

# Dynamically-Orthogonal Parabolic Equations for Probabilistic Ocean Acoustics in the New England Seamounts

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Underwater sound propagation is sensitive to specific features, scales, and gradients in the ocean environment, from turbulent processes at acoustic wavelengths to large-scale circulations at ocean basin scales. However, due to the limited ocean observations, wide range of scales, and dynamic ocean processes, it is challenging to model and predict all these acoustics-relevant ocean features at sufficient levels of accuracy. In addition, the dominant sensitivities are themselves not always well known or understood [1, 2], especially for strongly nonlinear effects. Finally, acoustics sensitivities depend on the sound frequency, source-receiver configuration, and many other operational and environmental factors. To further scientific understanding and augment acoustics modeling capabilities, both process studies of nonlinear sensitivities and stochastic modeling are useful. The former enables targeted studies of complex processes using data and models while the latter augments deterministic modeling with probabilistic environmental conditions and stochastic forcing inputs. The results can capture the environmental inputs that matter and organize them by their acoustic importance, both dynamically and probabilistically [3, 4].

In this work, we demonstrate the use of our deterministic [5] and stochastic Dynamically-Orthogonal Parabolic Equations (DO-ParEq) [6–9] to determine and quantify the features of ocean fields that most affect underwater sound propagation. We illustrate results for idealized seamounts and for the New England Seamounts off the eastern US coastline. A long-term goal is to utilize our nonlinear stochastic DO analysis to quantify acoustics dynamical regimes and complete global dynamical analyses of ocean acoustics sensitivity.

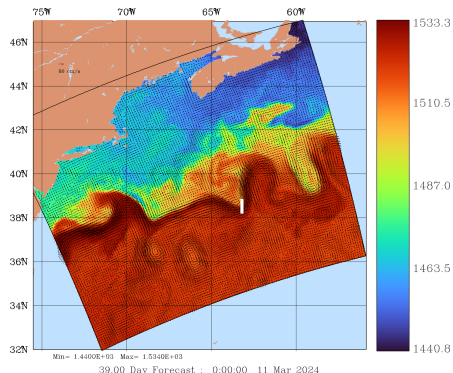
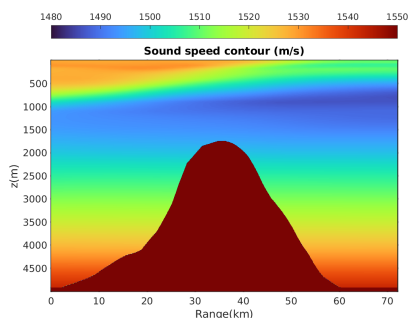


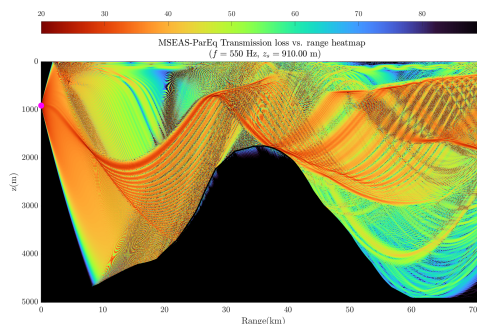
Fig. 1: Sea surface sound speed overlaid with current vectors, as hindcast by our MIT-MSEAS system on 00Z, March 11, 2024. The white line denotes the location of the section in Figure 2.

The New England Seamounts are a chain of seamounts to the southeast of Georges Bank. The Gulf Stream (separating the Shelf and Sargasso water masses) passes through this chain occasionally impinging upon an individual seamount. Figure 1 shows the sea surface sound speed field in this Gulf Stream and New England Seamounts region, overlaid with current vectors, for 00Z, March 11, 2024. The ocean fields are estimated by our MIT-MSEAS data-assimilative Primitive-Equation (PE) submesoscale-to-regional-scale ocean-modeling system [10, 11] and initialized by downscaling from global models. Our ensemble forecasts are initialized with 3D PE-field perturbations from Error Subspace Statistical Estimation (ESSE) and forced with stochastic tides and stochastic air-sea fluxes. The probabilistic forecasts of ocean sound speed and density fields are then input to the DO-ParEq.

Figure 2 illustrates sound propagation in this NE Seamount region for the Atlantis II seamount on 00Z, March 11, 2024.



(a) Sound speed field (m/s).

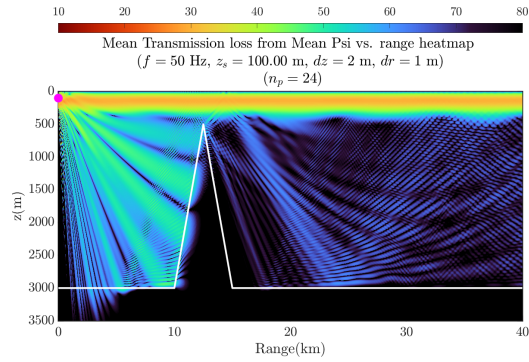


(b) Transmission Loss (TL) field (dB). Source Depth = 910 m, Frequency = 550 Hz. Cylindrical spreading was removed from TL.

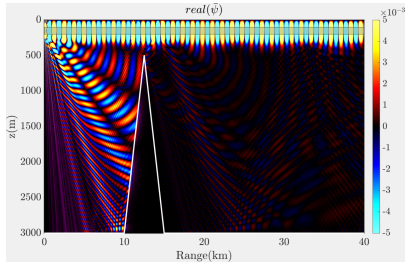
Fig. 2: MIT-MSEAS sound propagation simulation around the Atlantis II Seamount on 00Z, March 11, 2024. The effect of the stronger and sloping sound channel in the Slope water (right side of panels) can be seen by the mid-depth minimum in TL.

On that day, the Gulf Stream is impinging on Atlantis II. We compute the transmission loss fields (TL) in several 2D sections across the different seamounts using our MIT-MSEAS ParEq deterministic system [5]. We study the sensitivity to different source-receiver configurations (source/receiver depths, their separation, their number, etc.), source frequency, and ocean features (Gulf Stream state, eddy field, etc.).

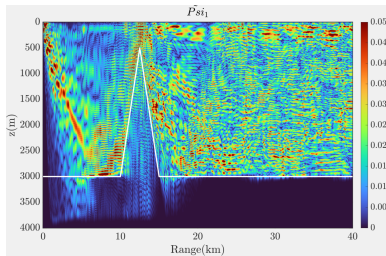
We employ our DO-ParEq to find and quantify the uncertain or variability features in the ocean fields and in the source depth and frequency that most affect underwater sound propagation. Figure 3 illustrates this use of the DO-ParEq for an idealized seamount case in deep water with a 40 km range propagation. The uncertain source depth is a random variable uniformly distributed between 50 m and 250 m. The other parameters are here deterministic. Using the DO-ParEq, we compute the DO decomposition of  $\psi$  and TL, i.e., the mean TL, mean pressure solution  $\psi$ , and  $\psi$  and TL DO modes and stochastic coefficients.



(a) Mean Transmission Loss (dB)



(b) Real part of the mean pressure solution (Pa)



(c) First DO mode of the pressure solution (non-dimensional)

Fig. 3: Stochastic DO-ParEq sound propagation simulation in an idealized seamount case using our MSEAS Dynamically-Orthogonal Parabolic WAPE Equations [8]. The source depth is a random variable distributed as  $Z_s \sim U[50, 250]m$ . Frequency = 50 Hz.

Finally, we evaluate the utilization of the DO-ParEq to simulate, decompose, and characterize the acoustic responses to variability and stochasticity in the ocean features, source depth, and location in our realistic stochastic ocean fields. Our results indicate that the DO-ParEq approach is a promising technique for classifying the environmental inputs that matter most for specific source-receiver parameters and configurations.

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