Rigid Sets and Coherent Sets in Realistic Ocean Flows

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Abstract

This paper focuses on the extractions of Lagrangian Coherent Sets from realistic velocity fields obtained from ocean data and simulations, each of which can be highly resolved and non volume-preserving. Two classes of methods have emerged for such purpose: those relying on the flow map diffeomorphism associated with the velocity field, and those based on spectral decompositions of the Koopman or Perron-Frobenius operators. The two classes of methods are reviewed, synthesized, augmented, and compared numerically on three velocity fields. First, we propose a new "diffeomorphism-based" criterion to extract "rigid sets", defined as sets over which the flow map acts approximately as a rigid transformation. Second, we develop a matrix-free methodology that provides a simple and efficient framework to compute "coherent sets" with operator methods. Both new methods and their resulting *rigid sets* and *coherent sets* are illustrated and compared using three numerically simulated flow examples, including a realistic, submesocale to large-scale dynamic ocean current field in the Palau Island region of the western Pacific Ocean.

Keywords: LCS, Rigid sets, Koopman operator, Arnoldi Iterations, Ocean Modeling, Lagrangian transport, Realistic data
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1. Introduction

The pioneering concept of Lagrangian Coherent Structures (LCS) has emerged [1] to offer visualization and understanding of material transport in time-dependent fluid flows. The terminology was born from direct observations of realistic flows and refers to the persistence of distinguished material sub-domains over time [2, 3, 4, 5]. Extracting LCS is expected to allow for improved Lagrangian hazard predictions; typical ocean applications include pollution tracking [6, 7, 8], search and rescue [9], or ecosystem characterizations [10, 11, 12]. To date, several definitions of LCS that do not fully coincide have been proposed [2, 13, 4, 14, 15, 16, 17, 18, 19], and there are as many computational methodologies to extract them from time-dependent (*non-autonomous*) velocity fields $\boldsymbol{v}(t,\boldsymbol{x})$. Here, the variable $\boldsymbol{x}$ denotes the spatial position over a two or three dimensional computational domain $\Omega \subset \mathbb{R}^n$ ($n = 2$ or $n = 3$). These approaches can be classified broadly into two categories [20].

The first category of methods [21, 22] focuses on the motion of individual particles whose location $\boldsymbol{x}(t)$ satisfies the Ordinary Differential Equation (ODE),

$$\begin{cases} \dot{\boldsymbol{x}} = \boldsymbol{v}(t,\boldsymbol{x}(t)) \\ \boldsymbol{x}(0) = \boldsymbol{x}_0, \end{cases} \tag{1}$$

or equivalently on the relevant feature of the flow map $\boldsymbol{\phi}^t : \Omega \to \mathbb{R}^n$ associated to (1) which is defined for any $\boldsymbol{x}_0 \in \Omega$ by $\boldsymbol{\phi}^t(\boldsymbol{x}_0) = \boldsymbol{x}(t)$. This first category of methods seeks simplified visualizations of the diffeomorphism

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