A non-hydrostatic two-dimensional unstructured finite volume model for tsunami waves

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As the recent Indian Ocean tsunami demonstrated it is important to develop accurate tsunami models, that can be incorporated into early warning systems. Many tsunami models employ the hydrostatic assumption. This is because the tsunami wave is generally much longer than the ocean depth. However, there is the potential to introduce inaccuracies into the numerical solution in regions where non-hydrostatic effects may be important. For example, in areas with complex sea topography, dispersion needs to be taken into account in order to accurately model the propagation and shoaling of the tsunami waves.

Recently, a two-dimensional unstructured finite-volume model has been developed by Cui et al. (2010). This model has been shown to be mass and momentum conservative. The model produces accurate solutions in the simulation of flooding and drying. Based on this model, we use a non-hydrostatic correction method to incorporate the dispersive effects. The non-hydrostatic algorithm utilizes a non-hydrostatic pressure term to describe weakly dispersive waves. A fractional step numerical procedure proposed by Stelling and Zijlema (2003) is employed. In the first step, the model uses the conventional shallow water equations to compute the velocities and water levels. Then in the second step, the non-hydrostatic pressures are calculated implicitly by using the Poisson equation, which guarantees the new velocity field is divergence free. The velocities and water levels are then updated using the resulting non-hydrostatic pressure field.

The original implementation of the non-hydrostatic algorithm results in a sparse matrix with a large bandwidth, because the non-hydrostatic pressure gradient is approximated by the four node values within the two neighboring triangles of each edge. The solution of this matrix requires significant computational effort. To improve the efficiency of the model, the non-hydrostatic pressure gradient is only approximated by the three node values within one triangle when substituting the new velocities into the Poisson equations. In so doing, the bandwidth of the matrix is reduced by half. The new model is validated against several classic tsunami test cases, such as a standing wave, the propagation of solitary wave, and wave run-up onto a conical island.

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