensity

$$E\{I(\mathbf{x})\} \simeq \sum_{m=1}^{M} I_0(s_m) F_0(s_m) F_i(s_m) + 2 \sum_{m=1}^{M-1} \sum_{l=m+1}^{M} I_0^{1/2}(s_m) I_0^{1/2}(s_l) \times F_0^{1/2}(s_m) F_0^{1/2}(s_l) F_i^{1/2}(s_m) F_i^{1/2}(s_l) \times F_p(s_m) F_p(s_l) \cos[k_0(S_{0m} - S_{0l})] \quad (36)$$

$$\underset{F_p \to 0}{\longrightarrow} \sigma_I, F_p = 0. \quad (37)$$

When $F_0(s)$ and $F_i(s)$ are essentially unity (which occurs outside a pronounced sound-speed frontal region as shown in Sec. III and Ref. 2), Eq. (36) reduces to

$$E\{I(\mathbf{x})\} \simeq \sum_{m=1}^{M} I_0(s_m) + 2 \sum_{m=1}^{M-1} \sum_{l=m+1}^{M} I_0^{1/2}(s_m) I_0^{1/2}(s_l) F_p(s_m) \times F_p(s_l) \cos[k_0(S_{0m} - S_{0l})], \quad (38)$$

which is Eq. (49) of Ref. 1.

In a similar manner σ_I of Eq. (24) can be evaluated as follows. Equation (51) of Ref. 1 is replaced by

$$\frac{p(\mathbf{x})}{[c(\boldsymbol{\alpha})]^{1/2}} \doteq \sum_{m=1}^{M} I_0^{1/2}(s_m) H_\alpha^{1/2}(s_m) \times F_\alpha^{1/2}(s_m) G_\alpha(s_m) e^{i\omega t} \quad (39)$$

which can be shown to yield[1,4–6]

$$\left| E\left\{ \frac{p(\mathbf{x})}{\{c(\mathbf{x})\}^{1/2}} \right\} \right|^2 \doteq \sum_{m=1}^{M} I_0(s_m) F_0(s_m) F_i(s_m) F_p^2(s_m) + 2 \sum_{m=1}^{M-1} \sum_{l=m+1}^{M} I_0^{1/2}(s_m) I_0^{1/2}(s_l) F_0^{1/2}(s_m) F_0^{1/2}(s_l) \times F_i^{1/2}(s_m) F_i^{1/2}(s_l) F_p(s_m) F_p(s_l) \cos[k_0(S_{0m} - S_{0l})]. \quad (40)$$

Therefore Eqs. (24), (36), and (39) yield

$$\sigma_I \doteq \sum_{m=1}^{M} I_0(s_m) F_0(s_m) F_i(s_m) [1 - F_p^2(s_m)] \quad (41)$$

$$\underset{F_p \to 0}{\longrightarrow} E\{I\}, F_p = 0. \quad (42)$$

When $F_0(s)$ and $F_i(s)$ are essentially unity (i.e., outside a pronounced sound-speed frontal region), Eq. (41) reduces to

$$\sigma_I \doteq \sum_{m=1}^{M} I_0(s_m) [1 - F_p^2(s_m)], \quad (43)$$

which is Eq. (56) of Ref. 1.

Note that the incoherent limits of $E\{I\}$ and σ_I are theoretically equal, i.e.,

$$E_i\{I\} \equiv E\{I_p(F_p = 0)\} \sim \sigma_I(F_p = 0) \equiv \sigma_{Ii}. \quad (44)$$

However, during sound propagation scattering smooths out the higher order statistics (i.e., it essentially lowers the tails of the probability distribution) so that σ_{Ii} is reduced from its theoretical value and gives

$$E_i\{I\} \sim a\sigma_{Ii} \quad (45)$$

in practice; $a \gg 1$ and depends on frequency. In Ref. 7, $a = 6.5$ at 1030 Hz.

III. CONCLUSIONS

In Ref. 1 the effect of the phase fluctuation on the mean multipath ray theory intensity in a random ocean was developed. This resulted in the introduction of the partial coherence factor of Eq. (32). In this paper the amplitude fluctuation behavior was retained as well. This resulted in the two new stochastic factors of Eqs. (26) and (27) that become important (i.e., differ from unity) only in regions of strong horizontal sound-speed gradients that can occur in an ocean front.[2,8,9] These stochastic factors occur when the approximation of Eq. (46) of Ref. 1 is not made. Instead Eqs. (25) and (32) are applied to Eq. (22) to give the mean multipath intensity relation of Eq. (36). In a similar manner, Eqs. (36) and (40) are applied to Eq. (24) to give the standard deviation σ_I about $E\{I\}$ in Eq. (41).

Figure 1 shows some typical sound-speed contours in the North Pacific ocean between Hawaii and Alaska.[2] Note that the sound-speed gradients that impact $F_0(s)$ and $F_i(s)$ [see Eqs. (26)–(31)], and hence $E\{I\}$ of Eq. (36), are negligible except for the sharp horizontal sound-speed gradient above about 300 m at about 40 °N which constitutes a pronounced ocean front. Outside this frontal region, Eq. (36) reduces to Eq. (38). However, for sound propagating through this ocean front all factors in Eq. (36) prove necessary.[2]

We cannot determine the random variable $\mu(s)$ from the averaged data of Fig. 1 so we cannot use Eqs. (29) and (31) to determine $R_{a3}Q_3$ and $R_{a1}Q_1$, respectively. However, the sound speed changes by about 5 m/s across the front in Fig. 1

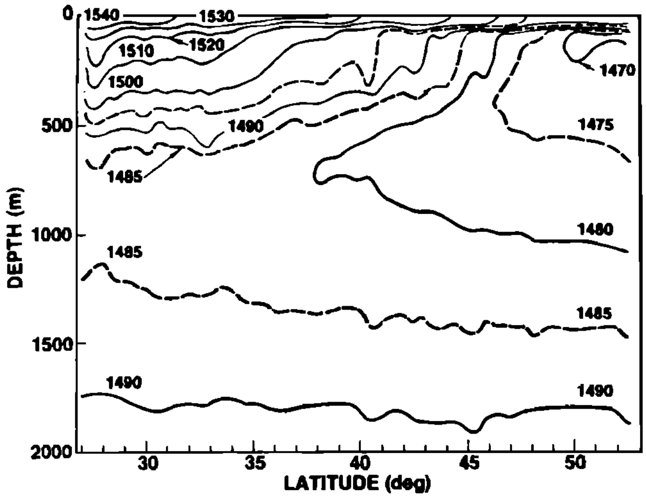

FIG. 1. Sound-speed contours (m/s) between Hawaii and Alaska.

so it can be estimated that $\alpha \approx 3 \times 10^{-3}$. From Ref. 2 for a cw source depth of 500 ft, a receiver depth of 300 ft and an acoustic frequency near 200 Hz, an intensity drop of about 8 dB occurs over a frontal distance of about 100 km. Assuming one dominant path eliminates the second term in Eq. (36) leaving $F_0(s)F_i(s)$ to account for the 8-dB drop over $s \approx 100$ km. Since a relatively sharp drop (i.e., 8 dB in 100 km) is seen in Fig. 5 of Ref. 2, it will be assumed that $F_0(s)$ dominates $F_i(s)$ so that d_0 of Eq. (26) equals about 74 m. Then Eq. (28) gives $R_{a3}Q_3 \approx 10/\text{m}^2$ which, of course, agrees with the 8-dB loss via Eq. (26). If it is assumed $R_{a1}Q_1$ is of the same order of magnitude as $R_{a3}Q_3$ and Eq. (27) is used, $10 \log_{10} F_i(s) \approx 0.08$ dB which is indeed negligible compared to $10 \log_{10} F_0(s) \approx -8$ dB from Eq. (26). Hence the relative s dependence in Eqs. (26) and (27) should lead to $F_0(s)$ dominating $F_i(s)$ in Eq. (36).

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