To: ARR
From: PJH
Date: October 27, 2005 – 3:24 pm
Re: "Pacanowski & Philander Mixing" in HOPS
CC: PM

Original Formulation

Pacanowski & Philander (1981), following empirical studies of Robinson (1966) and Jones (1973), express the vertical eddy viscosity, ν, and the vertical eddy diffusivity, κ, as functions of the Richardson number, R_i, according to the formulae

$$\nu = \nu_b + \frac{\nu_0}{(1+\alpha R_i)^n}$$

and

$$\kappa = \kappa_b + \frac{\nu}{(1+\alpha R_i)}$$

where

$$R_i = \frac{\beta g \frac{\partial T}{\partial z}}{\left|\left|\frac{\partial \vec{u}}{\partial z}\right|\right|^2} \quad ,$$

and the coefficient of thermal expansion of water, β, is given by

$$\beta \sim 8.75\left(10^{-6}\right)(T+9).$$

Here T is the temperature in Celsius, $\vec{u}$ is the horizontal velocity and g is the acceleration due to gravity. This leave Pacanowski & Philander with 5 tunable parameters: the background viscosity, ν_b, the background diffusivity, κ_b and the "adjustable parameters" ν, α and n. They conclude with the following recommendations:

$$\begin{aligned}
\nu_b &= 1\frac{\mathrm{cm}^2}{\mathrm{s}} \\
\kappa_b &= 0.1\frac{\mathrm{cm}^2}{\mathrm{s}} \\
\nu_0 &= O(50)\frac{\mathrm{cm}^2}{\mathrm{s}} \\
\alpha &= 5 \\
n &= 2 \quad .
\end{aligned}$$

HOPS Implementation

In the HOPS PE model[1] , the Pacanowski & Philander coefficients are expressed as

$$\nu = \nu_b + \frac{\nu_0}{(1+5R_i)^2}$$

[1] The expressions for vertical eddy viscosity and diffusivity appear to be a straight port from GFDL MOM version 1.1.

and

$$\kappa = \kappa_b + \frac{\nu_0}{(1+5R_i)^3}$$

where

$$R_i = \max\left(\frac{-g\frac{\partial \rho_\theta}{\partial z}}{\left|\left|\frac{\partial \vec{u}}{\partial z}\right|\right|^2}\ ,\ 0\right)$$

and ρ_θ is the potential density. Since HOPS is on an Arakawa B-grid, two slightly different Richardson numbers are evaluated. For viscosity, we are evaluating on the velocity grid, so the gradient of potential density is an average of the 4 neighboring tracer grid values. Similarly for diffusivity, the velocity shear is evaluated from the 4 neighboring velocity grid values.

Comparison

There are 2 main differences between the Pacanowski & Philander formulae and the HOPS implementation.

(1) The HOPS vertical eddy diffusivity is missing the term

$$\frac{\nu_b}{(1+5R_i)} \quad .$$

(2) The Pacanowski & Philander factor $\beta\frac{\partial T}{\partial z}$ has been replaced[2] by $-\frac{\partial \rho_\theta}{\partial z}$.

References

Jones, J.H. (1973) "Vertical mixing in the Equatorial Undercurrent". *J. Phys. Oceanogr.*, **3**, 286–296.

Pacanowski, R.C. and G.H. Philander (1981) "Parameterization of Vertical Mixing in Numerical Models of Tropical Ocean". *J. Phys. Oceanogr.*, **11**, 1443–1451.

Robinson, A.R. (1966) "An investigation into the wind as the cause of the Equatorial Undercurrent". *J. Mar. Res.*, **86**, 1903–1916.

[2] This also appears to be a straight port from GFDL MOM version 1.1.