a) Separate ∂D into:

i. Boundaries, ∂D^{iu}, along which the streamfunction values, $\Psi_{C^{iu}}$, are uncertain

ii. Boundaries, ∂D^{c}, along which the streamfunction values, $\Psi_{b^e} \cup \Psi_{C_k^{ic}}$, are certain

b) Compute "certain coast" streamfunction, $\Psi_{(0)}$, (table 2, eq. 12)

$$\nabla \cdot \left(\omega \nabla \Psi_{(0)}\right) = \left[\nabla \times \left(\omega H \vec{U}_{(0)}\right)\right] \cdot \hat{k} \qquad \Psi_{(0)}\Big|_{\partial D^c} = \Psi_{b^c} \equiv \begin{cases} \Psi_{b^e} & \text{if} \quad s \in \partial D^e \\ \Psi_{C_k^{ic}} & \text{if} \quad s \in C_k^{ic} \end{cases}$$

Split the optimization functional into $J_{b^u}^{uu}, J_{b^u}^{uc}, J_{b^u}^{uo}$

Constructing $J_{b^u}^{uu}$

Transport between all pairs of coasts in ∂D^{iu}

a) Find s_{nm}^{uu}, s_{mn}^{uu} on C_n^{iu}, C_m^{iu} s.t.

$$[s_{nm}^{uu}, s_{mn}^{uu}] = \underset{[s_{iu,n}, s_{iu,m}]}{\arg\min} \left| \Psi_{(0)}(s_{iu,n}) - \Psi_{(0)}(s_{iu,m}) \right|$$

b) Set $\Delta_{nm}^{uu}\Psi_{(0)} = \Psi_{(0)}(s_{nm}^{uu}) - \Psi_{(0)}(s_{mn}^{uu})$

c) Functional for minimizing transport between C_n^{iu}, C_m^{iu}:

$$\varpi_{nm}^{uu}(\Psi_{C_n^{iu}} - \Psi_{C_m^{iu}} - \Delta_{nm}^{uu}\Psi_{(0)})^2$$

d) Combine the functionals for all pairs of C_n^{iu}, C_m^{iu} to obtain (eq. D.6)

$$J_{b^u}^{uu}\left(\Psi_{C_1^{iu}}, \dots, \Psi_{C_{N^{iu}}^{iu}}\right) = \frac{1}{2}\sum_{n=1}^{N^{iu}}\sum_{m=n+1}^{N^{iu}}\left[\varpi_{nm}^{uu}(\Psi_{C_n^{iu}} - \Psi_{C_m^{iu}} - \Delta_{nm}^{uu}\Psi_{(0)})^2\right]$$

Constructing $J_{b^u}^{uc}$

Transport between all pairs of coasts in ∂D^{iu} and ∂D^{c}

a) Find s_{nk}^{uc} on C_n^{iu} s.t.

$$s_{nk}^{uc} = \underset{s_{iu,n}}{\arg\min} \left| \Psi_{(0)}(s_{iu,n}) - \Psi_{C_k^c} \right|$$

b) Set $\Delta_{nk}^{uc}\Psi_{(0)} = \Psi_{(0)}(s_{nk}^{uc}) - \Psi_{C_k^c}$

c) Functional for minimizing transport between C_n^{iu}, C_k^{c}:

$$\varpi_{nk}^{\mathrm{uc}}(\Psi_{C_n^{iu}} - \Psi_{C_k^c} - \Delta_{nk}^{uc}\Psi_{(0)})^2 = \varpi_{nk}^{\mathrm{uc}}(\Psi_{C_n^{iu}} - \Psi_{(0)}(s_{nk}^{uc}))^2$$

d) Combine the functionals for all pairs of C_n^{iu}, C_k^{c} to obtain (eq. D.7)

$$J_{b^u}^{uc}\left(\Psi_{C_1^{iu}}, \dots, \Psi_{C_{N^{iu}}^{iu}}\right) = \frac{1}{2}\sum_{n=1}^{N^{iu}}\sum_{k=1}^{M^c}\left[\varpi_{nk}^{\mathrm{uc}}\left(\Psi_{C_n^{iu}} - \Psi_{(0)}(s_{nk}^{uc})\right)^2\right]$$

Constructing $J_{b^u}^{uo}$

Transport between all coasts in ∂D^{iu} and open boundaries ∂D^{o}

a) Find s_{nb}^{uo} on C_n^{iu} and s_{bn}^{ou} on ∂D^o s.t.

$$[s_{nb}^{uo}, s_{bn}^{ou}] = \underset{[s_{iu,n}, s_{o,b}]}{\arg\min} \left| \Psi_{(0)}(s_{iu,n}) - \Psi_{(0)}(s_{o,b}) \right|$$

b) Set $\Delta_{nb}^{uo}\Psi_{(0)} = \Psi_{(0)}(s_{nb}^{uo}) - \Psi_{(0)}(s_{bn}^{ou})$

c) Functional for minimizing transport between C_n^{iu} and ∂D^o:

$$\varpi_{nb}^{\mathrm{uo}}\left(\Psi_{C_n^{iu}} - \Psi_{(0)}(s_{bn}^{ou}) - \Delta_{nb}^{uo}\Psi_{(0)}\right)^2 = \varpi_{nb}^{\mathrm{uo}}\left(\Psi_{C_n^{iu}} - \Psi_{(0)}(s_{nb}^{uo})\right)^2$$

d) Combine the functionals for all islands to obtain (eq. D.8)

$$J_{b^u}^{uo}\left(\Psi_{C_1^{iu}}, \dots, \Psi_{C_{N^{iu}}^{iu}}\right) = \frac{1}{2}\sum_{n=1}^{N^{iu}}\left[\varpi_{nb}^{\mathrm{uo}}\left(\Psi_{C_n^{iu}} - \Psi_{(0)}(s_{nb}^{uo})\right)^2\right]$$

Combine the optimization functionals (eq. D.5)

$$J_{b^u}\left(\Psi_{C_1^{iu}}, \dots, \Psi_{C_{N^{iu}}^{iu}}\right) = J_{b^u}^{uu}\left(\Psi_{C_1^{iu}}, \dots, \Psi_{C_{N^{iu}}^{iu}}\right) + J_{b^u}^{uc}\left(\Psi_{C_1^{iu}}, \dots, \Psi_{C_{N^{iu}}^{iu}}\right) + J_{b^u}^{uo}\left(\Psi_{C_1^{iu}}, \dots, \Psi_{C_{N^{iu}}^{iu}}\right)$$

Minimization: Standard Least Squares Problem

Solve the system of N^{iu} linear equations (table 2, eq. 15) to obtain the $\Psi_{C_n^{iu}}$

$$\left[\sum_{\substack{m=1\\m\neq n}}^{N^{iu}}\varpi_{nm}^{uu} + \sum_{k=1}^{M^c}\varpi_{nk}^{\mathrm{uc}} + \varpi_{nb}^{\mathrm{uo}}\right]\Psi_{C_n^{iu}} - \sum_{\substack{m=1\\m\neq n}}^{N^{iu}}\varpi_{nm}^{uu}\Psi_{C_m^{iu}} = \sum_{\substack{m=1\\m\neq n}}^{N^{iu}}\varpi_{nm}^{uu}\Delta_{nm}^{uu}\Psi_{(0)} + \sum_{k=1}^{M^c}\varpi_{nk}^{\mathrm{uc}}\Psi_{(0)}(s_{nk}^{uc}) + \varpi_{nb}^{\mathrm{uo}}\Psi_{(0)}(s_{nb}^{ou})$$