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Abstract

We obtain, solve, and verify fundamental differential equations for energy-time path planning in dynamic flows. The equations govern the energy-time reachable sets, optimal paths, and optimal controls for autonomous vehicles navigating to any destination in known dynamic environments, minimizing both energy usage and travel time. Based on Hamilton-Jacobi theory for reachability and the level set method, the resulting methodology computes the Pareto optimal solutions to the multi-objective path planning problem, numerically solving the exact equations governing the evolution of reachability fronts and optimal paths in the augmented energy and physical-space domain. Our approach is applicable to path planning in various dynamic flow environments and energy types. We first validate the methodology through a benchmark case of crossing a steady jet for which we compare our results to semi-analytical optimal energy-time solutions. We then consider unsteady flow environments and solve for energy-time optimal missions in a quasi-geostrophic double-gyre flow field. Results show that our theory and schemes can provide all the energy-time optimal solutions and that these solutions can be strongly influenced by unsteady flow conditions.

Keywords: Path planning, Time-optimal, Energy-optimal, Multi-Objective Optimization, Hamilton-Jacobi, Reachability

1. Introduction

The growth of autonomous vehicles has been staggering in the last decade. Self driving cars, autonomous drones, and underwater vehicles have all seen a surge of interest (e.g. Griffiths [2002], Schofield et al. [2010], Floreano and Wood [2015], Bagloee et al. [2016], Lermusiaux et al. [2017b], Murphy [2019]). Central to the effective operation of all autonomous vehicles is efficient and accurate motion control which falls under the purview of path planning. Path planning, in the most general sense, corresponds to a set of rules to be provided to an autonomous robot for moving from one configuration to another in some optimal fashion (Lolla et al. [2014b]). The metric for optimality depends on the application and specific objectives of the user. It includes optimizing for travel time, energy employed, vehicle safety, or quality of collected data to name a few (Lolla [2016], Subramani and Lermusiaux [2016]). Increasingly, the requirements for vehicles to operate autonomously for longer duration in harsh conditions (Bellingham and Rajan [2007]) and the need to sustain the health of our planet (Karakoc et al. [2019]) have resulted in the ever-increasing importance of energy optimization. In particular, one can optimize vehicles and propulsion systems, or better use the environment to minimize energy consumption. There are indeed many applications where the environment can play a significant role in the sustainable navigation of marine, land, and air vehicles (Webb et al. [2001]). The novel focus of this work is on exact equations and computational methods for the joint energy-time optimal path planning of autonomous vehicles navigating in known strong and dynamic environments. The theory and schemes are verified in the ocean context but the results are applicable to other environments.

The energy requirements of marine vehicles are diverse. In addition to the power required for the propulsion system itself, power is also needed for non-propulsive purposes — what is known as the “hotel” load — for components such as the sonar system for mapping and systems for data transmission and reception (Reader et al. [2002]). When navigating from a start point to the desired destination, energy efficiency ultimately comes from two major sources: (1) The vehicle itself such as new designs, propulsion systems,