Dynamically Orthogonal Narrow-Angle Parabolic Equations for Stochastic Underwater Sound Propagation

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Abstract: Accurate underwater acoustic propagation requires precise knowledge of the ocean physics, bathymetry, seabed, and acoustic parameters. However, in realistic ocean applications, such information is typically incomplete due to the sparse and heterogeneous measurements as well as to the complex ocean physics, nonlinearities, multiscale interactions, and large dimensions. Robust informative acoustic predictions thus require efficient techniques for quantifying these uncertainties and predicting the stochastic acoustic wave fields. In this work, we derive and implement stochastic partial differential equations that predict the acoustic pressure fields and their probability distributions. We start from the acoustic narrow-angle parabolic equation (NAPE), a widely used acoustic modeling technique, and employ the instantaneously optimal Dynamically Orthogonal (DO) equations framework. The derived DO-NAPEs capture the multi-dimensional uncertainties using a reduced-order model that respects the nonlinear governing equations and non-Gaussian statistics. We showcase their applications in both range-independent and range-dependent test cases with uncertain sound speed, source location, and bathymetry fields. We validate our results against Monte Carlo predictions of the transmission loss statistics, realizations, and probability distributions. We also highlight the computational advantages of our framework and analyze the stochastic convergence properties of our solutions.
I. INTRODUCTION

Reliable acoustic exploration and navigation in the ocean require precise knowledge of the environmental state (e.g., ocean physics, bathymetry, seabed) and acoustic parameters (e.g., source location and frequencies). When all such information is available, sound waves can be reliably used to explore the ocean and seabed (Baggeroer et al., 1993; Becker et al., 2009; Firing and Gordon, 1990; Gartner, 2004; Medwin and Clay, 1998), to locate underwater objects and animals (Blondel, 2010; Bonnel et al., 2014; Jagannathan et al., 2009; Lavery et al., 2007; MacLennan and Simmonds, 2013; Makris et al., 2006; Quazi, 1981), and to communicate via the oceanic waveguide (Akyildiz et al., 2005; Benjamin et al., 2010; Stojanovic, 1996). However, in these real-world applications, such information is typically incomplete (Lermusiaux et al., 2006; Tollefsen, 2021) due to the sparse and heterogeneous data collected (Etter, 2018), as well as to the complex ocean physics and acoustics dynamics, multiscale interactions, and large dimensions (Brekhovskikh and Lysanov, 1982; Duda et al., 2019).

This incomplete knowledge leads to several sources of uncertainty in the acoustic models. The ocean currents, temperature, salinity, and pressure fields, and as a result the sound speed fields, are themselves often outputs of stochastic ocean physics models with uncertain initial and boundary conditions, and numerical approximations (Lermusiaux et al., 2006; Rixen et al., 2012). Finally, the exact bottom topography and properties are not available which adds uncertainties to the acoustic predictions (Dosso et al., 2014; Etter, 2018; Jakobsen et al., 2017; Tolstoy, 1996). To represent these uncertainties and incomplete knowledge, stochastic environmental fields force the acoustic models and sound propagation becomes stochastic. In this work, we use a principled probabilistic approach to quantify the effects of