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Real-time Probabilistic Reachability Forecasting for Gliders in the Gulf of Mexico

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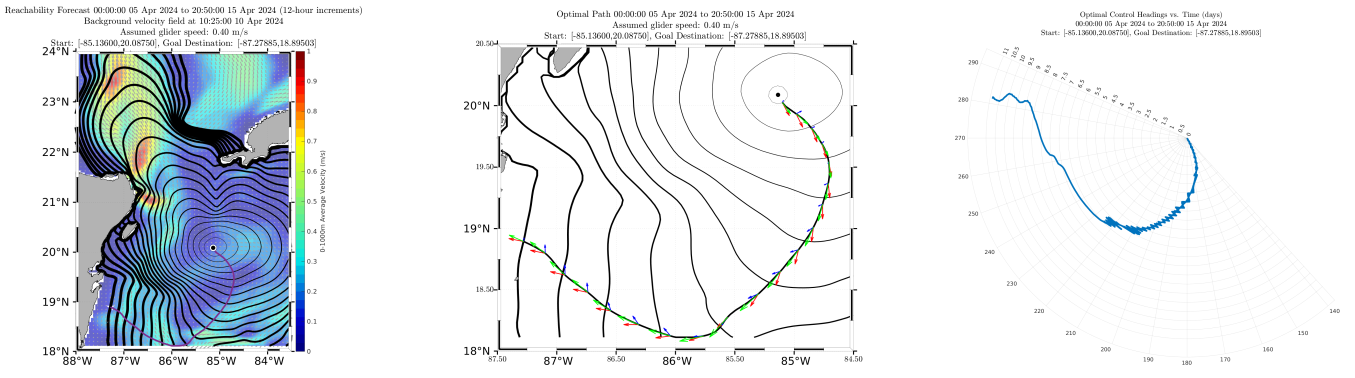
Principled optimal path planning for autonomous marine vehicles made major advances in the past decade [1, 2]. However, rigorous planning theory and schemes are not often used in real-time with vehicles at sea [3, 4], especially with ocean gliders [5, 6]. In this work, we demonstrate for the first time the use of differential reachability analysis and time-optimal path planning forecasts to help guide a fleet of four ocean gliders in the Gulf of Mexico (GoM) region for more than two months. This real-time effort was part of the Understanding Gulf Ocean Systems (UGOS) program of the U.S. National Academies of Sciences, Engineering, and Medicine.

The collaborative sea experiment, the Mini-Adaptive Sampling Test Run (MASTR), occurred from February to April 2024 [7]. Using the MIT-MSEAS data-assimilative Primitive-Equation (PE) submesoscale-to-regional-scale ocean-modeling system [8], we issued daily deterministic and probabilistic forecasts of ocean fields and derived quantities in real-time [7, 9]. We provided multi-resolution ensemble forecasts. They were forced with stochastic tides and air-sea fluxes and initialized by downscaling from two global models with 3D PE-field perturbations using Error Subspace Statistical Estimation (ESSE). We issued mutual information forecasts, optimal adaptive sampling guidance for air and sea platforms, and reachability and path planning forecasts for underwater vehicles. The latter were used by four gliders that sampled the western Caribbean Sea including the Yucatan Current (YCu) and the mesoscale eddies in the region, clearly showing the presence of subsurface salinity maxima. The data was processed, quality-controlled, and assimilated in real-time [7].

Because ocean gliders are relatively slow (0.3–0.5 m/s) with respect to the prevailing currents (upwards of 1 m/s in portions of the YCu), it can be difficult to intuitively predict what areas are accessible to the gliders and what trajectories are time-optimal. To quantitatively answer these questions, we built upon our rigorous level set differential equations [1, 10] to compute exact reachability fields and time-optimal paths for glider operations, given the uncertain future currents. These in-



(a) Reachability fronts (at 12-hour intervals, increasing thickness for later times) with time-optimal path overlaid on forecast currents (m/s)

(b) Optimal path with forecast current (blue), optimal vehicle propulsion (red), and net velocity (green) vectors, overlaid of reachability fronts

(c) Polar diagram of time-optimal glider control headings vs. time

Fig. 1: Reachability fronts, time-optimal path, and optimal controls for the recovery of glider Stommel, as forecast for 5 to 15 April. The optimal path takes advantage of a forecast mesoscale eddy and its strong southwestward and then northwestward currents to the south of the starting location, whereas a straight line path is slowed down by opposing currents (as seen by tightly grouped reachability fronts).

put currents and their probabilities were the real-time forecasts of the MSEAS-PE system. The gliders were piloted using a 1000 m yo-yo pattern. Accordingly, 0-1000 m depth-averaged currents and horizontal glider velocities of 40-50 cm/s (RU38) and 30-40 cm/s (SG625, SG652, and Stommel) were used for reachability computation.

Figure 1 illustrates the reachable sets, time-optimal paths, and optimal heading forecasts issued for the four gliders. Reachability fronts shown in Fig. 1(a) were issued daily, often displayed at 12-hour intervals, with line thickness increasing for later times, and overlaid on the forecast 0-1000 m averaged current velocity magnitude and vectors (at the temporal midpoint of the forecast). A reachability front is here the boundary of the set of points a glider can reach within a certain duration. The shape and growth of the reachable front are driven by the glider's horizontal propulsion and the dynamic local currents. Generally, strong northward currents in the YCu near the Mexican coast cause the reachable fronts to spread more quickly in that region, especially near Arrowsmith Bank.

To account for uncertainties in forecast currents, we computed probabilistic reachability fields using our ESSE ensemble forecasts [9]. After computing each ensemble member's reachable set, the overall probability field was calculated by taking the union of each member's reachable set and normalizing by the number of members. A March 22-27, 2024, forecast is shown in Figure 2, where the colored field is the forecast probability that the location is within the reachable set of the glider on March 27, 2024, overlaid with the ensemble mean velocity vectors (at the temporal midpoint of the forecast). These probabilistic forecasts provide additional clarity on which regions are reachable under most current conditions, and which regions are possibly reachable but only under more extreme currents. For the example shown in Figure 2, the glider could likely reach many locations to the north through the Yucatan Strait under any current condition (within the expected statistical distribution). However, the probability that the glider could reach locations directly to the south near Cozumel is much lower and would require more extreme currents (in this case, weak northward currents).

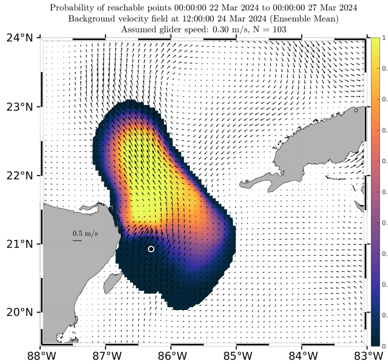


Fig. 2: Forecast probability (pdf) of reachable points after 5 days for a 0.3 m/s glider stating at the white dot location on 00Z March 22, using MIT-MSEAS ensemble forecasts (N=103). The pdf field is overlaid on the ensemble mean current vectors at $t = 2.5$ days.

Reachability forecasts were also provided to assist in the localization and retrieval of gliders in distress. In these cases, the glider model was modified in real time to represent the real glider. For example, some glider motions were modeled predominantly due to advection with surface currents. Other gliders had limited motions and were affected only by currents at specific depths. We also generated reachability forecasts for "virtual" gliders near Key West and Tampa to assist in determining future optimal deployment strategy [7].

To conclude, we note that these reachability forecasts, along with uncertainty forecasts [7, 9], were not just intellectual exercises but were used in real-time by the glider operators to achieve their mission. This confirms that principled optimal path planning is not only feasible in real time but is also an important tool for operations.

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