A sigma–coordinate, discontinuous Galerkin method for the three–dimensional shallow water equations

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In this presentation, we describe the development, implementation, and application of a novel discontinuous Galerkin (DG) method for the three–dimensional shallow water equations. A key feature of the developed DG method is the discretization of all the primary variables using discontinuous polynomial spaces of arbitrary order, including the free surface elevation. In a standard Cartesian–coordinate system, this results in elements in the surface layer having mismatched lateral faces (a staircase boundary). This difficulty is avoided in the current method by employing a sigma–coordinate system in the vertical, which transforms both the free surface and bottom boundaries into coordinate surfaces. The top sigma–coordinate surface, which corresponds to the free surface, is discretized using a combination of triangular and quadrilateral elements that are extended in the vertical direction to produce a three–dimensional mesh of one or more layers of triangular prism and hexahedral elements. The polynomial spaces over these elements are constructed using an orthogonal basis, which results in a matrix–free implementation of the method. New symmetric quadrature rules for the integration of complete polynomials over triangular prisms are also developed, which require fewer integration points than other available methods of numerical integration over triangular prisms. The h (mesh) and p (polynomial order) convergence properties of the method are demonstrated on a set of analytic test cases for the three-dimensional shallow water equations.

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