Dynamically Orthogonal Narrow-Angle Parabolic Equations
for Stochastic Underwater Sound Propagation. Part II: Applications

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Abstract:

The stochastic dynamically orthogonal (DO) narrow-angle parabolic equations (NAPEs) are exemplified and their properties and capabilities are described using three new 2D stochastic range-independent and range-dependent test cases with uncertain sound speed field, bathymetry, and source location. We validate results against ground-truth deterministic analytical solutions and direct Monte Carlo predictions of acoustic pressure and transmission loss fields. We verify the stochastic convergence and computational advantages of the DO-NAPEs and discuss the differences with normal mode approaches. Results show that a single DO-NAPE simulation can accurately predict stochastic range-dependent acoustic fields and their non-Gaussian probability distributions, with computational savings of several orders of magnitude when compared to direct Monte Carlo methods. With their coupling properties and their adaptation in range to the dominant uncertainties, the DO-NAPEs are shown to predict accurate statistics, from mean and variance to multiple modes and full probability distributions, and to provide excellent reconstructed realizations, from amplitudes and phases to other specific properties of complex realization fields.
I. INTRODUCTION

In Part I of this two-part paper (Ali and Lermusiaux, 2023), we derived, discretized, and implemented stochastic differential equations that (i) capture the dominant input uncertainties in the environment (e.g., ocean, bathymetry, and seabed) and in the acoustic parameters (e.g., source location, frequency, and bandwidth), and (ii) predict the acoustic pressure fields and their probability distributions, respecting the nonlinear governing equations and non-Gaussian statistics. Starting from the acoustic Parabolic Equation (PE), we derived Dynamically Orthogonal (DO) differential equations for range-optimal acoustic uncertainty quantification. Using DO expansions for the input uncertainties, we developed the reduced-order DO-PEs theory and applied it to derive the DO Narrow-Angle PE (DO-NAPE) stochastic partial differential equations (PDEs).

In the present study, we illustrate and analyze the properties and capabilities of the DO-NAPEs in a wide range of test cases with varying sources of uncertainty and increasing complexity. We focus on stochastic acoustic propagation in 2D space (depth $z$, range $\eta$; stochastic parameter $\xi$), within ocean environments with uncertain sound speed, bathymetry, and source depth. The goal is to predict the DO decomposition $\psi_{DO}$ and $TL_{DO}$ of the stochastic complex envelope pressure field $\psi(z, \eta; \xi)$ and transmission loss $TL(z, \eta; \xi)$, respectively. The sources of uncertainties are the stochastic squared effective index of refraction $n_{eff}^2(z, \eta; \xi)$, stochastic bathymetry $b(\eta; \xi)$, and stochastic source depth $Z_s(\xi)$. The boundary conditions (BCs) are assumed deterministic, as well as the source frequency and