Shelf Break Experiment

1 Introduction

The original objective of this project was to experimentally investigate the shelf-break front, comparing the experimentally obtained angle of the density surface to theoretical values. The actual setup of the experiment, however, proved to challenging in itself. Therefore, this project focuses on documenting the various attempts to simulate the shelf-break front, describing in detail the final setup with most of the kinks worked out, and quantifying the results of the experiment. Some theory is presented, however it is no longer the focus of this report.

2 Theory

The shelf-break experiment is formulated in terms of a geostrophic adjustment problem. Consider a two-layer system of fluid governed by:

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} - fv = -g'\frac{\partial h}{\partial x} \quad (2.1)$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + fu = 0 \quad (2.2)$$

$$\frac{\partial h}{\partial t} + \frac{\partial hu}{\partial x} = 0 \quad (2.3)$$

$$q = \frac{f + \frac{\partial v}{\partial x}}{h(x)} \quad (2.4)$$

where $g' = \frac{\rho_s - \rho_f}{\rho}$. Here we have neglected all variation in y, but allow for a velocity v in that direction. The problem is assumed to be axis-symmetric, which means only a two-dimensional slice of the solution is considered, and the v velocity essentially becomes the tangential velocity. Note that due to the hydrostatic assumption, the problem reduces to a one-dimensional problem. We consider the initial condition as depicted in Figure [1a], that is, an initial step-function of denser and lighter fluid. For the final state (Figure [1b]) we assume steady-state, and no radial velocity $u = 0$.

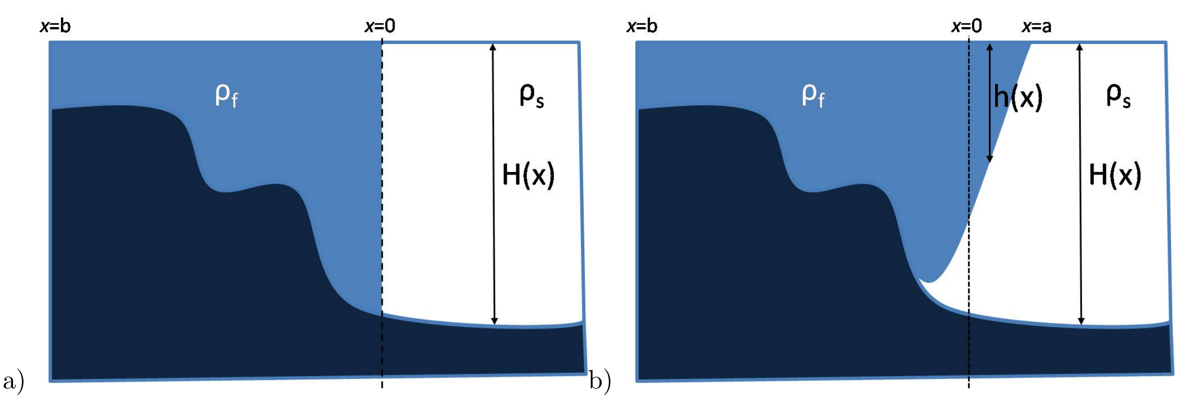


Figure 1: Schematic for theoretical calculations

At the final time the equations above reduce to:

$$-fv = -g'\frac{\partial h}{\partial x} \tag{2.5}$$
$$0 = 0 \tag{2.6}$$
$$0 = 0 \tag{2.7}$$
$$q = \frac{f + \frac{\partial v}{\partial x}}{h(x)} \tag{2.8}$$

since we assume steady state and no u velocity at the final time. That leaves two equations to solve the problem. Applying conservation of PV on a column of fluid we can write:

$$q_i = q_f \tag{2.9}$$
$$\frac{f}{H(x)} = \frac{f + \frac{\partial v}{\partial x}}{h(x)} \tag{2.10}$$

Taking $\frac{\partial}{\partial x}$ of eq[2.5], and substituting into eq[2.10] we have an expression for $h(x)$ in the interior:

$$\frac{\partial^2 h}{\partial x^2} - \frac{f^2}{gH(x)}h = -\frac{f^2}{g'} \tag{2.11}$$

If $H(x)$ was a constant, we could solve these equations analytically, but because we are considering a generally shaped geometry we have to solve it numerically. What remains is to specify appropriate boundary conditions. At $x = b$, the center of the axis of rotation, we apply a symmetry boundary condition, and at $x = a$ we let the curvature of $h(x)$ be zero, that is $\frac{\partial^2 h}{\partial x^2} = 0$. This system is can only be determined up to an arbitrary constant, therefore we include the integral constraint that follows from mass conservation, $\int_{-b}^{0} H(x)dx = \int_\Omega h(x)dx$. In summary we solve:

$$\frac{\partial^2 h}{\partial x^2} - \frac{f^2}{gH(x)}h = -\frac{f^2}{g'} \quad \text{On} \quad \Omega \tag{2.12}$$
$$\frac{\partial h}{\partial x} = 0 \quad \text{at} \quad x = 0 \tag{2.13}$$
$$\frac{\partial^2 h}{\partial x^2} = 0 \quad \text{at} \quad x = wall \tag{2.14}$$
$$\int_{-b}^{0} H(x)dx = \int_\Omega h(x)dx \tag{2.15}$$

Numerically we use a second order finite difference scheme implemented in MATLAB. The scripts are included at the end of this report.

An important scaling parameter for this problem is the radius of deformation, defined as $R_d = \frac{\sqrt{gH(x)}}{f}$. For the geostrophic adjustment problem with flat boundaries, R_d gives the distance a as defined in Figure [1b]. Also, to check that the small Rossby number approximation (made when deriving the governing equations) is valid, we calculate it as $R_o = \frac{|v|}{H_b + R_d}$.

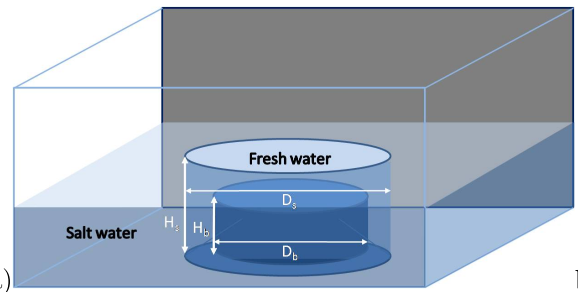
3 Experimental setup

A number of setups were experimented with. The first set of three experiments attempted to reproduce the shelf-break front as a geostrophic adjustment problem over topography. The second set of four experiments used a constant flowrate of freshwater as the coastal freshwater source, mimicking the setup by Cenedese(2001) [1]. Each experiment within a set was slightly different, and each is documented below, with the optimal setup described in detail.

For all the experiments, a square tank of dimension 406×406[mm] was used.

3.1 Geostropic Adjustment Setup

This experimental setup is similar to the geostrophic adjustment experiment, with some changes. The basic setup is depicted in Figure [2]. The tank is placed on the rotating table and centered, with an error of approximately ±3[mm]. A bowl or other cylindrical object, which will be referred to as the "shelf," is placed inside the tank and centered with an additional error of approximately ±3[mm]. A cylinder with two open ends is place on top or over the shelf, and the bottom is sealed with Vaseline. Saline water of density ρ_s is added to the tank, up to a height of $H_s + H_b$, and fresh water is added to the region inside the sealed cylinder. Food coloring is used to color the fresh water in order to distinguish between the saline and fresh water. The diameter and height of the bathymetry (or shelf) are represented by D_b and H_b respectively, and the diameter and height of the sealed cylinder are represented by D_s and H_s respectively. The different parameters for the three experiments are summarized in Table [1].

a)


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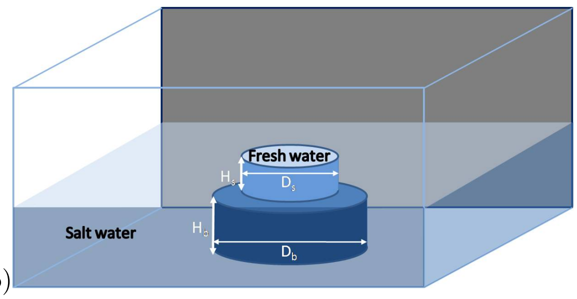


Figure 2: Geostrophic adjustment setup for shelf-break front experiment with a) fresh water only over shelf b) fresh water surrounding shelf.

Parameters	Experiment 1	Experiment 2	Experiment 3
f[s$^-1$]	2.5	2.5	2.3/2.5
ρ_s[kg/m^3]	1008	1012.5	1015
H_s[mm]	50	50	50
D_s[mm]	123	163	163
H_b[mm]	64	51	51
D_b[mm]	150	127	127
R_d[mm]	38	45	49
R_o[mm]	0.08	0.092	0.098

Table 1: Experimental parameters for geostrophic adjustment attempts. Note, freshwater density was always $\rho_f = 1000$[kg/m^3].

The experimental procedure consisted of two basic steps:

1. Start table rotation, and wait approximately 1 hour (or more) for water to reach solid-body rotation (spin-up)
2. Remove cylinder separating fresh and salt water

For the last experiment, an additional step was included. After reaching solid-body rotation, the rotation rate was slightly increased before removing the cylinder.

3.2 Constant Fresh Water Source Setup

This experimental setup is similar to the setup found in Cenedese (2001)[1]. The basic setup is depicted in Figure [3]. Multiple setups were attempted, and these are discussed in the next section. Here the final setup is described in detail.

WARNING: The edges of the cut soda can are sharp and can cause small cuts to fingers.

The tank is placed on the rotating table and centered, with an error of approximately ± 3[mm]. Next to the tank, a funnel is affixed some distance above the tank using buckets, an adjustable clamp, a metal cylinder, and duct-tape (see [3a]). Sufficient height of this funnel is required so that the gravity-fed fresh water can be delivered and the tube does not bend or touch the free surface.

Parameters	Experiment 4	Experiment 5	Experiment 6	Experiment 7
f[s^{-}1]	2.5	2.1	2.5	2.5
ρ_s[kg/m^3]	1005	1005	1005	1007
H_s[mm]	30	30	30	26/33
D_s[mm]	66	66	66	66
H_b[mm]	102	102	102	103
D_b[mm]	127	127	127	127
Flowrate	medium	medium	fast	1.25cm^3/s (slow)
R_d[mm]	32	39	32	39
R_o	0.059	0.074	0.059	0.074

Table 2: Experimental parameters for constant fresh water source attempts. Note, freshwater density was always $\rho_f = 1000$[kg/m^3].

Next, the shelf is placed inside the tank and centered with an error of approximately ± 3[mm]. A cut soda can is centered and placed on the shelf with an additional error of approximately ± 1[mm]. The soda can (cut to an appropriate height) is affixed to the shelf using duct tape which was folded sticky side out (to create double-sided tape). This is not enough to keep the can from moving due to buoyancy when empty, thus it needed to be weighed down. To weigh down the can, either loose change or a metal cylinder was used. For the final setup, a metal cylinder with a hole is placed in the can. To deliver fresh water to the shelf, one end of a plastic tube (see [3]) is placed inside the hole of this metal cylinder. The tube has an adjustable valve, which can be used to control the flowrate. The other end of the tube is connected to a funnel which is fixed some distance above the tank. The only problem with this setup was that the connection between the tube and the funnel was not completely sealed, and some water fed into the funnel would leak.

The flowrate of the freshwater is adjusted until it takes approximately 1 minutes for 100[ml] of water to flow through the system. This step additionally fills the can with fresh water to start. Note that colored water should not be used at this stage. A sponge is affixed to the edges of the can, and it serves multiple purposes. The sponge distributes the fed freshwater more evenly around the system, it removes radial momentum from the fed flow, and it adds rotational momentum (spins up the fresh water).

The salt water is prepared by mixing the appropriate amount of salt with 2 litres of fresh water. The density of the salt crystals is approximately 2[g/cm^3], however due to the packing ration of the crystals it ends up being closer to 1[g/cm^3]. Using approximately 100[ml] of salt gives a density of 1005[kg/m^3] for Experiments 1-3, and, while using approximately 150[ml] of salt gives a density of 1007[kg/m^3] for Experiment 4.

Salt water of density ρ_s is added to the tank, up to a height of $H_s + H_b$, such that no water leaks into the soda can. The diameter and height of the bathymetry (or shelf) are represented by D_b and H_b respectively, and the diameter and height of the can are represented by D_s and H_s respectively. The different parameters for the four experiments are summarized in Table [2].

The experimental procedure consisted of two basic steps:

1. Start table rotation, and wait approximately 1 hour (or more) for water to reach solid-body rotation (spin-up)
2. Pour colored-fresh water into the funnel, keeping the funnel filled till the fresh water runs out

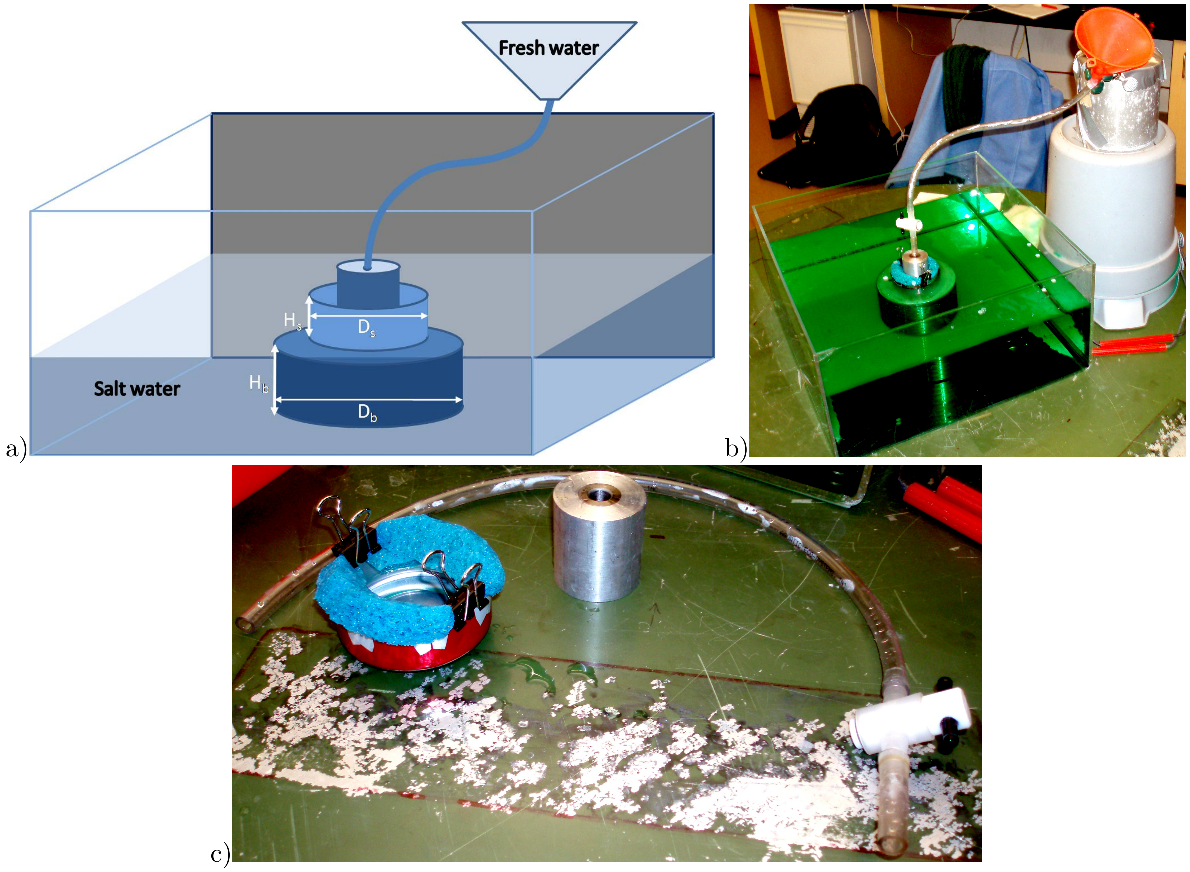


Figure 3: Constant fresh water source setup for shelf-break front experiment. a) Schematic with notation b) Photograph of actual setup c) Details of equipment

4 Results

Movies of the side and top view of Experiments 1, 4-7 can be found at http://mseas.mit.edu/download/mpuecker/12p804/ShelfBreak/

4.1 Experiment 1: Observations and results

Figure [4] shows side-views of Experiment 1, and the result from the theoretical numerical model is shown in Figure [5]. Note that in Figure [4] the free surface acts as a mirror (because the angle of incidence is small for the camera) so the density profile is reflected in the free surface.

It was observed that initially a current is formed around the shelf, and the fresh water stays locked around the shelf. However, the current is not maintained, and after about 1 minute (Figures [4c-d]) the fresh water detaches from the shelf as dense water flows on top of the shelf, and the problem reduces to essentially a geostrophic adjustment without the shelf. The result from the numerical model supports this argument, showing that the radius of deformation is too large, so it was expected that the fresh water would not stay locked around the shelf.

4.2 Experiment 2-3: Observations and results

The results from the theoretical numerical model is shown in Figures [6-7].

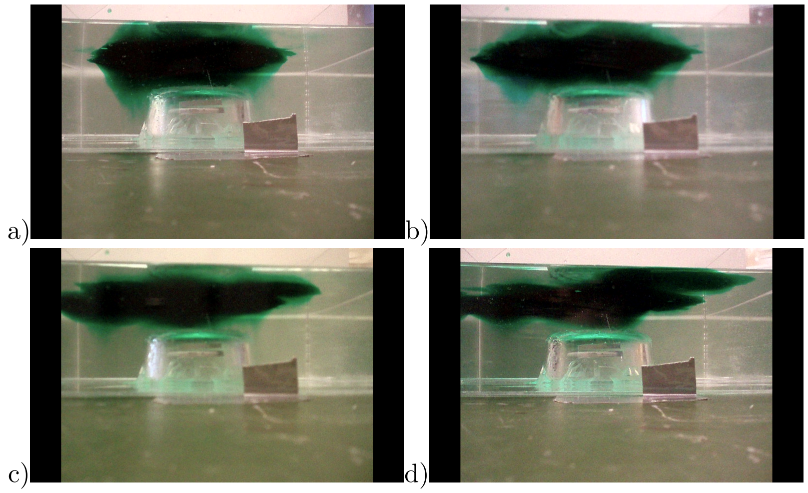


Figure 4: Experiment 1 results at four different times from the start a) to the end d) of the experiment

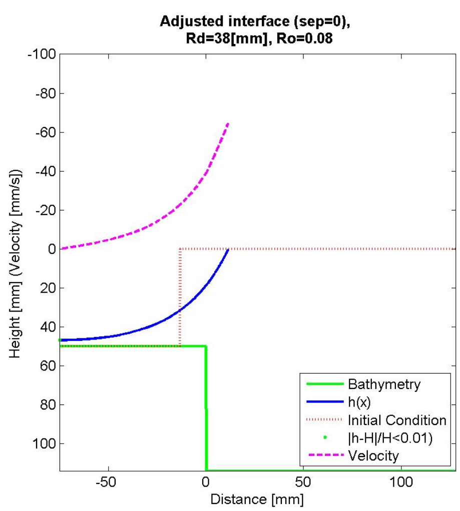


Figure 5: Theoretical numerical result for Experiment 1

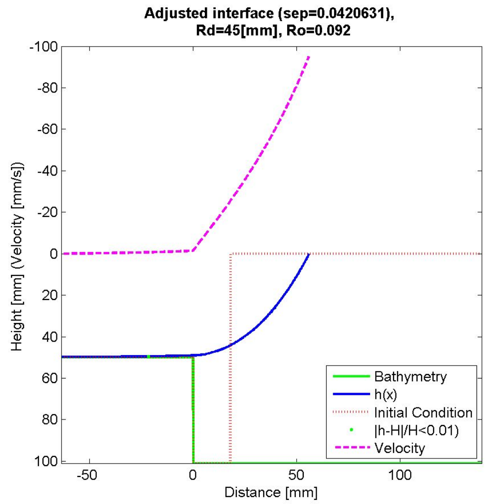


Figure 6: Theoretical numerical result for Experiment 2

Both of these experiments yielded similar results to each other and to Experiment 1. In both cases the current did not remain. The second case, where the relative rate of rotation was changed, resulted in faster break-down of the current.

4.3 Experiment 4: Observations and results

Figure [8] shows four side-views and two top views of Experiment 4, and the result from the theoretical numerical model is shown in Figure [9].

The fresh water is not uniformly delivered around the coke can, but the majority of fresh water is delivered around one side of the coke can. Also, the delivered fresh water has a significant radial velocity component. The tracers indicate that flow is not locked at the shelf edge, (i.e. velocities not completely tangential), but instead move across it, notably near the region where the fresh water is introduced. The major motion of the tracers, however, indicate an anti-cyclonic current that is strongest near the edge of the shelf, and decays away from the shelf. Dye the same color as the fresh water was added to better visualize the flow structures from above. This was unfortunate because after this it was no longer possible to distinguish fresh from salt water. However, the additional tracer did show backward breaking waves present in the flow (Figure [8e-f]). These structures were not present throughout the experiment, but only developed after some time. Also, the size of the waves grew after the supply of fresh water stopped. From the side views, the inner side of the fresh water slowly progresses down the side of the coke can, and then across the shelf. The angle between the fresh and salt water seem to remain constant, and the profile seems to be concave down. Finally, similar to Experiment 1, the dense water would eventually make its way back on top of the shelf, and the fresh water would spread across the top near the free-surface.

The theoretical model predicts a concave up density interface, and a sharp velocity gradient at the shelf edge. The predicted angle is much steeper than that observed in the experiment.

4.4 Experiment 5: Observations and results

Figure [10] shows four side-views and two top views of Experiment 5, and the result from the theoretical numerical model is shown in Figure [11]. Note that this experiment re-used the salt water solution from Experiment 5 (to save time while tweaking the experimental setup).

The fresh water is not uniformly delivered around the coke can, but the fresh water is delivered biased toward one side of the coke can. Note, the non-uniformity of the fresh water deliverance is not as great as

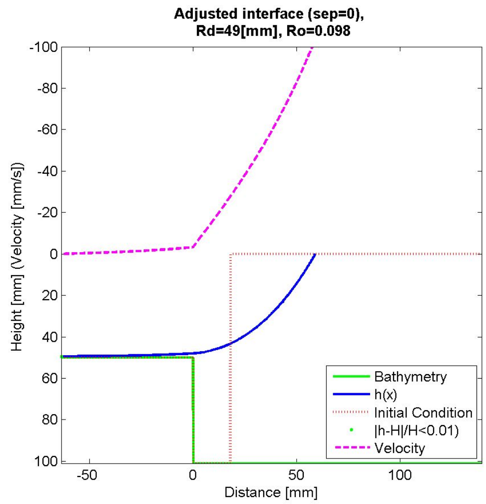


Figure 7: Theoretical numerical result for Experiment 3

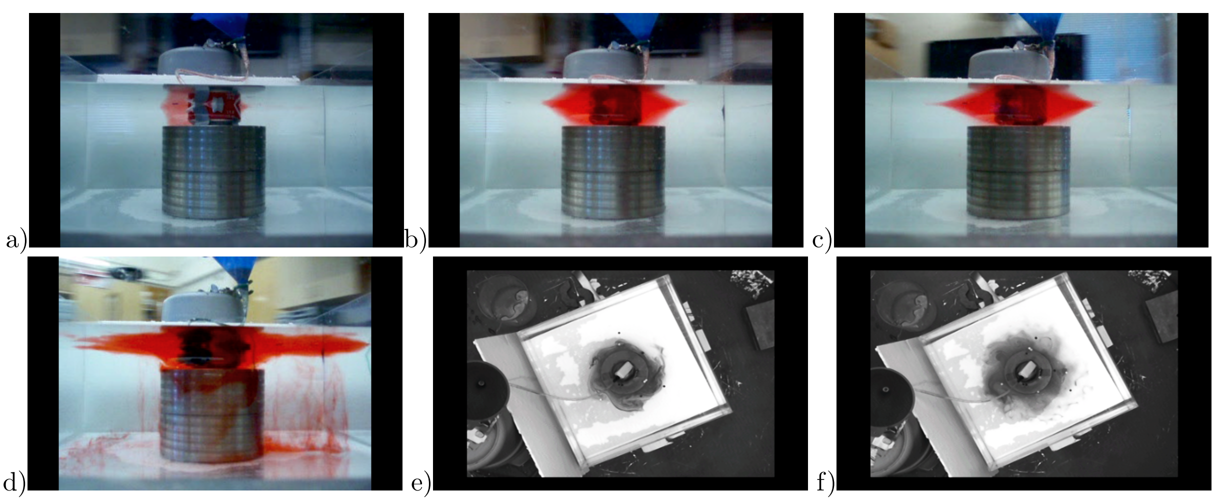


Figure 8: Experiment 4 results at four different time from the start a) to the end d) of the experiment, and a top view e) just after water flow is stopped and f) at the end of the experiment

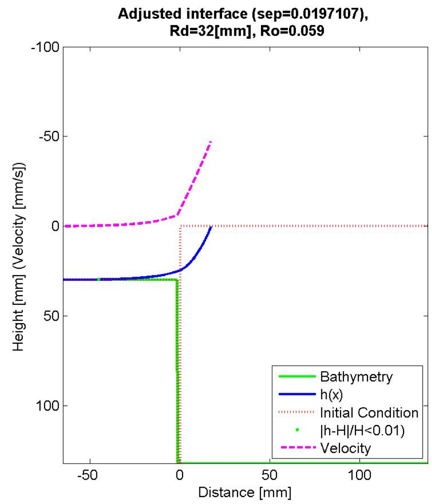


Figure 9: Theoretical numerical result for Experiment 4

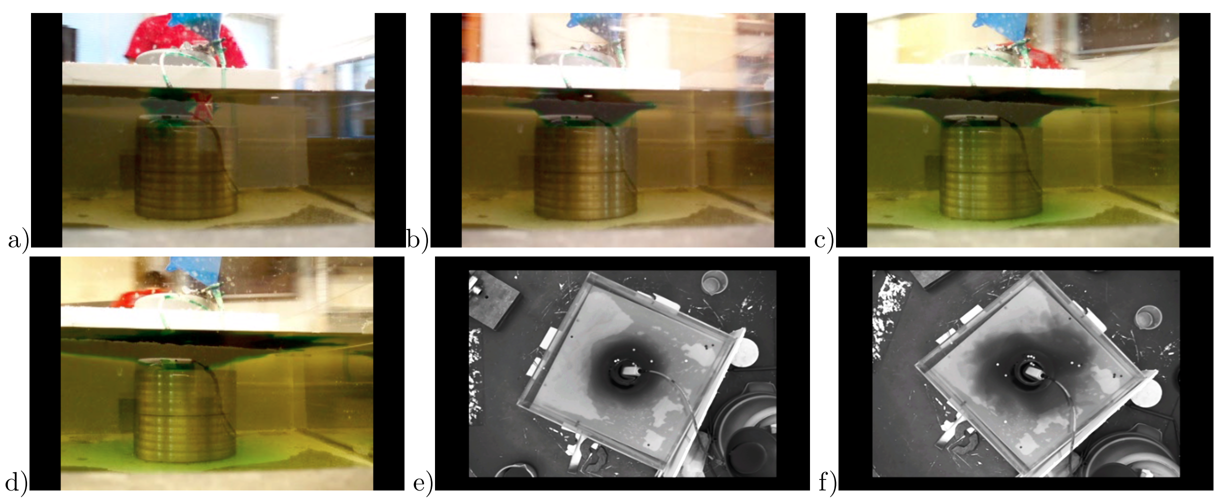


Figure 10: Experiment 5 results at four different time from the start a) to the end d) of the experiment, and a top view e) just after water flow is stopped and f) at the end of the experiment

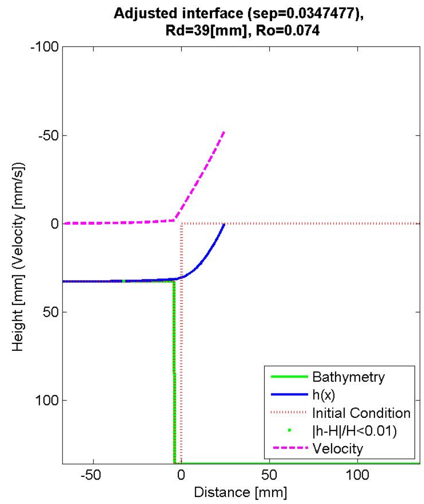


Figure 11: Theoretical numerical result for Experiment 5

in Experiment 4. The delivered fresh water still has a significant radial velocity component. The tracers indicate that flow is not locked at the shelf edge, (i.e. velocities not completely tangential), but instead move across it, notably near the region where the fresh water is introduced. The major motion of the tracers, however, indicate an anti-cyclonic current that is strongest near the edge of the shelf, and decays away from the shelf. Backward breaking waves were again present in the flow (Figure [10e-f]), but seem to develop only after the supply of fresh water had stopped. Finally, similar to Experiment 1, the dense water would eventually make its way back on top of the shelf, and the fresh water would spread across the top near the free-surface. In this case the current would break down, no longer present at the shelf.For this experiment, the flow was restarted after the current was no longer present to see whether or not it could be re-established. It was found that the current would form once again. From the side views, the inner side of the fresh water slowly progresses down the side of the coke can, and then across the shelf, but the bottom of the profile never crosses the shelf or moves down the shelf. The angle between the fresh and salt water seem to remain constant, and the profile seems to be concave down. However, after the fresh water stops, salt water makes it on top of the shelf, and as the current breaks down, the interface becomes concave up. Finally, similar to Experiment 1, the dense water would eventually make its way back on top of the shelf, and the fresh water would spread across the top near the free-surface.

The theoretical model predicts a concave up density interface, and a sharp velocity gradient at the shelf edge. The predicted angle is much steeper than that observed in the experiment.

4.5 Experiment 6: Observations and results

Figure [12] shows four side-views and two top views of Experiment 6, and the result from the theoretical numerical model is shown in Figure [13].

The fresh water is not uniformly delivered around the coke can (Figure [12g]), but the majority of fresh water is delivered around one side of the coke can. The delivered fresh water still has a significant radial velocity component, the largest for any experiment. The flow rate in this case was also very high, which caused an eddy of fresh water to detach from the main source around the shelf (Figure [12f] The tracers indicate that flow is not locked at the shelf edge, (i.e. velocities not completely tangential), but instead move across it, notably near the region where the fresh water is introduced. The major motion of the tracers, however, indicate an anti-cyclonic current that is strongest near the edge of the shelf, and decays away from the shelf. Backward breaking waves were again present in the flow (Figure [12e-f]), with a wavenumber of 4-5 in this case, but these breaking waves did not seem to develop until after the supply of fresh water had

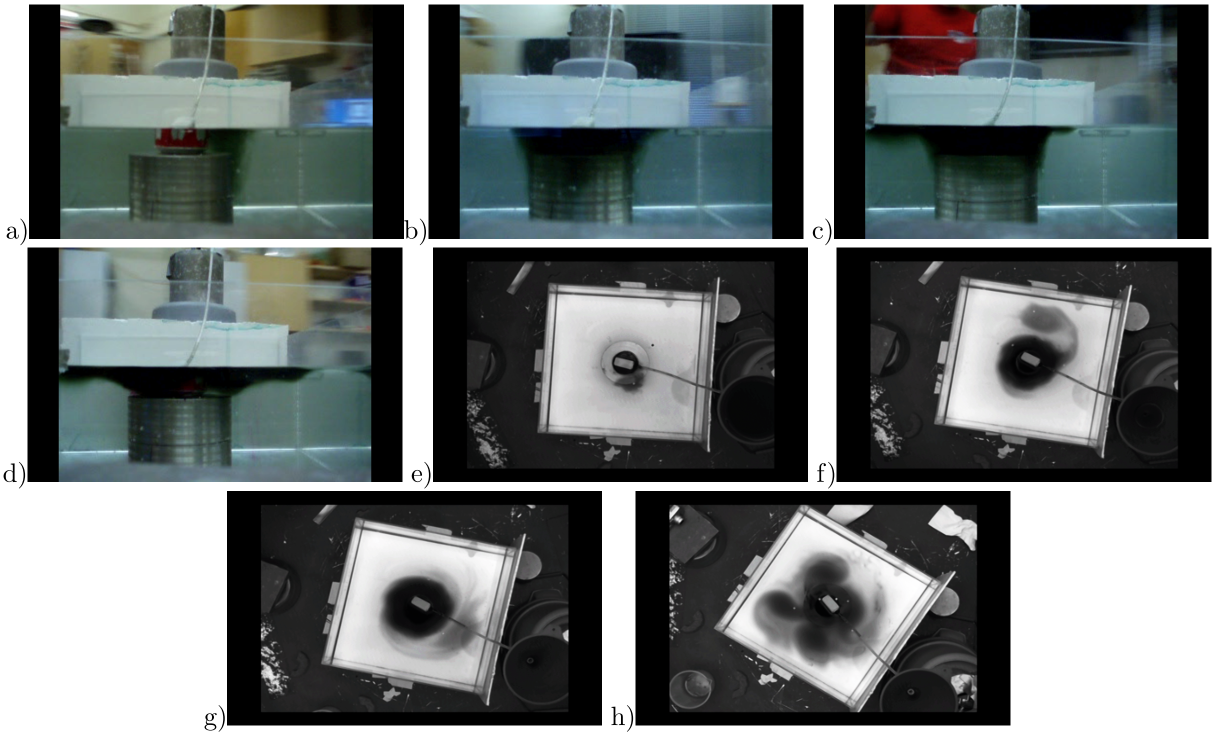


Figure 12: Experiment 6 results at four different time from the start a) to the end d) of the experiment, and a top view e) at the start of the experiment f) after about 1 minute g) just after water flow is stopped and h) at the end of the experiment

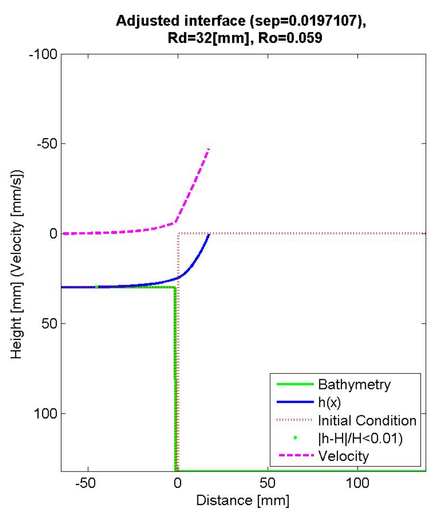


Figure 13: Theoretical numerical result for Experiment 6

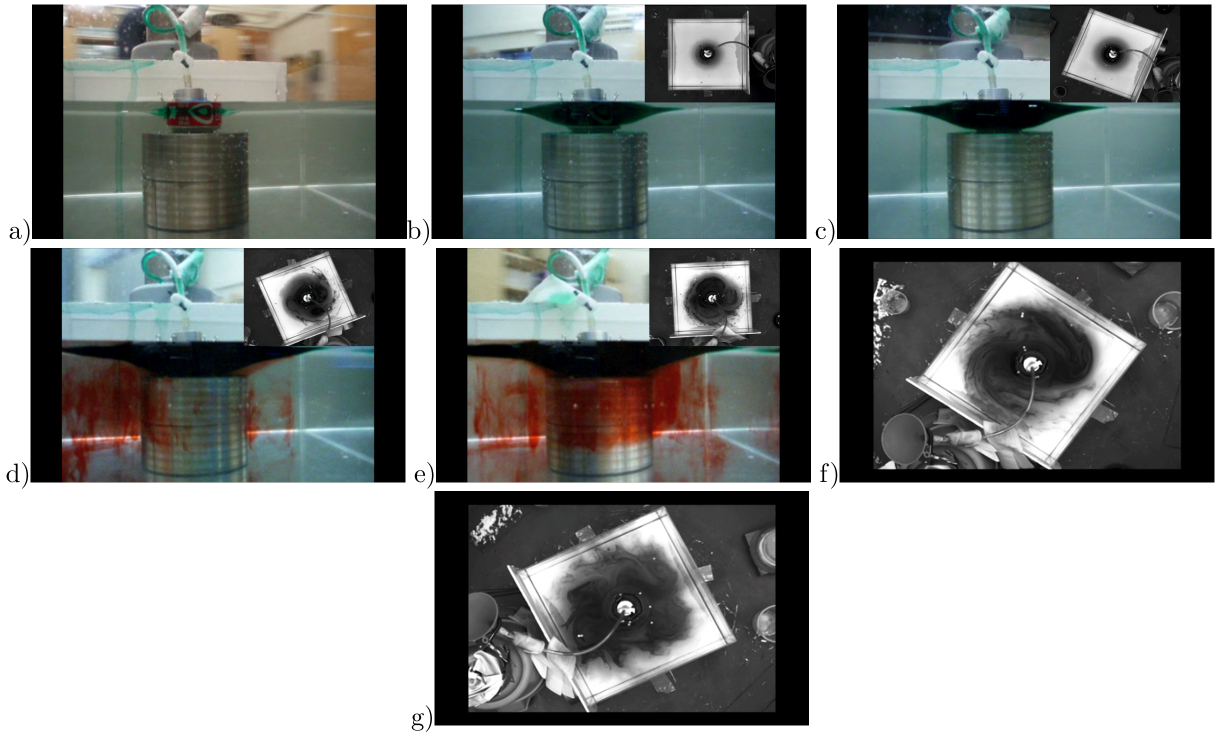


Figure 14: Experiment 7 results at four different times from the start a) to the end e) of the experiment, and a top view f) just after water flow is stopped and g) at the end of the experiment

stopped. Finally, similar to Experiment 1, the dense water would eventually make its way back on top of the shelf (some evidence in Figure [10d]), and the fresh water would spread across the top near the free-surface. The current breaks down and is no longer present at the shelf. From the side views, the inner side of the fresh water quickly progresses down the side of the coke can, and then across the shelf. The bottom of the profile crosses the shelf edge moving partly down the shelf. The fresh water that moves down the shelf does not do so uniformly, but a wave seems to form. That is, there seems to be a vertical oscillation around the shelf edge. The angle between the fresh and salt water seems to remain constant while fresh water is being fed, and the profile seems to be concave down. However, after the fresh water stops, salt water makes it on top of the shelf, the current breaks down, the interface becomes concave up, and the angle becomes shallower.

The theoretical model predicts a concave up density interface, and a sharp velocity gradient at the shelf edge. The predicted angle is much steeper than that observed in the experiment.

4.6 Experiment 7: Observations and results

Figure [14] shows four side-views and two top views of Experiment 7, and the result from the theoretical numerical model is shown in Figure [15]. Particle tracker results measuring the tangential and radial velocities are given in Figure [16] for early in the experiment, and [17] for later in the experiment.

This experiment is the only one where the fresh water is uniformly delivered around the coke can (Figure [14a]), and the delivered fresh water only has a small radial velocity component. The flow rate in this case was measured, and compared to the other experiments was the slowest. The tracers indicate that velocities are mostly tangential, forming an anti-cyclonic current that is strongest near the edge of the shelf and decays away from the shelf. Backward breaking waves were again present in the flow (Figure [14f-g]), with a wavenumber of 2-3 in this case, and these breaking waves developed while fresh water was being supplied.

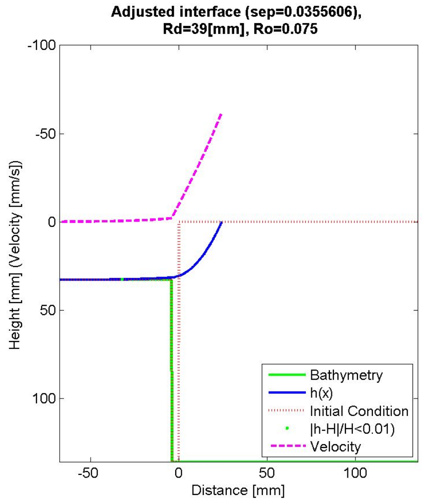
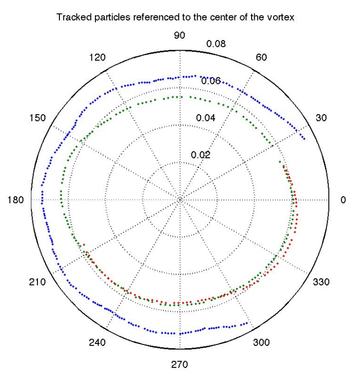
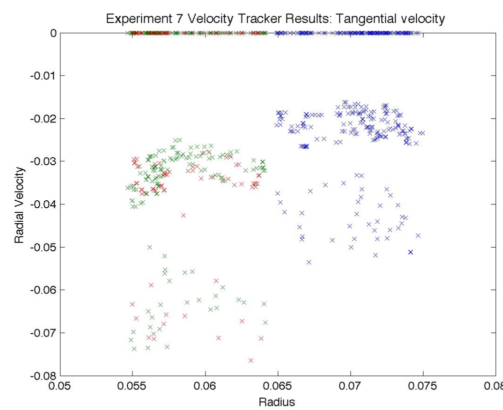


Figure 15: Theoretical numerical result for Experiment 7



a)



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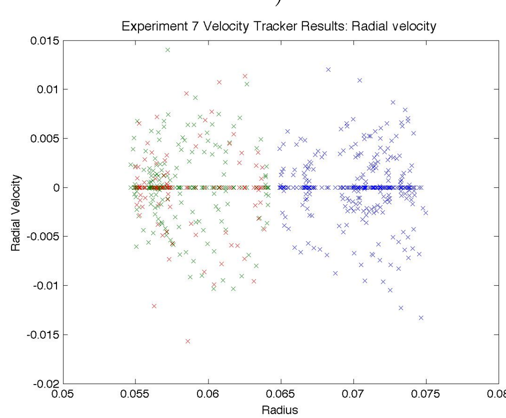
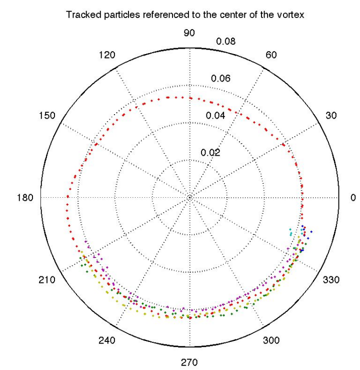
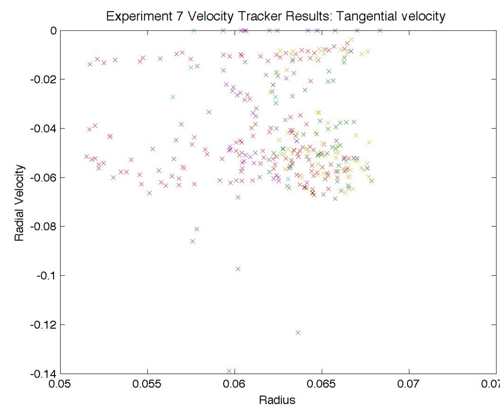


c)

Figure 16: Particle tracker results during the first 4 minutes of the experiment. Velocities are in [m/s] and radius is in [m]. a) Particle tracks b) Tangential velocities at different radii c) Radial velocities at different radii



a)



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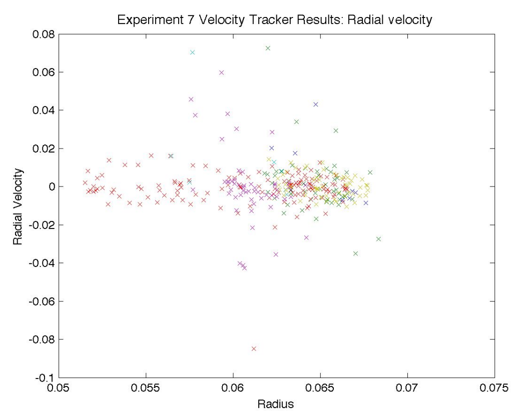

c)

Figure 17: Particle tracker results during between 4-6 minutes of the experiment. Velocities are in [m/s] and radius is in [m]. a) Particle tracks b) Tangential velocities at different radii c) Radial velocities at different radii

Similar to Experiment 1, the dense water would eventually make its way back on top of the shelf, and the fresh water would spread across the top near the free-surface. In this case the current would break down, no longer present at the shelf. From the side views, the inner side of the fresh water slowly progresses down the side of the coke can, and then slowly across the shelf. It seems that the progression down the side of the can is slowed by the tapered lip of the bottom of the can, but after a sudden jump of the profile past this taper, the profile then continues to progress across the shelf. The interface angle between the fresh and salt water seems to start at 21° becoming shallower to approximately 16° as fresh water is continued to be supplied. The density interface profile seems to be concave up near the bottom of the interface, and transitions to concave down near the free-surface. The injected dye indicated little baroclinic motion, since Taylor columns were observed.

The theoretical model predicts a concave up density interface, and a sharp velocity gradient at the shelf edge. The predicted angle is much steeper than that observed in the experiment.

4.7 Summary

The angles of the interfaces for each of the experiments are summarized in Table [3].

Parameters/Experiment	Exp. 1	Exp. 4	Exp. 5	Exp. 6	Exp. 7
Measured Angle (degrees)	19-28	20-30	35-51	51-64	16-21
Predicted Angle (degrees)	28-58	25-63	28-61	25-63	29-61
Flowrate	N/A	medium	medium	fast	1.25cm^3/s (slow)
R_d[mm]	38	32	39	32	39
R_o	0.08	0.059	0.074	0.059	0.074

Table 3: Experimentally measured and predicted angles for Experiment 1, 4-7. Note that for the experimental angles, the range or largest and smallest angle is given (measured by eye), while for the predicted angle the first angle is measured from the point of separation to the h=0 point.

5 Discussion

5.1 Experiments 1-3

The first set of three experiments were not successful in reproducing a sustained shelf-break front. The first experiment gave the best results, forming a front initially, but then breaking down. It is likely that energy is being taken out of the flow due to small scale turbulent and diffusive effects. Because the density gradient is not very large (in order to keep the radius of deformation small) the interface between the two fluids is not very sharp, allowing for easier momentum transport between light and dense fluid.

The theoretically measured angle was not a good predictor of the experimentally measured angle. In general the experimental angle was considerably smaller than the predicted angle. It was difficult to measure the angle of the interface accurately, but within the large error tolerance, the measured and predicted angle do not overlap. One possible explanation is that the real bathymetry was not a step, for this case, as was idealized by the theoretical result. The other explanation could be that the theoretical model is oversimplified, and does not capture the necessary physics, which is likely true. The Rossby number was small enough to make the assumptions reasonable, however the assumption of no frictional or transient effects is likely poor in this problem.

5.2 Experimental setups for Experiments 4-7

The experimental setups in the first three attempts (experiments 4-6) had a number of problems. In the 4^{th} experiment, it was found that the funnel needs to be high enough so that the flow is not restricted, and that the tube does not touch the free surface. A mistake was made in that dye of the same color was added to visualize vertical structures in the flow. Also, the non-uniform delivering of fresh water was noted as problematic. What ends up happening is similar to a river entering the ocean: with the strong jet of fresh water emerging from a single location, the jet bends as it is geostrophically adjusted. In the 5^{th} attempt it was discovered that in order to maintain the current, a constant flow of fresh water was needed. By the 6^{th} attempt it was thought that all the kinks were worked out, but it became obvious that a better flow control was necessary to maintain a constant flowrate of fresh water. Also, a method to more evenly distribute the fed water, as well as remove radial momentum from the flow was required. For Experiment 7, a tube with an adjustable value was used, and the initial flowrate was measured. Also, a sponge was fixed to the rim of the soda can to more evenly distribute the fresh water. This final setup gave reasonably good results, which will be discussed next. The only mistake in this experiment was using a syringe to inject the red-colored dye when trying to visualize vertical structures. In the end, dropping a drop of dye using an eye-dropper seemed to be the best approach.

The position of the side-view camera is also important. In the experiments 1, 4-5 the camera was placed directly on the rotating table, with the focal centre far down the shelf. The result is that reflections from the free-surface are observed, making it difficult to measure the angle of the interface. By placing the camera on a raised platform until the focal center is level with the free surface, more accurate measurements of the density interface can be obtained.

5.3 Experiment 7 Discussion

Comparing the model and the theory, the angle predicted by the theory is not a good indication of the angle of the front found from the experiment. Not even the concavity of the shape is correctly predicted. The reason for the wrong shape may be due to the constant addition of fresh water to the system. And accurate theoretical model would need to account for the volume flow rate of water. While the radial velocity was small for this case due to damping from the sponge, the free surface height will change at the center and drive a radial velocity. This may explain why the density interface changes from concave up to concave down: the radial flow at the free surface causes the interface to be concave down in that region, contrary to the prediction. The predicted maximum velocity is within 20 percent of the measured velocity during the first 4 minutes, and within 50 percent of the maximum measured velocity. This result will depend on the volume of the fluid used to initialize the theoretical experiment, and the value chosen was close to the volume of fresh water before the first 4 minutes of the experiment, if a larger volume was chosen, the velocity would be larger. Therefore, the velocity prediction is reasonably good.

Also, examining the movies, it can be seen that a significant velocity gradient exists at the shelf-break, demonstrating that the front was, in fact, formed. This suggests that the experimental setup is adequate for demonstrating the shelf-break front phenomena.

The backward breaking waves present during all the experiments show that an instability develops around the shelf break. Different wave numbers were found during these experiments, and it may be interesting to see if the wave numbers of these instabilities can be predicted.

6 Recommendations and conclusions

The final setup described in this report seems to be a good setup for studying the shelf break front. Improvements can be made by using a more sophisticated flow control, using a more precisely constructed ring source for the fresh water, and replacing the soda can with a cylinder that does not have a taper at the bottom.

The theoretical model discussed is not adequate to predict the angle of the interface, but it does give reasonable approximations of the maximum velocity.

For laboratory demonstrations, a series of experiments could be performed with varying radii of deformation. The radius of deformation could be changed either by changing the density, or the rate of rotation.

References

[1] C. Cenedese and P. F.Linden, "Stability of a buoyancy-driven coastal current at the shelf break," *J. Fluid. Mech.*, Vol. 452, 2001, pp. 92-121.

[2] B. Cushman-Roisin, Introduction to Geophysical Fluid Dynamics. *Prentice Hall*: Englewood Cliffs, New Jersey. 1994.

[3] J. Pedlosky, Geophysical Fluid Dynamics: Second Edition. *Springer*: New York, New York. 1987.

Appendix: MATLAB code

```
%Shelf script
%Creates plots and everything for all the shelf-break experiments MPU ran
Bathyd=[0.127,.127,.127,.127]; %m
Bathyh=[0.102,.103,0.102,0.103];
Cand = [0.13,0.135,0.13,0.135];
Canh = [0.03,0.033,0.03,0.033];
rho2 = [1005,1005,1005,1007];
f    = [2.5,2.1,2.5,2.5];

for i=1:length(f)
    clf
    [Rd(i) Ro(i)]=shlf2(Bathyd(i),Bathyh(i),Cand(i),Canh(i),rho2(i),f(i));
    drawnow,pause(0.5)
    eval(sprintf('print -djpeg80 img/Expb_%d',i))
end

function [Rd Ro] = shlf(Bathyd,Bathyh,Cand,Canh,rho2,f,m)
% function [Rd Ro] = shlf(Bathyd,Bathyh,Cand,Canh,rho2,f,m)
% SHLF calculates the geostrophically balanced state over either a step or
% sloping geometry starting from an initial step-function state
%
%INPUTS
%       Bathyd:     Diameter of the step bathymetry, or diameter at which
%                     shelf starts to slope [m]
%       Bathyh:     Height of step geometry [m]
%       Cand:       Diameter of can where fluid is released [m]
%       Canh:       Height of shelf, or height of freshwater in cylinder [m]
%       rho2:       Density of heavier fluid (light fluid assumed rho1=1000 [kg/m^3])
%       f:          Coriolis parameter
%       m:          Shelf slope (optional) ifnan or not specified, step
%                     geometry is used.
%OUTPUTS
%       Rd:         Radius of deformation = max(sqrt(g'H(x))/f)
%       Ro:         Rossby number = max(abs(v)/(f*Cand/2))
%
% Written by Matt Ueckermann for 12.804. Fall 2009
%
% See also: taylor.m, getsalt.m, ShelfScript.m
%
if nargin<7;m=NaN;end
%% Setup Parameters (Used for debugging)
% Bathyd=0.127; %mm
% Bathyh=0.051*2;
% Cand = 0.163;
% H1=0.03;
% rho2=1005;
% f=2.5;
% m=NaN;

% Other parameters
```

```
H1=Canh;
Tankw=0.406; %mm
NX = 1000;
Nx = NX;
dx = Tankw/2/(Nx-1);
X = 0:dx:Tankw/2;
x=X;
rho1=1000;
%Find reduced gravity
g = 9.81*(rho2-rho1)/rho1;

%Set "b"
b=Bathyd/2;

%Create function for geometry. Supports sloped function and step function.
if ~isnan(m)
    H = @(x) H1 + heaviside(x-b+eps).*(x-b)*m;
else
    H = @(x) H1 + Bathyh*heaviside(x-b+eps);
end
b=-Cand/2;

%Calculate radius of deformation
Rd = sqrt(g*max(H(x)))/f;

%Numerically calculate the new height of the rho1 density fluid (height
%measured form free surface)
h=-1;iter=0;

%Iterative solve the solution. Iterations are required because this was not
%phrased as an optimization problem, and the problem does not know of the
%constraint h=>0 everywhere. Therefore we iterate to find the answer, each
%time solving a smaller domain.
while min(h)<0 && iter<Nx/2
    %Keep track of iterations
    iter=iter+1;

    %Use second order central difference for second derivative in domain
    %interior
    Coeffs = taylor([-dx dx],2);
    A = sparse(toeplitz([Coeffs(2:3);zeros(Nx-2,1)]))...
        +sparse(diag(ones(Nx,1).*(-f^2./g./H(x)')));

    %boundary conditions
    %Remove contributions from first row
    A(1,:)=0;
    %Add Neumann boundary condition contributions to first row
    A(1,1)=1;
    A(1,2)=-1;

    %Remove contributions from last row
    A(end,:)=0;
    %Add second order zero derivative boundary condition to last row
    A(end,end)=1;
    A(end,end-1)=-2;
    A(end,end-2)=1;

    %Add additional row to include integral constraint (using midpoint rule
    %to do the integration)
    A(end+1,:)=dx;
    A(end,1)=dx/2;
    A(end,end)=dx/2;

    %Create right-hand-side vector
    rhs = ones(Nx,1).*-f^2/g;
```

```
    %Adjust rhs vector for boundaries
    rhs(1) = 0;
    rhs(end) = 0;

    % Need to figure out value for integral constraint.
    % Integrate over the volume to find amount of initial rho1 fluid
    Ii=sum(  (  H(0:dx:dx*abs(round(Cand/2/dx))) + H(dx:dx:dx*abs(round(Cand/2/dx))+dx)  )/2*dx);

    %Add contribution to rhs vector
    rhs(end+1)=Ii;

    %Solve numerical system
    h = A\rhs;

    %Check to see if h<0 everywhere (that we have a valid solution If there
    %is a region where h>0, we contract the domain, repeat the calculation
    %setting the second order boundary condition to the new lhs node in the
    %smaller domain.

    %Pick a smart way to contract the domain, use this way until we're
    %within 10 grid points of the correct value
    tmp=find(h<0,1);
    if (Nx-1-tmp)/NX>10/NX
        Nx=tmp;
        x(Nx+1:end)=[];
    end
    %Contract the domain one gridcell at a time (will happen when we're
    %within 10 gridpoints of correct answer
    Nx=Nx-1;
    x(end)=[];
end

%Plot the geometry
HH(5)=plot(1000*(X+b),1000*H(X),'g');hold on

%Plot the numerical solution
x(end+1)=x(end)+dx;Nx=Nx+1;
HH(1)=plot(1000*(x+b),1000*(h),'-');
axis ij
axis equal


% Need to check if volume is conserved
% Integrate over the volume to find amount of final rho1 fluid
I=sum((h(2:end)+h(1:end-1))/2*dx);

%Now integrate over initial Height to find the point at which initial and
%final mass is the same
I0=0;i=0;
while (I0<I)
    I0 =I0 + (H(x(1)+i*dx)+H(x(1)+(i+1)*dx))/2*dx;
    x0 =  x(1) + i*dx+dx/2;
    i=i+1;
end
Hi = H(X); Hi(i:end)=0;
%Plot the initial distribution (as calculated from the volume conservation
HH(2)=plot(1000*(X+b),1000*(Hi),':r');

%Find the point at which the adjusted fluid depth is within 1% of the
%initial fluid depth
ids=find( (abs(h-H(x)')./H(x)'<0.01),1,'last');
if isempty(ids),ids=1;end;
%Plot the result
HH(3)=plot(1000*(x(ids)+b),1000*(h(ids)),'*g');
```

```
%Calculate the velocity using second order central differences (staggered
%grid)
v = g/f*(h(2:end)-h(1:end-1))/dx;
min(v)
%calculate the theoretical Rossby number
Ro = max(abs(v')./(2.*f)./(Bathyd+Rd));
x2=dx/2:dx:Tankw/2-dx/2;
%Plot the result
HH(4)=plot(1000*(x2(1:Nx-1)+b),v*1000,'--m');
xlim([1000*b,1000*(Tankw/2+b)])
ylim([-1000*0.1,1000*(H1+Bathyh)])
TT(3)=xlabel('Distance [mm]');
TT(4)=ylabel('Height [mm] (Velocity [mm/s])');
TT(1)=title(sprintf('Adjusted interface (sep=%g), \n Rd=%0.2g[mm], Ro=%0.2g',x(ids),Rd*1000,Ro));
TT(2) = legend('Bathymetry','h(x)','Initial Condition','|h-H|/H<0.01)','Velocity','Location','SouthEast');
formatplot(HH,gca,TT,TT(1))
```