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Session 1aAO: Acoustical Oceanography**

1aAO3. Transverse coherence lengths, processing limits and implications

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How does scattering in sound channels (deep and shallow waters) limit coherent array processing or what is the limitation of resolution in terms of the mutual coherence function and its temporal and spatial coherence lengths? The resolution of an array is limited by the mutual coherence function; but estimation in a partially coherent noise background with a multipath signal is difficult using the normalized cross power spectral density, magnitude squared coherence, because of the properties of both signals and noise. The measurement of magnitude-squared coherence is a poor statistical estimator since it is a function of the signal-to-noise ratio and multipath interference with large confidence bounds. Array gain measurements and a wave-theoretic coherence functional form can provide estimates of temporal and spatial coherence lengths defined as the $1/e$ value of this function. This paper reviews single path coherence results and those derived from array measurements over the low- to mid-frequency range in deep and shallow water. Representative coherence lengths are discussed in terms of boundary interactions, internal wave scattering, and coastal mesoscale features. The implications for arrays used to estimate geoacoustic properties, mammal locations, and scattering from the boundaries are presented.

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Spatial Coherence and array signal gain were measured across this front!



Surface manifestation of the The South Korean
Coastal Front due Tsushima Current front in
the Strait of Korea



What is the limit coherent horizontal array processing terms of the mutual coherence function and length in deep and shallow-water-sound channels ?

Resolution is limited by the mutual coherence function; but estimation in a partially coherent noise and a multipath signal is difficult since magnitude squared coherence estimator measures the properties of both signals and noise.

C. Carter * has determined the magnitude squared coherence is a poor statistical estimator since it is a function of the signal-to-noise ratio with large confidence bounds.

However, array signal gain measurements and a wave-theoretic coherence functional form can provide estimates of the horizontal coherence length defined as the 1/e value of this function..

* G. C. Carter, C. H. Knapp, and A. H. Nuttall, " Estimation of the magnitude squared coherence function via overlapped FFT processing," IEEE Trans. on Audio and Electroacoustics, AU-21(4), pp.337-344,1973.,E. R. H. Scannell and G. C. Carter," Confidence bounds for magnitude-squared coherence estimates," Proc. IEEE Int. Conf. on Acoustics, Speech, and Signal Processing, pp.670-673, 1978.

Talk Outline

- **Deep Water Horizontal Coherence**
 - Coherence Measurement
 - Pairwise Coherence Results
 - Array Signal Gain results
- **Shallow Water- Downward Refraction- Sandy-Relative Signal Gain Results**
- **Basic Conclusion:** *Water borne paths in deep water have coherence lengths on the order of 100λ at a frequency of 400 Hz at a range of 400 km. While shallow water (100m) with sandy bottoms have coherence lengths on the order of 30λ at a frequency of 400 Hz at a range of 40 km.*

What do we mean by coherence?

How is it estimated from measurements?

What bearing does it have on array performance?

What do we mean by coherence?

- **The Mutual Coherence function of a propagating wave is well defined**

$$\begin{aligned}\Gamma_{12} &= \Gamma[R, r_1, r_2, \omega_1, \omega_2, t_1, t_2] \\ &= \langle p(R, r_1, \omega_1, t_1) \cdot p(R, r_2, \omega_2, t_2)^* \rangle\end{aligned}$$

- **The Spatial Coherence Function**

$$\Gamma_{12} = \langle p(R, r_1) \cdot p(R, r_2)^* \rangle = \langle p(R, r_1) \cdot p(R, r_1 + r_{\perp})^* \rangle$$

Born and Wolf, 1959, Principles of Optics, Pergamon Press, Ny, NY, Chapt. 10 pp 401-544.
 Beran, M. J., Parrent, G.B., M.J. Beran, Theory of Partial Coherence, Prentice Hall, Englewood, Cliffs, NJ, pp1-11.
 Uscinski, B.J., 1977, The elements of Wave Propagation, McGraw-Hill, NY, NY
 J. J. McCoy, and B. B. Adams, 1975, " Effects of a fluctuating temperature field on the spatial coherence of acoustic signals,"
 NRL, Washington, D.C. 20375, NRL Tech. Rept. 7809, 1975 -(Avail DTIC).
 R. F. Dashen, S. M. Flatte, W. H. Munk, and F. Zachariasen, 1977, " Limits on Coherent Processing Due to Internal Waves,"
 Stanford Research, Menlo Park, Ca, 94025, Stanford Research Rep. Tr-JSR-76-14, 1977-(Avail DTIC).
 Flatte, S.M., R. F. Dashen, W. H. Munk, K.E. Watson and F. Zachariasen, 1977, Sound Transmission through a Fluctuating Ocean, Cambridge University press, Cambridge, U.K. pp 126-149.

Volume scattering relations

The Beran-McCoy-Adams Formulation:

Transverse Coherence Function is

$$\begin{aligned}\Gamma(\Delta y) &= R_p(R, f, \Delta y, z) / I(z) = \exp\left(-(\Delta y / L_{hc})^{3/2}\right) \\ &= \exp\left(-E_f f^{5/2} R(\Delta y)^{3/2}\right) = \exp\left(-E_k k^{5/2} R(\Delta y)^{3/2}\right)\end{aligned}$$

where $E_f = 1.136 \cdot 10^{-6}$ $E_k = 1.136 \cdot 10^{-6} (\epsilon \cdot 10^{-10})$
 and $L_{hc}^{-1} = f^{5/2} (E_f R)^{2/3}$.

The Flatte-Dashen Formulation:

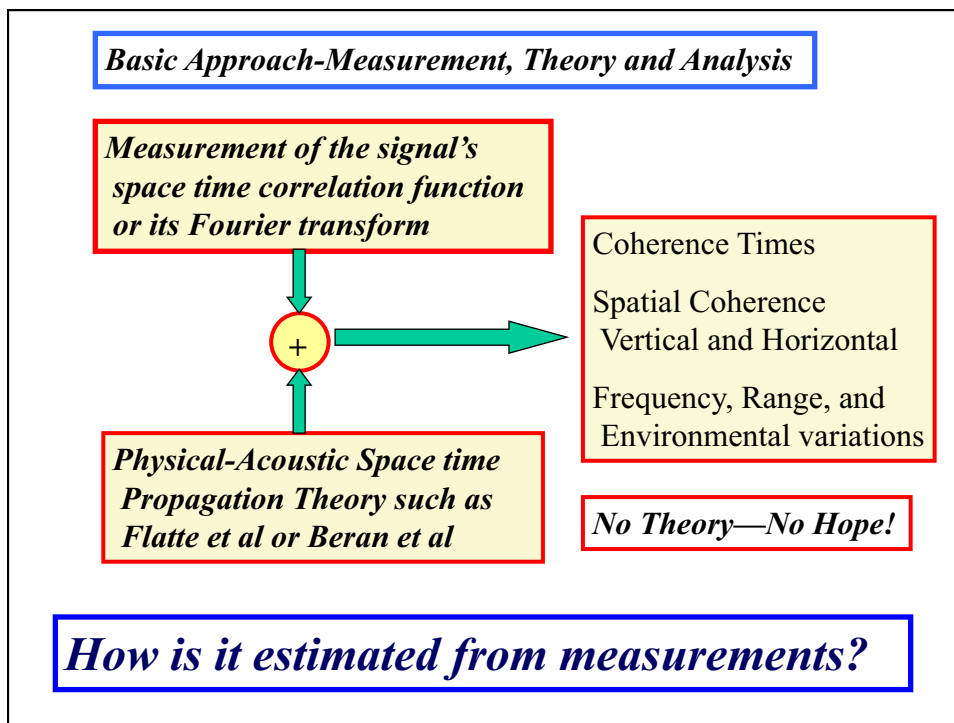
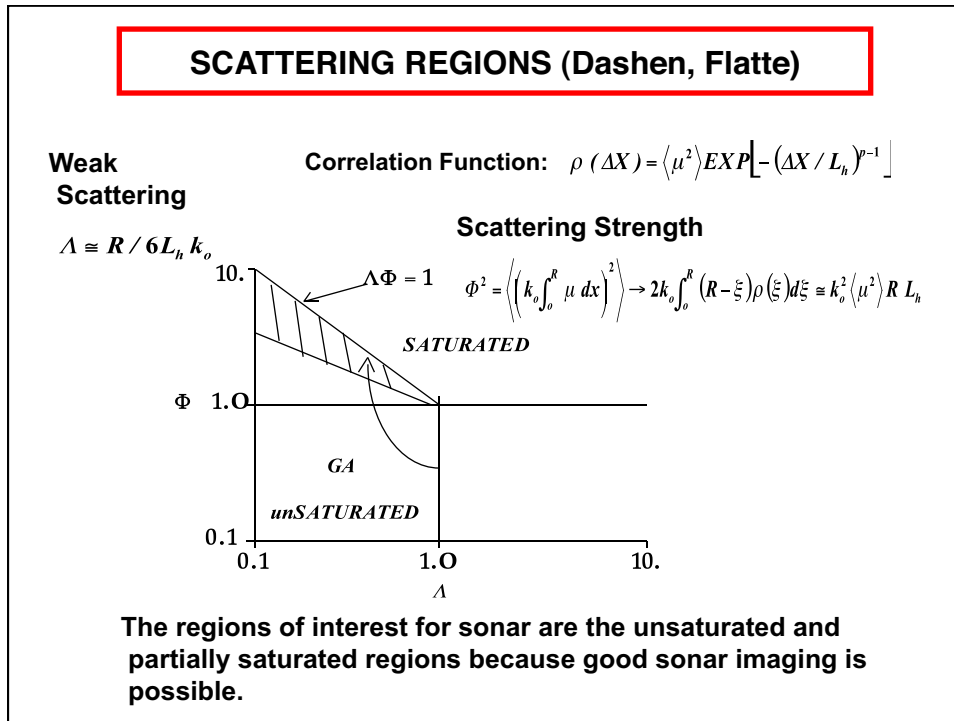
$$\Gamma(\Delta y) = R_p(R, f, \Delta y, z) / I(z) = \exp(-D_{1,2} / 2) = \exp\left(-(\Phi \Delta y / L_{hc} \sqrt{2})^2\right)$$

Generally we write: $\Gamma(\Delta y) = \exp\left(-(\Delta y / L_{hc})^n\right)$.

M.J. Beran, J. J. McCoy, and B. B. Adams, " Effects of a fluctuating temperature field on the spatial coherence of acoustic signals," NRL, Washington, D.C. 20375, NRL Tech. Rept. 7809, 1975 -(Avail DTIC).

R. F. Dashen, S. M. Flatte, W. H. Munk, and F. Zachariasen, " Limits on Coherent Processing Due to Internal Waves," Stanford Research, Menlo Park, Ca, 94025, Stanford Research Rep. Tr-JSR-76-14, 1977-(Avail DTIC).

W. M. Carey and W. B. Moseley, " Space-time processing, environmental-acoustic effects," in Progress in Underwater Acoustics, ed. H. M. Merklinger, Plenum Pub., pp. 743-758, 1987 (IEEE J. Ocean. Eng. 16(3), pp. 285-301, 1991).



Measurement options

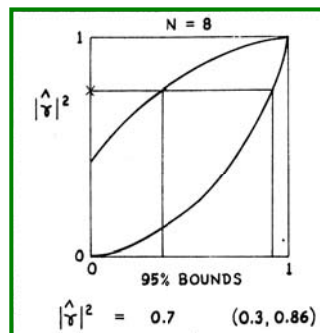
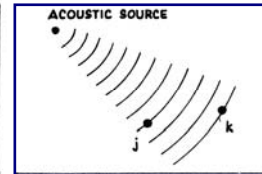
- **Pair-wise coherence: narrowband/broadband signals**
- **Direct measurement of signal gain with a filled aperture with arrivals in the broadside direction.**
- **Steered beam response with a filled or sparse aperture with arrivals near broadside.**
- **Coded Signals (M Sequence) and Replica Correlation: Multi-Path and Modal separation with high signal to noise ratio.**

• What is coherence?
 • How and how accurately do you estimate it?

$$\gamma_{ab}(f) = \frac{G_{ab}(f)}{[G_a(f)G_b(f)]^{1/2}}$$

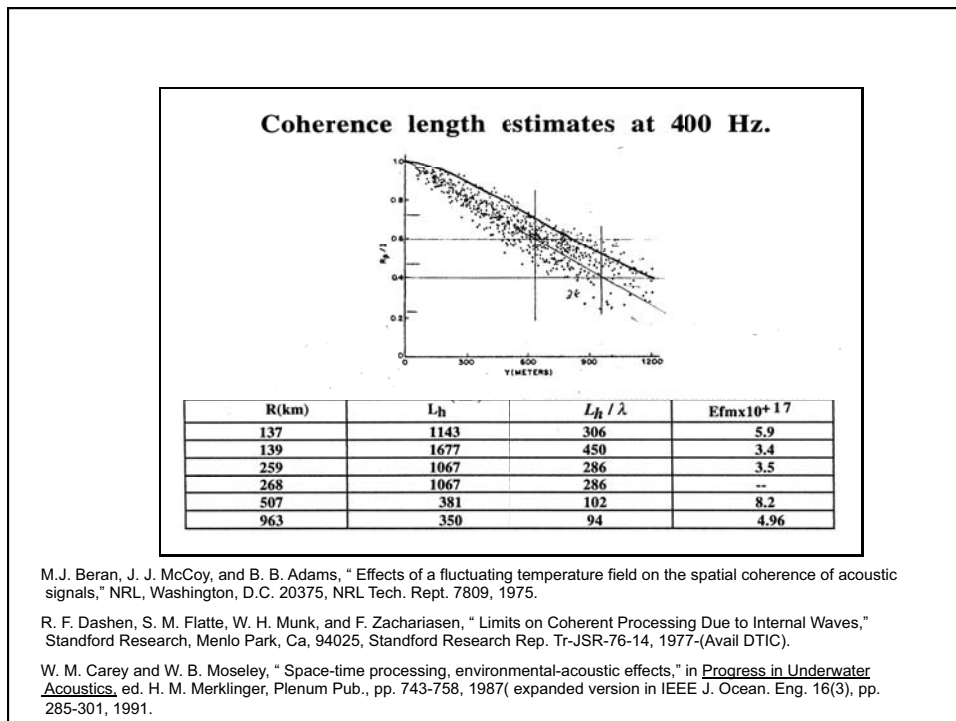
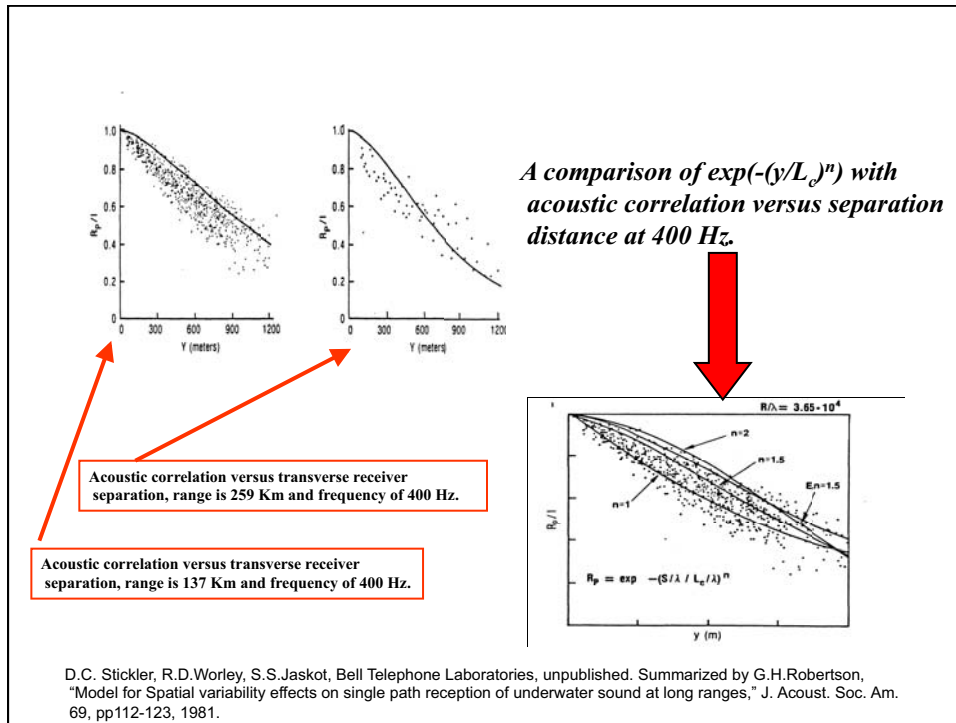
$$0 \leq |\gamma_{ab}(f)|^2 \leq 1, \forall f$$

a, b either source, receiver pair or receiver, receiver pair

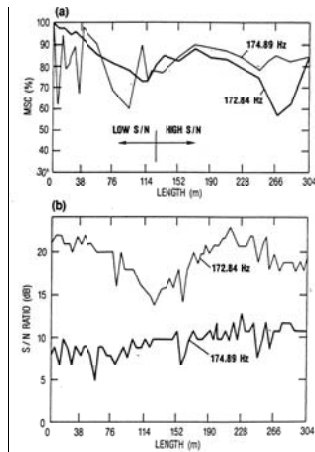


- ### CONCLUSIONS
- COHERENCE
 - NORMALIZED CROSS SPECTRUM
 - SIGNAL TO NOISE MEASURE
 - LINEARITY MEASURE
 - ESTIMATION
 - DIFFICULT
 - BOUNDS LARGE

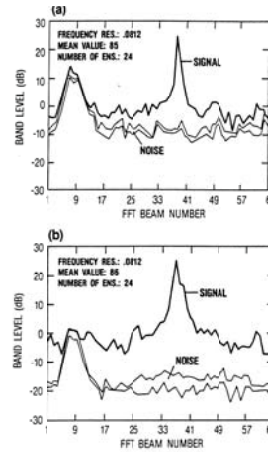
G. C. Carter, C. H. Knapp, and A. H. Nuttall, "Estimation of the magnitude squared coherence function via overlapped FFT processing," IEEE Trans. on Audio and Electroacoustics, AU-21(4), pp.337-344, 1973., E. R. H. Scannell and G. C. Carter, "Confidence bounds for magnitude-squared coherence estimates," Proc. IEEE Int. Conf. on Acoustics, Speech, and Signal Processing, pp.670-673, 1978.



MSC, S/N, and Beam Signal level



MSC is a function of S/N and Noise Coherency and multi-path effects !



Beam signal levels can be separated from noise sources!

W. M. Carey and W. B. Moseley, "Space-time processing, environmental-acoustic effects," in *Progress in Underwater Acoustics*, ed. H. M. Merklinger, Plenum Pub., pp. 743-758, 1987(expanded version in IEEE J. Ocean. Eng. 16(3), pp. 285-301, 1991.

THE EXPECTATION VALUE OF THE MAGNITUDE SQUARED RESPONSE FUNCTION:

$$\langle f_{ip}(\psi_s) \cdot f_{ij}^*(\psi_s) \rangle = \int_{-L/2}^{+L/2} [4P_p \cdot P_j^* / L^2] \exp\left(-\left[\frac{y_p - y_j}{L_h}\right]^2\right) \cdot \exp(i2\psi_s(y_p - y_j)/L) dy_p dy_j$$

This expression may be written in matrix notation:

$$P(k \sin \theta, \omega) = \langle f_{ip}(\psi_s) \cdot f_{ij}^*(\psi_s) \rangle = C^T \cdot R_{CPS} \cdot C$$

where:

$$R_{CPS} = \left\| \left[\frac{4P_p \cdot P_j^*}{L^2} \exp\left(-\left[\frac{y_p - y_j}{L_h}\right]^2\right) \right] \right\|$$

W. M. Carey, "The determination of signal coherence length based on signal coherence and gain measurements in deep and shallow water", J. Acoust. Soc. Am. 104 (2, pt.1), August 1998, 831-837.

MATCH THE ENVIRONMENTAL ANGLE SPREAD TO THE BEAMWIDTH OF THE HALF POWER POINTS!

$$S(k \sin \theta) = \int_{-\infty}^{\infty} \exp(-y/L_h)^n \exp(iky \sin \theta) dy$$

where $\exp(-y/L_h)^n$ is the coherence function

	n = 1	n = 1.5	n = 2	$\Delta\theta_{hp}$
BW(radian)	0.318 λ/L	0.457	0.530	0.886 λ/L
BW(degrees)	18.2°	26.2°	30.36°	50.76° λ/L
L_a/L_h	2.72	1.89	1.64	

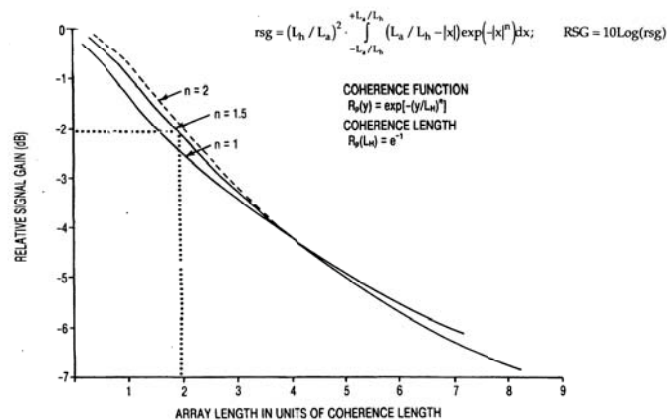
The relative signal gain is:

$$rsg = (L_h/L_a)^2 \int_{-L_a/L_h}^{+L_a/L_h} (L_h/L_h - |x|) \exp(-|x|^n) dx; \quad RSG = 10 \text{Log}(rsg)$$

Thus the gain of the array is related to the coherence length—provided we have a criterion and a coherence functional form!

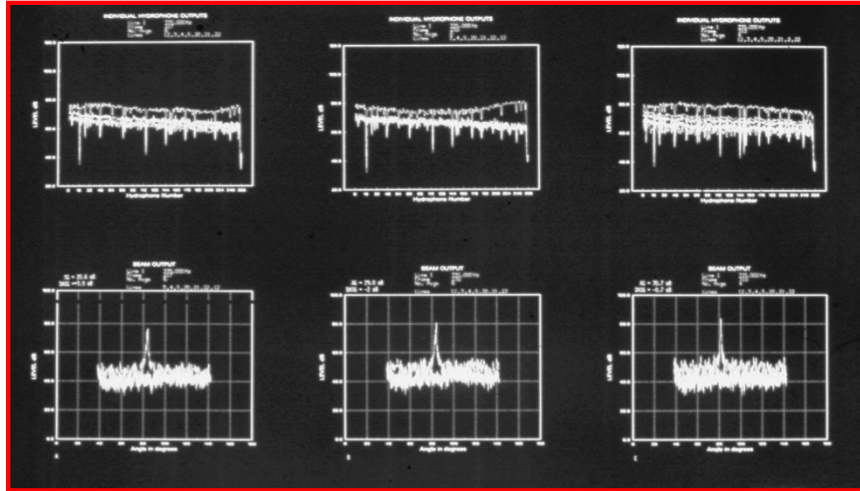
W. M. Carey and W. B. Moseley, "Space-time processing, environmental-acoustic effects," in *Progress in Underwater Acoustics*, ed. H. M. Merklinger, Plenum Pub., pp. 743-758, 1987(expanded version in IEEE J. Ocean. Eng. 16(3), pp. 285-301, 1991.

THE RELATIVE SIGNAL GAIN

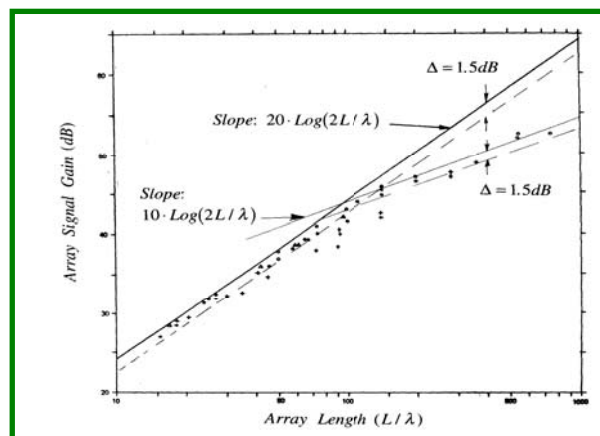


W. M. Carey and W. B. Moseley, "Space-time processing, environmental-acoustic effects," in *Progress in Underwater Acoustics*, ed. H. M. Merklinger, Plenum Pub., pp. 743-758, 1987(expanded version in IEEE J. Ocean. Eng. 16(3), pp. 285-301, 1991.

The illumination and beam pattern for a long range source of sound.

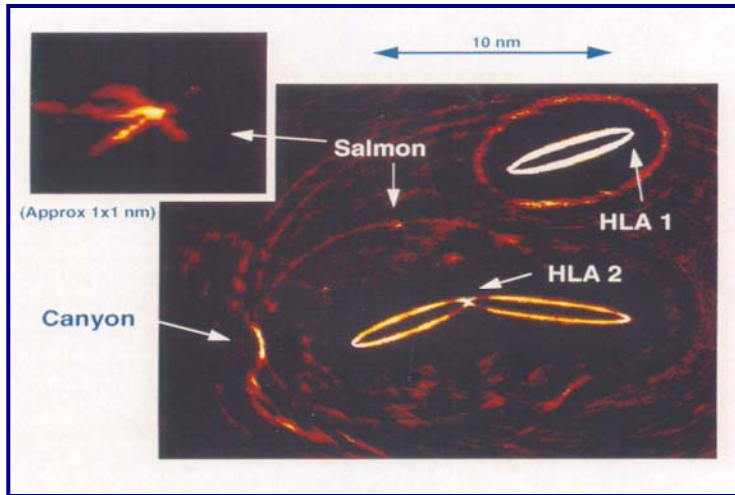


Signal Gain is an indicator of coherence and when interpreted with a wave theoretic model yields coherence length estimates

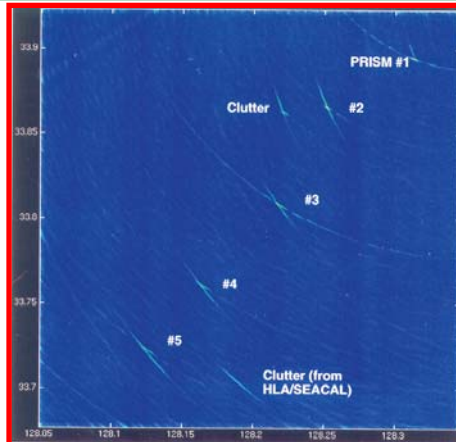
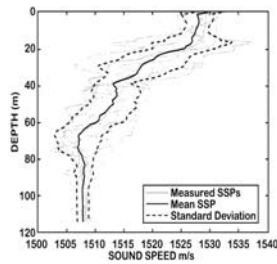


A summary of experimental array estimates

***Shallow water coherent array processing results
North of The Hudson Canyon***



This figure shows a range/cross range plot of echoes received on two experimental arrays ten miles apart. The explosives were detonated midway between the two arrays. The distance to T5 is approximately 20 nmi from the source location. In this 400 nmi square area we observe two target like clutter echoes one of which was cross fixed. However both clutter events had different frequency content than the targets.



These results are from the ACT series of experiments conducted by W. Carey, P. Cable, and J. O'Connor on the Florida Shelf in the Gulf of Mexico, on the Jersey Continental Shelf, and in the Straits of Korea. Explosive sources were developed and deployed by W. Marshall and the analysis was performed by Mike Steele, T. Kooij, J. Angle from BBN Laboratories. In addition the CW sources were deployed by W. Carey and G. Hunsaker, NRAD. Analysis of the Straits of Korea array data was performed by J. Reese, NRAD.

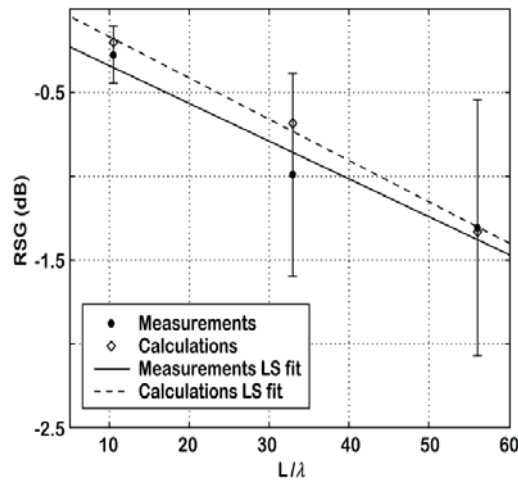
Shallow water coherence length results.

Ref.	[17]	[18]	[19]	[20]	[20]	[20]	[20]	[20]
Location	N. Sea	N.W. Atl.	GM/WFE	GM/FS	NWA/JS	SOK-1	SOK-1	SOK-2
SVP	ISV	DR	DR	DR	DR	DR	DR	DR
WD	65m	0.1-1 km	0.1-1km	200m	100m	100m	100m	100m
Bottom	S	S-SC	S-SC	S-SC	S-SC	S-SC	S-SC	SC-S
f1(Hz)	400	135	173-175	200-400	200-400	354	300	354
f2	800			400-800	400-600	600	500	604
Range km	7.4	100	25	9.3	4-22	7-11	5-45	14-24
L_c	18	31	21	30	23	27	29	38
L_c	10			32	25	30	31	54
Source	Exp.	CW	CW	Exp.	Exp.	CW	Exp.	CW
SD	21m	18m	100m	100m	52m	30 m	52m	33m
RD	15m	750m	400m	200m	100m	101m	101m	94m
COV	8%	4%	6%	4%	4%	4%	5%	2-4%

SVP=Sound Velocity Profile; ISV= Isovelocity; DR=Downward Refracting; WD= Water Depth; S= Sand; S-SC= Sandy- Silty- Clay; SD=Source Depth; RD= Receiver Depth. COV=coefficient of variation in measured results.

W. M. Carey, "The determination of signal coherence length based on signal coherence and gain measurements in deep and shallow water", J. Acoust. Soc. Am. 104 (2, pt.1), August 1998, 831-837.

Wave Theoretic Numerical Modeling provides for Extrapolation



Urlick's Coherence Summary

TABLE 2
TABULAR SUMMARY OF COHERENCE MEASUREMENTS

COHERENCE LENGTH r_c	FREQUENCY	BAND WIDTH	AVERAGING TIME	PROPAGATION PATH(S)	SOURCE DEPTH	RECEIVER DEPTH	SOURCE RECEIVER RANGE	HYDROPHONE ORIENTATION	REFERENCE
>3 km ($\lambda=0.8$, constant to 3 km)	7.5 kHz cw	Narrow	0.1 sec	Single, Near Surface	Shallow	---	0.6-12 km	Horizontal Transverse	16
6 meters 1 meter	4, 7, 15 kHz	Pulses	3 and 30 ms Pulses	Surface Reflection	Shallow Water, Depth 80 m.	---	600 m.	Horizontal Transverse Vertical	17
>4000 ft. 1600 ft.	400 Hz	Pulses	0.1 hr.	Single, Refracted	---	Artemis*	19 mi.	Horizontal Vertical	19
1300 ft. 700 ft.	400 Hz	Pulses	---	Single, Refracted	1200 ft	Artemis*	270 mi.	Horizontal Vertical	18
48 ft. 6 wavelengths	750 Hz cw	Narrow	106 ms	Surface Reflected	9500 ft	TVA**	24 mi.	Vertical	9
300 ft.	800 Hz Pulses	---	5 ms	Single, Refracted	9500 ft	TVA**	24 mi.	Vertical	20
8 wavelengths	50-100 Hz 100-200 Hz 200-400 Hz	Octave	Shot Duration	Multiple Refracted	800 ft.	TVA**	100-600 mi.	Vertical	21
300 ft. 1000 ft.	367 Hz	Narrow	840 sec 150 sec	Multiple Refracted (RSR)	---	Artemis*	700 mi.	---	22
2 wavelengths 8 wavelengths 10 wavelengths 30 wavelengths	2.1-3.3 kHz 1.5-1.7 kHz 0.7-0.9 kHz 0.3-0.5 kHz	200 Hz	Shot Duration	Shallow Water Paths	Shallow Water, Depth 65 m.	---	4 mi.	Horizontal Transverse	23
500 ft. ±	44-88 Hz 177-354 Hz 354-707 Hz	Octave	2 sec	Surface Bottom Reflections	15 ft.	4000 ft.	1 ky 2 ky 3 ky	Vertical	14

*Hydrophone field on bottom 2000 - 4000 feet depth on channel axis near Bermuda.
**Vertical string on bottom at 14,000 feet near Bermuda (TRIDENT Vertical Array).

Work performed before 1978-Darpa 1979-PO, Stefanick 1987

Conclusions

- Deep water (Sofar Propagation):
 - A summary of single path and array signal gain measurements show that coherence lengths of 100λ are achievable at a frequency near 400 Hz at ranges of 400 Km.
 - This length should scale with frequency to the five halves power and range to the two thirds power.
- Shallow Water (Downward Refraction, Coastal, Sandy):
 - RASG estimate show that a coherence length of 30λ at a range of 40 Km is a reasonable number.
 - Theoretical description is lacking.
 - Most likely cause is the combination of volume fluctuations in temperature, salinity and sound speed.*

*W. M. Carey, J.F. Lynch, W.L. Siegmann, Et al , "Sound Transmission and Spatial Coherence in Selected Shallow Water Areas: Measurements and Theory" , Theoretical and Computational Acoustics 2003, eds. A. Tolstoy et al, World Scientific, New Jersey, 2004, pp 38-71. (in Press, JCA 14 (2) , 2006).

A Key Sonar Issue :

How large of a transverse-horizontal aperture will be supported by the deep ocean and shallow water wave guide?

OR

What is the azimuthal spread of the signal due to long range propagation?

Presentation References

- [1] W.M. Carey and R.A.Wagstaff, "Low frequency noise fields," J. Acoust. Soc. Amer. 80(5), pp. 1523-1526,1986.
- [2] G. C. Carter, C. H. Knapp, and A. H. Nuttall, " Statistics of the estimate of the magnitude coherence function," IEEE Trans. on Audio and Electroacoustics, AU-21(4),pp.388-389,1973.
- [3] G. C. Carter, C. H. Knapp, and A. H. Nuttall, " Estimation of the magnitude squared coherence function via overlapped FFT processing," IEEE Trans. on Audio and Electroacoustics, AU-21(4), pp.337-344,1973.
- [4] E. R. H. Scannell and G. C. Carter," Confidence bounds for magnitude-squared coherence estimates," Proc. IEEE Int. Conf. on Acoustics, Speech, and Signal Processing, pp.670-673, 1978.
- [5] W. M. Carey and W. B. Moseley, " Space-time processing, environmental-acoustic effects," in Progress in Underwater Acoustics, ed. H. M. Merklinger, Plenum Pub., pp. 743-758, 1987(expanded version in IEEE J. Ocean. Eng. 16(3), pp. 285-301, 1991.
- [6] W.M. Carey, J. Reese, C. Stuart, " Mid-frequency measurements of array signal and noise characteristics" IEEE J. Ocean. Eng. 22(3), 548-565, 1997.
- [7] N. Hastings and J. Peacock, Statistical Distributions, John Wiley and Sons, New York, pp.13, 1974.
- [8] R. Urick, Principles of Underwater Sound, 2nd edition, McGraw Hill, New York, p-34,1974.
- [9] Y. S. Shifrin, Statistical Antenna Theory, Golem Press, 1971.
- [10] H. Cox, " Line array performance when the signal coherence is spatially dependent," J. Acoust. Am. 54, pp. 1743-1746, 1973.
- [11] M.J. Beran, J. J. McCoy, and B. B. Adams, " Effects of a fluctuating temperature field on the spatial coherence of acoustic signals," NRL, Washington, D.C. 20375, NRL Tech. Rept. 7809, 1975.
- [12] R. F. Dashen, S. M. Flatte, W. H. Munk, and F. Zachariasen, " Limits on Coherent Processing Due to Internal Waves," Stanford Research, Menlo Park, Ca, 94025, Stanford Research Rep. Tr-JSR-76-14, 1977-(Avail DTIC).

- [13] D.R.Morgan, T.M. Smith, "Coherence effects on the detection performance of quadratic array processors, with applications to large-array matched-field beamforming," J. Acoust. Soc. Am. 87(2), pp737-747,1990.
- [14] D.C. Stickler, R.D.Worley, S.S.Jaskot, Bell Telephone Laboratories, unpublished. Summarized by G.H.Robertson, "Model for Spatial variability effects on single path reception of underwater sound at long ranges," J. Acoust. Soc. Am. 69, pp112-123, 1981.
- [15] W.B.Moseley, D.R.Del Balso,"Horizontal random temperature structure in the ocean," J. Phys. Oceanog. 6, pp267-280, 1976.(Also NRL Rpt.7673,NRL,Wash. D.C.,1974)
- [16] W.B. Moseley,"Acoustic coherence in space time-an overview," in Proc. EASTCON'78 Also: "Geographic variability of spatial signal correlation and subsequent array performance," in Proc. Int. Symp. Underwater Acoustics, Tel Aviv, Israel,1981.
- [17] P. Wille and R.Thiele, "Transverse horizontal coherence of explosive signals in shallow water," J. Acoust. Soc. Am. 50 (1pt.2), pp. 348-353,1971.
- [18] W.M. Carey," Measurement of down-slope sound propagation from a shallow source to a deep ocean receiver," J. Acoust. Soc. Am. 79(1), pp. 49-59, 1986.
- [19] W. M. Carey, I. B. Gereben, and B. A. Brunson, " Measurement of sound propagation downslope to a bottom-limited sound channel", J. Acoust. Soc. Am. 81(2), pp. 244-257, 1987.
- [20] These results are from the ACT series of experiments conducted by W. Carey, P. Cable, and J. O'Connor on the Florida Shelf in the Gulf of Mexico, on the Jersey Continental Shelf, and in the Straits of Korea. Explosive sources were developed and deployed by W. Marshall and the analysis was performed by Mike Steele, T. Kooij, J. Angle from BBN Laboratories. In addition the CW sources were deployed by W. Carey and G. Hunsaker, NRAD. Analysis of the Straits of Korea array data was performed by J. Reese, NRAD.

- [21] R. Scholz, "Horizontal Spatial Coherence Measurements with Explosives and CW Sources in Shallow Water," in Aspects of Signal Processing, Part I, G. Tacconi (ed.), D. Reidel Publishing Co., Dordrecht-Holland, 95-108, 1977.
- [22] B. Gomes and J. Mathews, " West Florida Shelf Environment for the Area Characterization Test I (ACT I),"NRL/MR/7182-93-7061, NRL, Stennis Space Center, Ms., July 8,1994.
- [23] O. Marcia , " Act II Sea Test XBT Results," BBN Memorandum to W. Carey, Oct. 1993.
- [24] P. Bucca, J. Fulford, J.Lynch, and A.Newhall, " Environmental Variability During the Third Acoustic Characterization Test (ACT III) in the Strait of Korea," NRL/FR/7182-97-9667, NRL, Stennis Space Center, Ms., July 28,1997.
- [25] A. Wasiljeff, "Spatial Horizontal Coherence of Acoustical Signals in Shallow Water," SACLANTCEN SM-68, SACLANT ASW Research Centre, LaSpezia, Italy, May 1975.
- [26] W. M. Carey, "The determination of signal coherence length based on signal coherence and gain measurements in deep and shallow water", J. Acoust. Soc. Am. 104 (2, pt.1), August 1998, 831-837.
- [27] W. M. Carey, J.F. Lynch, W.L. Siegmann, Et al.,"Sound Transmission and Spatial Coherence in Selected Shallow Water Areas: Measurements and Theory" , Theoretical and Computational Acoustics 2003, eds A. Tolstoy et al, World Scientific, New Jersey, 2004, pp 38-71. (Also accepted for publication in the Journal of Computational Acoustics , 2004)