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1aAO2. Shear waves and the discrepancy between perceived and ideal frequency power laws for sediment attenuation

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Inverse techniques based on data for long range propagation in shallow water have recently inferred that the attenuation in certain marine sediments varies at low frequency as the 1.8 power. Idealized models predict the exponent to be exactly 2.0. The inverse inferences usually assume the bottom is a fluid, and this is ordinarily a good approximation because the shear wave speed in bottom sediments is typically very small. Direct numerical simulation [J. M. Collis et al., Proc. Oceans 2007, Aberdeen (2007)] indicates that shear waves make a sufficient contribution to shallow water attenuation that could account for the small discrepancy in exponents. To better assess whether this is the case, the present paper analyzes the effect of shear waves on modal attenuation. The Pekeris model with a lower elastic halfspace is used with the shear wave speed taken to be substantially less than the sound speed in water. The derived dispersion relation has complex roots for the horizontal wave number, and the imaginary part, found by a perturbation analysis, predicts that the shear wave contribution to the modal attenuation is proportional to the cube of the ratio of the shear wave to water sound wave speeds. [Work supported by ONR.]

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Shear waves and the discrepancy between perceived and ideal frequency power laws for sediment attenuation

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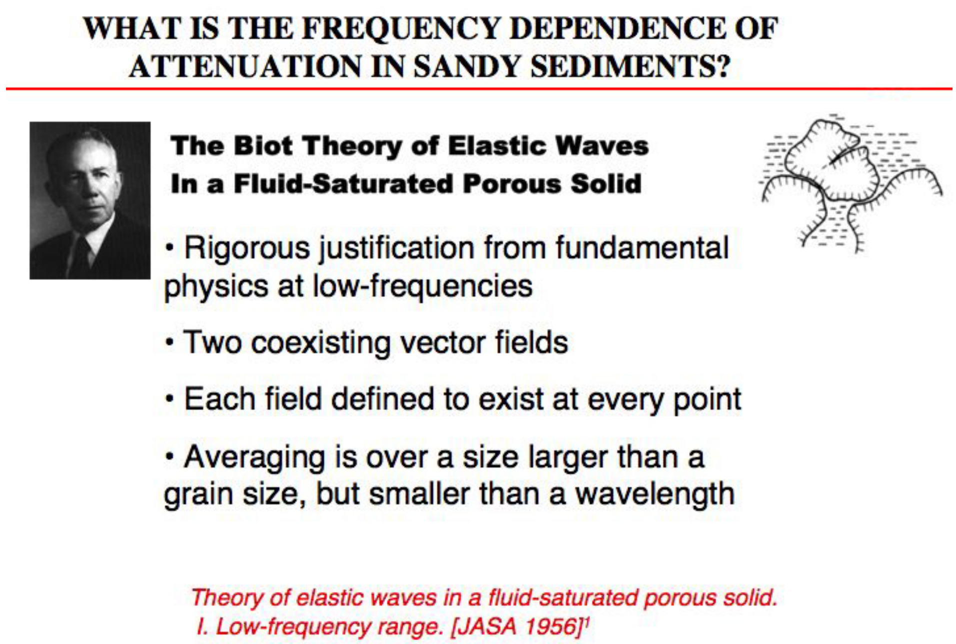
1. INTRODUCTION

The frequency dependence of the attenuation of acoustic waves in various types of materials continues to be a topic of research. The present discussion is prompted by recent papers that have inferred the low-frequency dependence of attenuation in broad categories of marine sediments. The specific focus of this discussion will be on sandy sediment types and frequencies less than 1 kHz.

A primary basis for the theory of elastic waves in a porous medium is that of Biot [1]. The theory was rigorously derived from fundamental physics, and has subsequently been rigorously verified using fundamental mathematics.



Considering a plane wave, and starting with Biot's wave equation one can derive a dispersion relationship for the wavenumber k, the imaginary part of which is the compressional wave attenuation. Using a power series expansion, it can been seen that the attenuation is proportional to the frequency squared.

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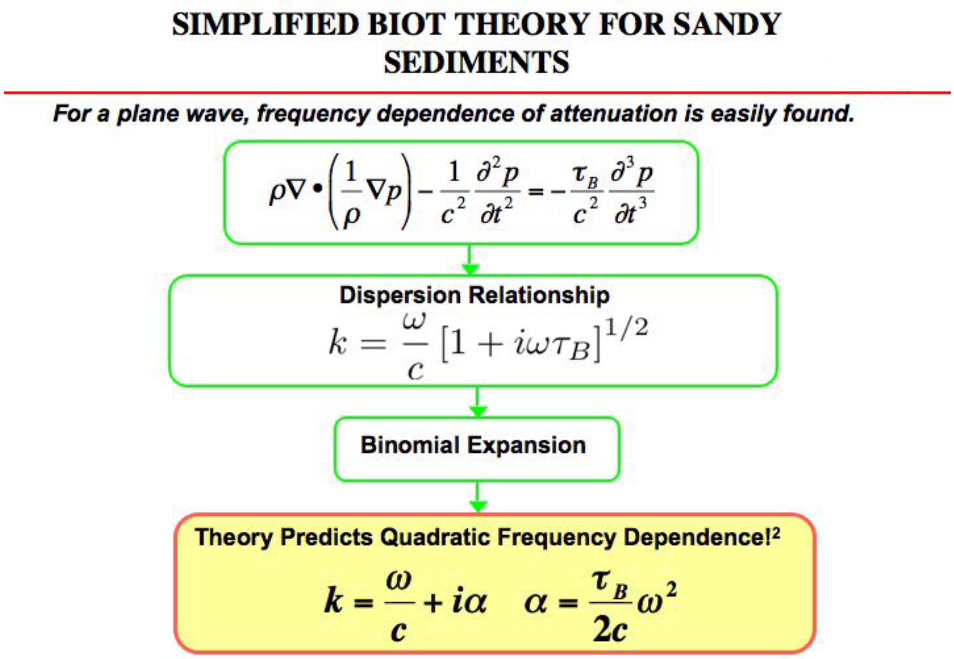
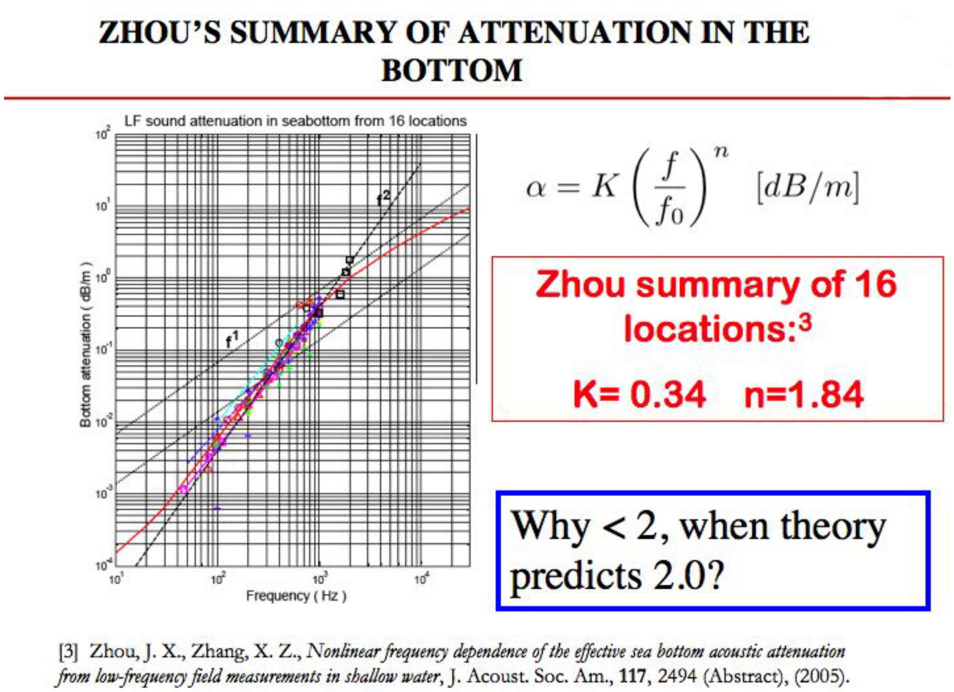


In a survey of experimentally measured datasets, a similar result (that compressional wave attenuation is nonlinearly dependent on frequency) has been reported by Zhou and Zhang [3]. If the compressional wave attenuation is written as

$$\alpha = K\,(f/f_0)^n, \tag{1}$$

for K an intrinsic sediment attenuation, f a given frequency, f_0 a reference frequency (taken here to be 1 kHz), and n the frequency exponent, transmission loss data can be fit to determine n. Shown below are Zhou's result and conclusion.



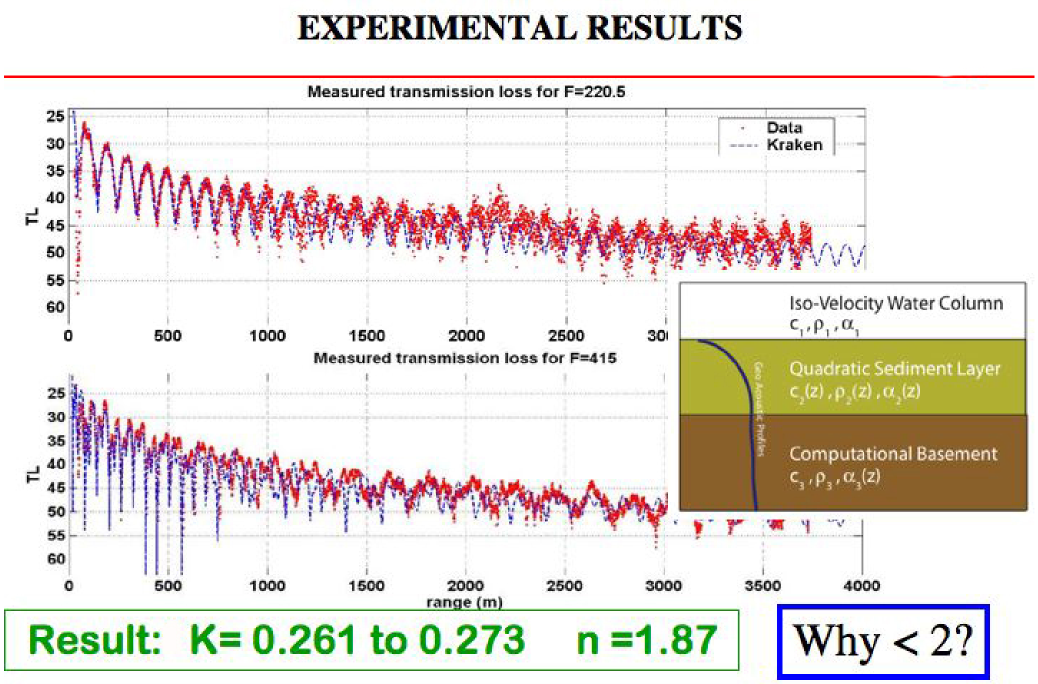
A natural question is why the experimentally determined exponent ($n = 1.84$) isn't as expected from theory ($n = 2.0$).

2. NONLINEAR FREQUENCY DEPENDENT ATTENUATION

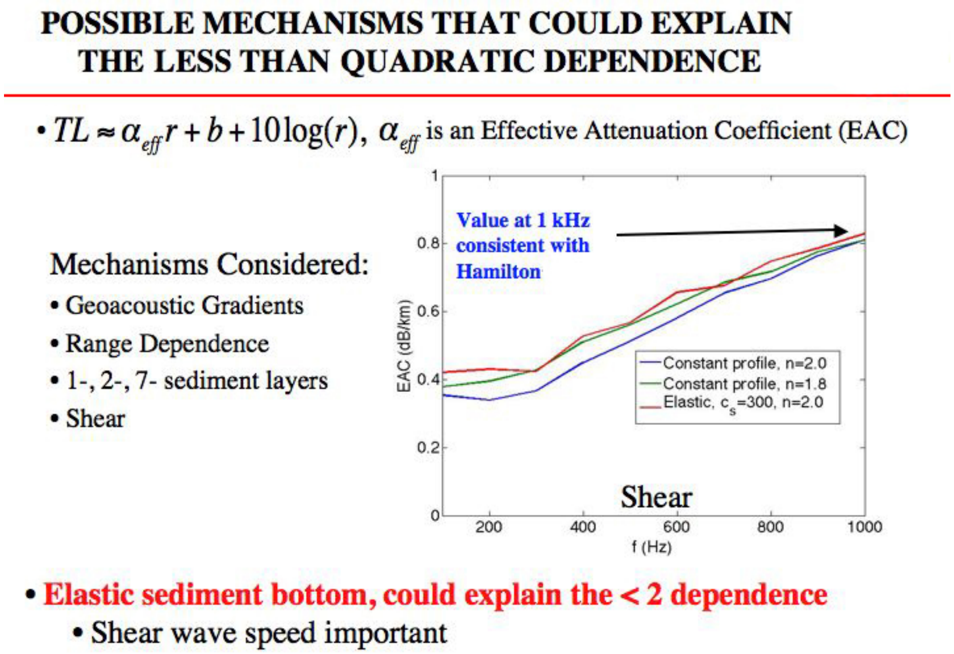
Subsequent to the work of Zhou, a careful study of experimental data was conducted by Holmes et al. [4], focusing specifically on sandy-silty sediment bottoms in propagation environments that are well known. The study found that the best fit to a large amount of data yielded

$$\alpha = 0.35 \left(\frac{f}{f_0} \right)^{1.8} . \tag{2}$$

As part of the study, Holmes conducted an experiment in Nantucket Sound, Massachusetts (an area that is well know), to collect transmission loss data over a range of frequencies. Using a simplified geoacoustic model of an isovelocity water layer, over a frequency dependent sediment layer, and computational basement, he concluded that the site-specific exponent was less than quadratic [5].



To help explain the apparent discrepancy between theory and experiment, numerical studies were conducted to consider various physical mechanisms that could explain the difference. Transmission loss data was range and depth averaged and then linear least squares fit to give an effective attenuation coefficient (EAC). By fixing the sediment attenuation in the uppermost layer at values of $n = 1.8$ and $n = 2.0$, the different physical mechanisms were considered and compared [6].

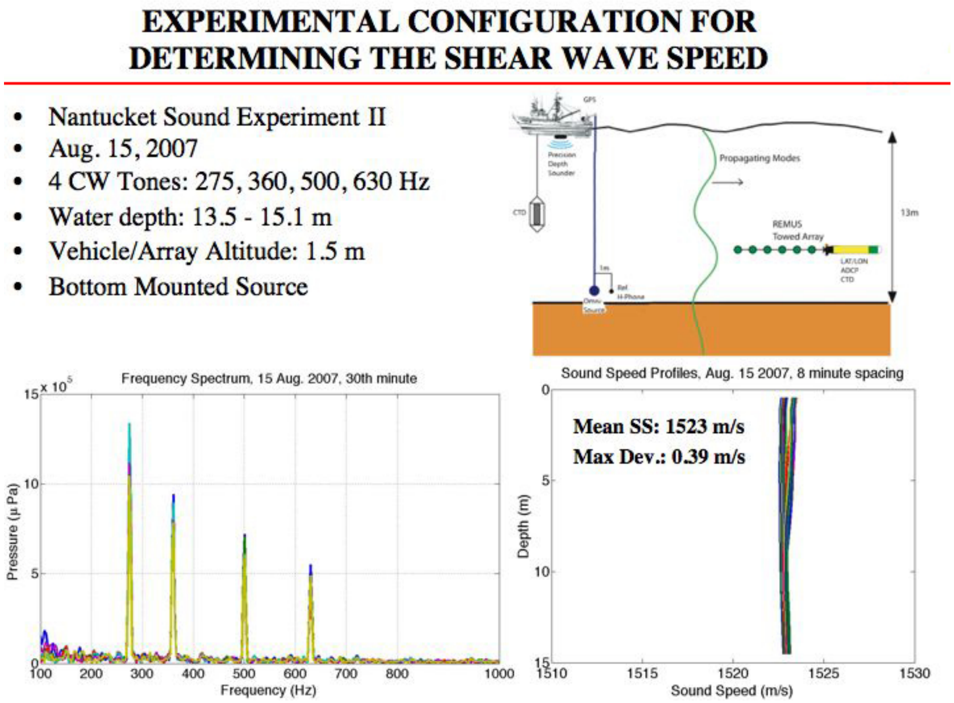


The only mechanism considered that came close to describing the less than quadratic dependence was that of shear in an elastic bottom. Important to the study was that the shear wave speed was much less than that of the water sound speed, on the order of 300 m/s.

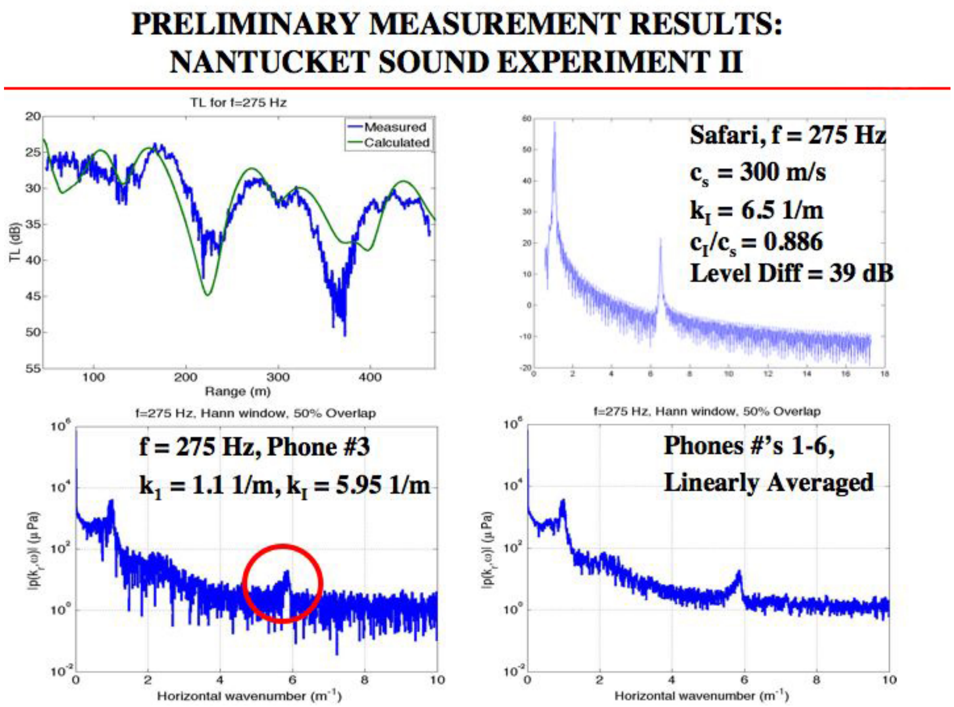
3. EXPERIMENTAL DETERMINATION OF SHEAR WAVE SPEED

A second experiment was conducted in the waters of Nantucket Sound, to try to implicitly determine the shear wave speed in the sandy sediment at the site by measuring a seismic interface wave along the ocean sediment interface. The shear wave field is evanescent in the water, and is expected to decay exponentially in the water away from the interface.

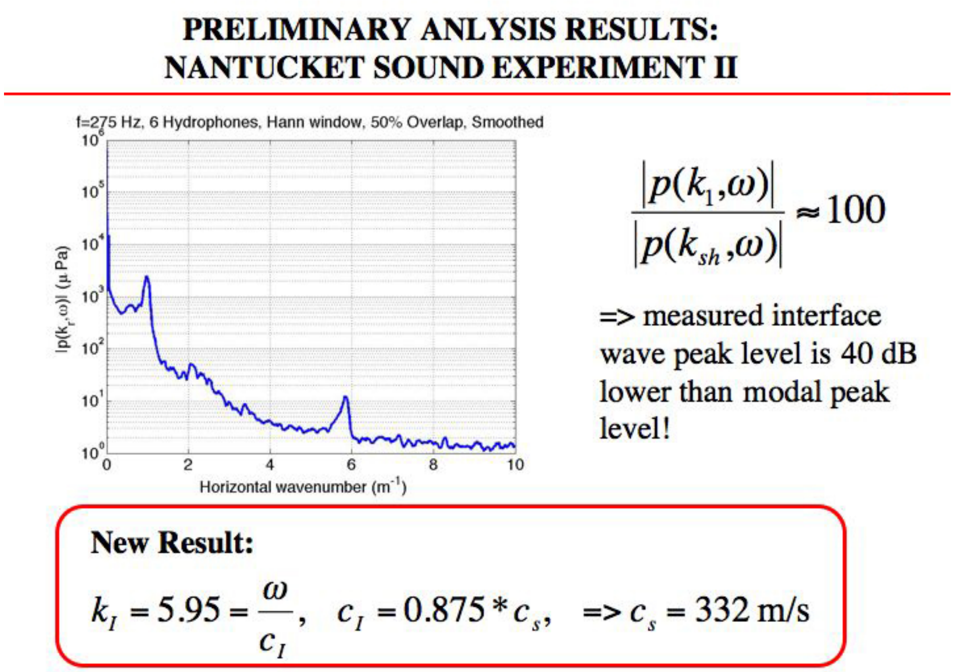
The idea behind the experiment was to mount a sound source on the bottom and have an autonomous underwater vehicle (AUV) tow a 6-element hydrophone array as close to the bottom and sound source as possible. The AUV track looped around the source, going out to a maximum range of 500 m before turning to make another run. Four continuous wave tones were put into the water by the source. The water was essentially isovelocity at 1523 m/s.



Preliminary results from the experiment are shown in the figure below. The top left pane shows transmission loss data from the experiment compared against calculations. The agreement provided a 'sanity check' that the data was of good quality. The top right pane shows the horizontal wavenumber spectrum computed prior to the experiment. The acoustic modal peak can be seen around $k = 1\ m^{-1}$ and the interface wave peak around $k = 6\ m^{-1}$. The bottom left pane shows the result of performing a Hankel transform on the transmission loss data for one element of the towed array data. The acoustic modal peak can be seen around $k = 1\ m^{-1}$, and another peak around $k = 6\ m^{-1}$. The bottom right pane shows the same result, but using an average of all of the elements in the array, yielding a cleaner plot with more clearly resolved peaks.



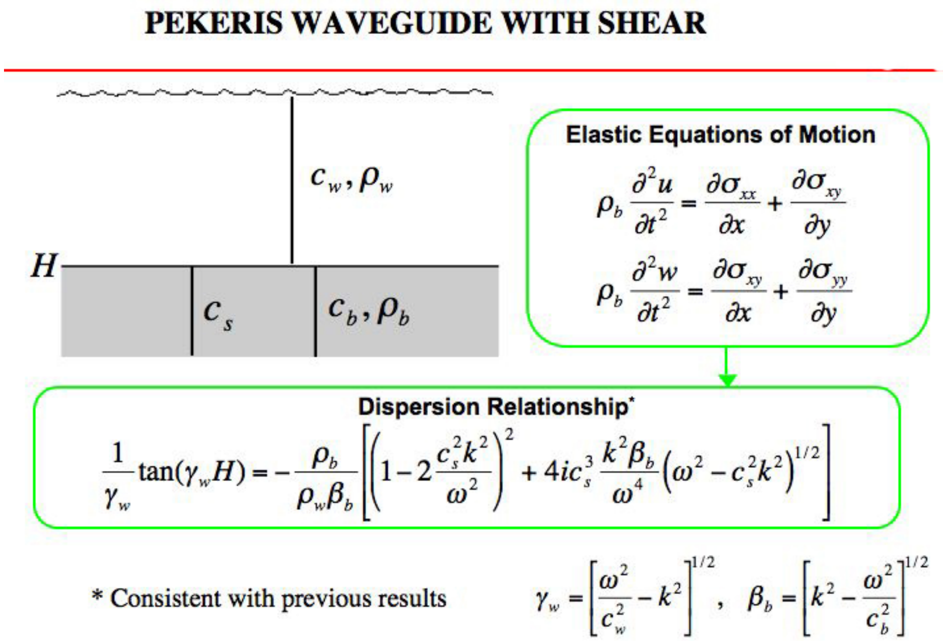
Using a moving window average on the 6-phone result, the wavenumber plot is smoothed to show the trapped mode peak around $k = 1\ m^{-1}$ and the interface wave peak around $k = 6\ m^{-1}$.



Note that the relative level differences of the water-borne modes to that of the interface wave peak are about 40 dB which is consistent with what was calculated numerically. Taking the value of the interface wave peak as 5.95 and using the fact that the interface wave speed is approximately 87.5% that of the shear wave speed, the shear wave speed is found to be consistent with what was expected from calculations. As the shear wave speed has not been measured in this manner before,the ability to measure the shear wave speed from a water borne vehicle presents a potentially powerful new survey technique.

4. PEKERIS WAVEGUIDE WITH SHEAR

The classic ocean waveguide problem presented by Pekeris [7] is considered and extended to include an elastic bottom. Starting with the elastic equations of motion, a dispersion relation is derived that relates the horizontal wavenumber k to the angular frequency ω. The result is consistent with the past work of Williams et al. [8].



To solve the dispersion relationship the parameter μ, defined as a nominal value plus a small perturbation, is introduced [9]. Grouping constants and defining a 'dimensionless frequency' produces a simpler-looking dispersion relationship that is now parameterized by μ.

PERTURBATION ANALYSIS (1)

- **To Solve Dispersion Relationship, Introduce Parameter:**

$$\mu = \mu_0 + \Delta\mu, \quad \text{and let,} \quad k = k(\mu,\omega) \quad => \quad k = k(\mu_0,\omega) + \Delta k(\mu_0,\omega)$$

- **Set:**

$$\gamma_w = \omega\left[\frac{1}{c_w^2} - \frac{1}{c_b^2}\right]^{1/2} \cos(\mu)$$

$$\beta_b = \omega\left[\frac{1}{c_w^2} - \frac{1}{c_b^2}\right]^{1/2} \sin(\mu)$$

- **Define "dimensionless frequency":**

$$\Omega = H\left[\frac{1}{c_w^2} - \frac{1}{c_b^2}\right]^{1/2} \omega$$

- **Dispersion Relation Becomes:**

$$b\tan(\Omega\cos(\mu)) = -\cot(\mu)[1 + pert], \quad b = \frac{\rho_w}{\rho_b}$$

- **Or, Writing Formally:**

$$F_0(k,\omega) + F_1(k,\omega) = 0, \quad F_1 = \text{perturbation}$$

Writing the dispersion relationship formally, and using the definition of the derivative, the relationship is expanded and terms of the same order are equated. Solving for the imaginary part of the horizontal wavenumber, an expression for the attenuation of individual modes over a shear-supporting bottom is derived.

PERTURBATION ANALYSIS (2)

- **Can write:**

$$F_0(k(\mu_0,\omega),\omega) + \Delta k \frac{\partial \mathrm{F}_0}{\partial \mathrm{k}}(k(\mu_0,\omega),\omega) + F_1(k(\mu_0,\omega),\omega) + \text{higher order terms} = 0$$

- **Giving:**

$$F_0(k(\mu_0,\omega),\omega) = 0,$$

$$\left[\frac{\partial \mathrm{F}_0}{\partial \mathrm{k}}(k(\mu_0,\omega),\omega)\right]\Delta k = -F_1(k(\mu_0,\omega),\omega)$$

- **Solving yields:**

$$\mathrm{Im}(\Delta k) = \frac{c_b}{T(\mu_0)H}\left(\frac{1}{c_w^2} - \frac{1}{c_b^2}\right)^{1/2}\left[\frac{\rho_w \Omega \sin^2(\mu_0)\cos^2(\mu_0)}{\rho_w + \rho_b \Omega \sin(\mu_0)\cos^2(\mu_0)}\right]\left[4\frac{c_s^3}{c_b^2}T^2(\mu_0)\sin(\mu_0)\left(\frac{(c_b^2 - c_w^2)(c_b^2 - c_s^2 T^2(\mu_0))}{c_b^4 c_w^2}\right)^{1/2}\right],$$

$$\text{using,} \quad k = \left(\frac{\omega}{c_b}\right)T(\mu_0), \quad T = \left(1 + \left(\left(\frac{c_b}{c_w}\right)^2 - 1\right)\sin^2(\mu_0)\right)^{1/2}, \quad \text{and,} \quad \Omega_n = \frac{1}{\cos(\mu_0)}\left[(n + 1/2)\pi + \tan^{-1}(b\tan(\mu_0))\right]$$

- **In Words: We Have an Expression for Modal Attenuation!**
- **Proportional to shear sound speed cubed**

Note that the attenuation is proportional to the shear sound speed cubed.

To assess the effects of shear, the pressure field is written as a modal sum, and individual modal attenuations are expressed as the sum of a compressional term and a shear term. The idea of this (continuing) work is to equate the compressional and shear terms to the frequency exponent n. Previous work gives modal attenuation values for a fluid bottom in a Pekeris waveguide.

COMPRESSIONAL AND SHEAR ATTENUATION

Modal Sum for the Pressure Field:

$$p(r,z) = \sum_{j=1}^{\infty} A_j \psi_j(z) \frac{e^{ik_j r}}{\sqrt{k_j r}}, \quad k_j = \frac{\omega}{v_j} + i\alpha_j$$

Modal Attenuation Coefficient, j^{th} mode:

$$\alpha_j(f) = \underbrace{\boxed{K_j(f)\alpha_{sed}}}_{\text{Intrinsic Compressional Term}} + \underbrace{\boxed{L_j(f)c_s^3}}_{\text{Shear Term}}$$

Earlier Work Provides (for a Pekeris waveguide):

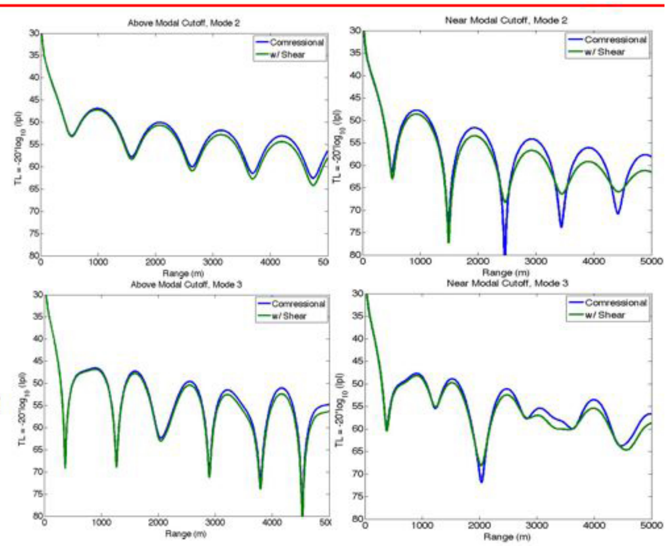
$$K_j = \frac{(v_j/c_b)b}{\Omega \sin(\mu_0)\left(1 + b^2 \tan^2(\mu_0)\right) + b\left(1 + \tan^2(\mu_0)\right)}$$

Idea: Relate K and L to the frequency exponent to quantify effects of shear

Initial transmission loss comparisons of modal attenuation coefficients for the Pekeris model with and without shear are presented in the figure below. The parameter μ_0 is varied from $\mu_0 = 0$ (mode cutoff) to $\mu_0 = \pi/2$. Near cutoff the greatest modal attenuation is expected.

SIMPLE COMPARISONS

- Calculated solution for Pekeris waveguide, MACs varied
- Vary parameter μ_0
  - $\mu_0 = 0$,
  - => mode cutoff
- Expect greater effective attenuation for higher order modes
- H=100 m
- r_{max}= 5 km
- c_s=300 m/s



Modal Attenuation is Greater With Shear

Initial calculations yield greater modal attenuation when shear is included in the model.

5. CONCLUDING REMARKS

The conclusions resulting from this study, based on theory, numerical analysis, and experimental data are summarized here.

CONCLUSIONS

- **Theory predicts quadratic nonlinear frequency dependent compressional wave attenuation**
- **Experimentally determined frequency dependent exponent is n=1.8±0.2**
- **Numerical studies show that the observed less than quadratic frequency dependence can be explained by shear wave leakage.**
- **Measured horizontal wavenumber spectrum yield shear wave speeds consistent with values required in the numerical studies.**
- **New expression derived for modal attenuation in a Pekeris waveguide with shear**
- **Modal attenuation is always greater when shear effects are allowed**

6. Ackowledgments

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