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A FORWARD REACHABILITY EQUATION FOR MINIMUM-TIME PATH PLANNING IN STRONG DYNAMIC FLOWS

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Abstract. A theoretical synthesis of forward reachability for minimum–time control of anisotropic vehicles operating in strong and dynamic flows is provided. The synthesis relies on the computation of the forward reachable set of states. Using ideas rooted in the theory of non–smooth calculus, we prove that this set is governed by the viscosity solution of an unsteady Hamilton–Jacobi (HJ) equation. We show that the minimum arrival time satisfies a static HJ equation, when a special local controllability condition holds. Results are exemplified by applications to a sailboat navigating in a uniform wind–field and autonomous underwater gliders operating in the Sulu Archipelago.

Key words. time–optimal, path planning, viscosity solutions, forward reachable set, gliders, autonomous underwater vehicles, sailboat, anisotropic, unmanned aerial vehicles, weather and ocean forecasts

AMS subject classifications.

1. Introduction.

This paper provides a theoretical synthesis for time–optimal control of anisotropic robotic units operating under the influence of strong and dynamic flow–fields. The deduction is based on a formulation of forward–reachability in terms of the viscosity solution to an unsteady Hamilton–Jacobi (HJ) equation. This problem is of great interest in several areas of engineering. For example, it is applicable to Unmanned Aerial Vehicles (UAVs), for exploratory, surveillance or imaging missions, transportation and delivery, as well as environmental and climate research. It is also pertinent to Autonomous Underwater Vehicles (AUVs) such as ocean gliders and surface crafts, for diverse missions including ocean mapping and sampling, naval reconnaissance and harbor protection. Flows encountered by these vehicles can often be comparable in magnitude to their nominal speeds (speed relative to the flows). For both air and sea vehicles, it is imperative to develop rigorous and efficient techniques for optimal routing, minimizing either the energy consumed, the travel time, or other performance criteria [29, 18]. Additionally, certain "anisotropic vehicles" are characterized by direction–dependent motion constraints, which must be accounted for. Hence, our focus is the deductive forward computation of minimum–time trajectories of anisotropic vehicles operating in complex time and space dependent flows.

Minimum–time problems have been well studied in the control theory and operations research literature. The first results are likely due to Galton who coined and employed isochrones for time-optimal ship routing problems [20, 47]. Since then, the most common approach to solve such problems uses the Dynamic Programming Principle (DPP) to derive an *unsteady* Hamilton–Jacobi–Bellman (HJB) equation for the globally optimal 'time–to–go', which is the minimum time required to drive the system to a *fixed target* state [2, 11]. The viscosity solution [14] of the governing

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