Gradient, divergence and laplacian discrete approximations for numerical ocean modelling

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Ocean modelling imply to handle the integration of the gradient and divergence, occasionally laplacian when it comes to explicitly diffusive problems, of possibly discontinuous functions, hence with their derivative not defined along discontinuity limits. It is generally done using some sort of finite differences analogy, but these approximations are mostly empirical and can lead to inconsistencies.

For first derivative operators (under integral sign), we propose new approximations (expressed as a correction term to the Rieman's integral) that master these inconsistencies and are compatible with standard integrations propriety, especially Liebniz's and Stokes' formulas. Using these approximations permits some unification of the finite differences, finite elements and finite volumes methods. However, the authors would like to open two fundamental questions:

1. The laplacian operator of a discontinuous function is not integrable, and an arbitrary re-formulation must be used in place of this operator (such as second order finite difference or an intermediary change of discretisation). Inappropriate re-formulation can lead to some difficulties in advection-diffusion applications as the efficiency of the re-formulated diffusion operator is not guaranteed to minor the velocity field variance in space. Our first question is: what would be then an optimal re-formulation?

2. The issue of discontinuities is somehow even more worrying in 3D modelling. In the finite volumes approach, the use of discontinuous elevation (hence discontinuous σ-layer discretisation), would, in theory, need extremely heavy derivation to rigorously take discontinuities into account, with a very significant additional numerical costs. In most of finite volumes models (structured or not structured), those discontinuities are simply ignored to maintain a low numerical costs (elevation/layer displacement is discontinuous in the mass conservation equation, but taken as continuous when deriving the pressure terms, and sigma layers are seen as continuous tilted surfaces to justify the horizontal transport from one column to another). The so-called "hydrostatic inconsistency" directly derive from this simplification (through the violation of Liebniz's rule). Our Second question is: shall we keep going with non-rigorous models (in case of finite volumes, cost-efficient and with well-known errors and limits) or shall we investigate and invest community efforts in more rigorous approaches, with a predictable significant impact on computational costs?