SLIM : a three-dimensional baroclinic finite-element model
Time and spatial discretizations

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This presentation describes both spatial and time discretizations of the three-dimensional baroclinic finite element SLIM model, based upon a Discontinuous Galerkin [Blaise *et al.*, 2010; Comblen *et al.*, 2010] . We solve the hydrostatic Boussinesq equations governing marine flows on a mesh made up of triangles extruded toward sea-bottom to obtain prismatic three-dimensional elements. Diffusion is implemented using the symmetric interior penalty method, with modified penalty coefficients to handle the anisotropy. The tracer equation is consistent with the continuity equation. A Lax-Friedrichs flux is used to take into account internal wave propagation. As a relevant illustration, a flow exhibiting internal waves in the lee of an isolated seamount on the sphere is simulated. This enables us to show the advantages of using an unstructured mesh, where the resolution is put in areas where the flow varies rapidly in space, the mesh being coarser far from the region of interest. The solution exhibits the expected wave structure. Linear and quadratic shape functions are used and the extension of to higher order discretization is straightforward.

The time stepper is based on an efficient mode splitting. To ensure compatibility between the barotropic and baroclinic modes in the splitting algorithm, we introduce Lagrange multipliers in the discrete formulation. The use of implicit-explicit Runge-Kutta methods enables us to treat stiff linear operators implicitly, while the rest of the nonlinear dynamics are treated explicitly. For such an implicit/explicit approach to be interesting, the discrete operators for the dynamics handled implicitly must be significantly stiffer than those for the explicit dynamics. Indeed, the time step allowed by the IMEX scheme must be significantly larger than the time step of a purely explicit discretization. Gowever, those time steps are much more expensive, as local linear systems are solved. To faster computations, the way is twofold. On the one hand, the computation of the discrete terms can still be improved, by recasting most of the operations into efficient matrix matrix products computed with highly optimized linear algebra subroutines. On the other hand, the time stepping strategy can itself be improved. IMultigrid methods have the potential to provide scalable solutions to large-scale discrete problems. Further such multigrid methods do not need the matrix of the linear system to be assembled, dramatically reducing the memory footprint of the algorithm. However, the design of an efficient multigrid algorithm is itself a whole domain of research.

References

S. Blaise, R. Comblen, V. Legat, J.-F. Remacle, E. Deleersnijder, J. Lambrechts, "A discontinuous finite element baroclinic marine model on unstructured prismatic meshes. Part I: spatial discretization", *Ocean Dynamics* (submitted)

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