An efficient method for the Incompressible Navier-Stokes Equations on Irregular Domains with no-slip boundary conditions, high order up to the boundary.

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Common efficient schemes for the incompressible Navier-Stokes equations, such as projection or fractional step methods, have limited temporal accuracy as a result of matrix splitting errors, or introduce errors near the domain boundaries, resulting in weakly convergent solutions. We recast the Navier-Stokes incompressibility constraint as a pressure Poisson equation with velocity dependent boundary conditions. Applying the remaining velocity boundary conditions to the momentum equation, we obtain a pair of equations, for the primary variables velocity and pressure, equivalent to the incompressible Navier-Stokes. Since in this recast system the pressure can be efficiently recovered from the velocity, this reformulation is ideal for numerical marching methods. The equations can be discretized using a variety of methods, in principle to any desired order of accuracy. In this work we illustrate the approach with a 2-D second order finite difference scheme on a Cartesian grid, and devise an algorithm to solve the equations on domains with curved (non-conforming) boundaries, including a case with a non-trivial topology (a circular obstruction inside the domain). This algorithm achieves second order accuracy in the L^∞ norm for both the velocity and the pressure. The scheme has a natural extension to 3-D.

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