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Range-Dynamical Low-Rank Split-Step Fourier Method for the Parabolic Wave Equation

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Numerical solutions to the parabolic wave equation are plagued by the curse of dimensionality coupled with the Nyquist criterion. As a remedy, a new range-dynamical low-rank split-step Fourier method is developed. Our integration scheme scales sub-linearly with the number of classical degrees of freedom in the transverse directions. It is orders of magnitude faster than the classic full-rank split-step Fourier algorithm and also saves copious amounts of storage space. This enables numerical solutions of the parabolic wave equation at higher frequencies and on larger domains, and simulations may be performed on laptops rather than high-performance computing clusters. By using a rank-adaptive scheme to further optimize the low-rank equations, we ensure our approximate solution is highly accurate and efficient. The methodology and algorithms are demonstrated on realistic high-resolution data-assimilative ocean fields in Massachusetts Bay for three-dimensional acoustic configurations with different source locations and frequencies. The acoustic pressure, transmission loss, and phase solutions are analyzed in geometries with seamounts and canyons across and along Stellwagen Bank. The convergence with the rank of the subspace and the properties of the rank-adaptive scheme are demonstrated, and all results are successfully compared with those of the full-rank method when feasible.

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I. INTRODUCTION

Wave propagation is pertinent to many scientific and engineering disciplines such as oceanography, seismology, acoustics, and optics. For propagation in complex media, both forward problems — where one predicts how a wave propagates in a known environment — and inverse problems — where one attempts to determine model/environmental parameters given observational data about wave propagation in the said environment — require numerical methods to provide accurate solutions. In this paper, we restrict our attention to the acoustic wave equation for propagation in the three spatial dimensions,

$$\rho \nabla \cdot \left(\frac{1}{\rho} \nabla p \right) - \frac{1}{c^2} \frac{\partial^2 p}{\partial t^2} = f, \tag{1}$$

where p is the acoustic pressure field, ρ the density field of the media, c is the sound speed field of the media, t time, and f some forcing field. Our specific domain of interest is underwater acoustics.

Unfortunately, numerically solving this hyperbolic partial differential equation (PDE) is often too computationally expensive in three dimensions and/or at mid-to-high frequencies. The Nyquist criterion requires we sample spatially at least twice per wavelength as well as temporally at least twice per period. This would be a minimal resolution in space-time but, nonetheless, a fine mesh for many problems, making the direct numerical simulation of the acoustic wave equation (1) in complex environments such as the ocean intractable. Several approximations can be made such as ray methods (Cervenỳ, 2001; Lichte, 1919), wavenumber integration (DiNapoli and Deavenport, 1980; Ewing *et al.*, 1957; Jardetzky, 1953; Kutschale, 1973), normal-mode methods (Ide *et al.*, 1947; Pekeris, 1945; Williams, 1970), and parabolic