A Lagrangian vortex method for the barotropic vorticity equation on a rotating sphere

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We present a Lagrangian vortex method for the barotropic vorticity equation (BVE) on a rotating sphere. The solution of BVE involves solving a conservative transport equation for the vorticity fields and a Poisson equation for the stream function. The vortex method tracks the flow map and absolute vorticity using Lagrangian particles and panels. The velocity is computed from the Biot-Savart integral on the sphere. An adaptive refinement strategy is implemented to resolve small-scale features and a treecode is used for efficient computation. A fourth-order Runge-Kutta scheme is used for time integration.

We start our investigation with point vortex method and the first test case is the Rossby-Haurwitz wave, which is the exact solution for BVE. Convergence study shows that the method is fourth order in time and first order in space for a uniform panel discretization of the sphere. Then we switch to vortex blob method for stability consideration. We also tested the evolution of vortex patch(s), which means the vorticity field is highly nonuniform on the surface of the sphere. Adaptive refinement strategy improves the computational efficiency.

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