Rigid Sets and Coherent Sets in Realistic Ocean Flows

Florian Feppon\textsuperscript{a,b} and Pierre F.J. Lermusiaux\textsuperscript{b}

\textsuperscript{a}Department of Mathematics, ETH Zürich, Rämistrasse 101, CH-8092 Zürich, Switzerland
\textsuperscript{b}Department of Mechanical Engineering, Massachusetts Institute of Technology, 77 Mass. Ave., Cambridge, MA - 02139

Correspondence: F. Feppon (florian.feppon@sam.math.ethz.ch), P.F.J. Lermusiaux (pierrel@mit.edu)

Abstract. This paper focuses on the extractions of Lagrangian Coherent Sets from realistic velocity fields obtained from ocean data and simulations, each of which can be highly resolved and non volume-preserving. We introduce two novel methods for computing two formulations of such sets. First, we propose a new “diffeomorphism-based” criterion to extract “rigid sets”, defined as sets over which the flow map acts approximately as a rigid transformation. Second, we develop a matrix-free methodology that provides a simple and efficient framework to compute “coherent sets” with operator methods. Both new methods and their resulting \textit{rigid sets} and \textit{coherent sets} are illustrated and compared using three numerically simulated flow examples, including a high-resolution realistic, submesoscale to large-scale dynamic ocean current field in the Palau Island region of the western Pacific Ocean.

1 Introduction

The pioneering concept of Lagrangian Coherent Structures (LCS) has emerged (Haller and Yuan (2000)) to offer visualization and understanding of material transport in time-dependent fluid flows. The terminology was born from direct observations of realistic flows and refers to the persistence of distinguished material sub-domains over time (Farazmand and Haller (2012); Haller and Beron-Vera (2012); Haller (2015); Balasuriya et al. (2018); Andrade-Canto et al. (2020)). Extracting LCS is expected to allow for improved Lagrangian hazard predictions; typical ocean applications include pollution tracking (Lekien et al. (2005); Coulliette et al. (2007); Lermusiaux et al. (2019); Nolan et al. (2020)), search and rescue (Serra et al. (2020)), or ecosystem characterizations (Scales et al. (2018); Doshi et al. (2019); Paris et al. (2020)). Various useful definitions of LCS have been proposed (Farazmand and Haller (2012); Froyland et al. (2007); Haller (2015); Karrasch (2015); Onu et al. (2014); Peacock and Haller (2013); Shadden et al. (2005); Tang et al. (2010); Allshouse and Thiffeault (2012)), and there are as many computational methodologies to extract them from time-dependent (\textit{non-autonomous}) velocity fields $v(t, x)$. Here, the variable $x$ denotes the spatial position over a two or three dimensional computational domain $\Omega \subset \mathbb{R}^n$ ($n = 2$ or $n = 3$). These approaches can be classified broadly into two categories (Feppon (2017)).

The first category of methods (Haller (2002); Haller et al. (2018)) focuses on the motion of individual particles whose location $x(t)$ satisfies the Ordinary Differential Equation (ODE),

\begin{equation}
\begin{aligned}
\dot{x} &= v(t, x(t)) \\
x(0) &= x_0,
\end{aligned}
\end{equation}