

## Hazard-Time Optimal Path Planning for Collaborative Air and Sea Drones

B. Schnitzler<sup>a,b</sup>, P. J. Haley, Jr.<sup>a</sup>, C. Mirabito<sup>a</sup>, E. M. Mule<sup>a</sup>, J.-M. Moschetta<sup>b</sup>, D. Delahaye<sup>c</sup>, A. Drouin<sup>c</sup> and P. F. J. Lermusiaux<sup>a,†</sup>

<sup>a</sup> Department of Mechanical Engineering, Massachusetts Institute of Technology, Cambridge, MA

<sup>b</sup> Department of Aerodynamics, Energy and Propulsion, Institut Supérieur de l’Aéronautique et de l’Espace, Toulouse, France

<sup>c</sup> OPTIM LAB, Ecole Nationale de l’Aviation Civile, Toulouse, France

<sup>†</sup>Corresponding author: pierrel@mit.edu

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The integrated optimization of autonomous air and marine platforms is becoming a grand challenge for the efficient utilization, monitoring, and protection of our environment and life on Earth. Applications include research, environmental monitoring, conservation, climate change mitigation, weather prediction, ocean forecasting, transport and distribution of goods, security, air-sea operations, communication, search and rescue, space and marine industry, and the blue economy. To achieve successful integrated air-sea autonomy in such applications, predicting hazards and reducing risks is critical, especially for autonomous vehicles with limited actuation or high costs. Much progress has been achieved in the past decade in either marine or air path planning [1–4]. Some efforts have included ocean risks [5–8] or air risks [9], but integrated air-sea applications are not yet commonplace [10].

In this work, we apply the MIT-MSEAS general partial differential equations for exact multi-objective reachability and optimal planning [3, 11–13] to guide autonomous air and sea drones that operate in fastest time in uncertain dynamic environments and steer clear of hazards along their path. For the first time, we combine weather, ocean, and environmental hazard forecasting with dynamic multi-objective optimal control to obtain hazard-time reachable sets, Pareto fronts, and optimal paths. Our first hazard-time optimal path planning application consists of an autonomous air drone that crosses the Atlantic Ocean optimizing travel time while avoiding hazardous rains. The second is the hazard-time optimal transport of an ocean vehicle by an air drone followed by a hazard-time optimal ocean mission. The air drone travels to a target location, drops the ocean vehicle, and the ocean vehicle completes its underwater mission in the fastest time, avoiding hazards. Other collaborative missions are also presented.

Our generic problem is that of computing the paths of drones that minimize both travel time in a dynamic flow environment  $\mathbf{V}(\mathbf{x}, t)$  and accumulated exposure to a dynamic hazard field  $\mathfrak{h}(\mathbf{x}, t)$ . Indeed we want to predict all Pareto-optimal paths for these two costs. The position vector in the physical environment space is denoted  $\mathbf{x}$ . To account for the hazard level, we add a dimension and denote the hazard level variable by  $\eta$ . In the  $(\mathbf{x}, \eta)$  augmented space [14], solving our problem consists in computing the augmented reachable set for the vehicle, *i.e.*, the set of all values  $(\mathbf{x}, \eta)$  that are reachable at some time  $t$ . We represent the reachable set by a scalar

function  $\phi(\mathbf{x}, \eta, t)$  whose subzero level set is the reachable set at time  $t$ . Denoting by  $v_{\max}$  the maximum speed of the drone or vehicle, this level set (value function) is governed by the following exact Hamilton-Jacobi-Bellman PDE [14, 15],

$$\frac{\partial \phi}{\partial t} + v_{\max} \left\| \frac{\partial \phi}{\partial \mathbf{x}} \right\| + \mathbf{V}(\mathbf{x}, t) \cdot \frac{\partial \phi}{\partial \mathbf{x}} + \mathfrak{h}(\mathbf{x}, t) \frac{\partial \phi}{\partial \eta} = 0. \quad (1)$$

In our applications, the physical space is the 2D planar space  $\mathbf{x} = (x, y)$ . The **rate of change of hazard field**  $\mathfrak{h}$  is either a spatiotemporal function of the hazardous precipitation field (rain)  $\mathfrak{p}(\mathbf{x}, t)$ , *i.e.*,  $\mathfrak{h} = \mathfrak{h}(\mathbf{x}, t, \mathfrak{p})$ , or a function of hazardous ocean currents or winds  $\mathbf{V}(\mathbf{x}, t)$ , *i.e.*,  $\mathfrak{h} = \mathfrak{h}(\mathbf{x}, t, \mathbf{V})$ .

The first application is the crossing of the Atlantic between Dakar, Senegal, and Natal, Brazil, by an unmanned aerial vehicle (UAV) with a cruising speed of  $23 \text{ m s}^{-1}$  100m above sea level. The hazard field is the ECMWF 3h-accumulated rain forecast issued on 2024-04-25 00:00Z. Snapshots of wind and rain fields are given in Fig. 1.

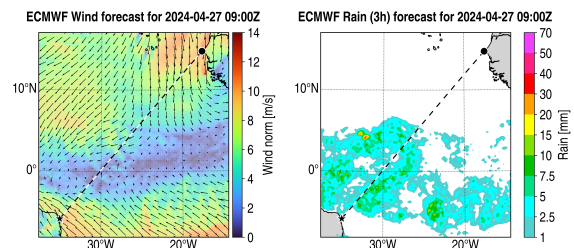


Fig. 1: Wind (100 m) and rain forecast snapshots from ECMWF. The path planning start point is depicted as a circle, the endpoint as a star, and between them the great circle is drawn. The projection is Lambert with one standard parallel at 5° north.

The air drone takes off on 2024-04-27, at 15:00Z. It flies to the destination the fastest possible while avoiding high accumulated exposure to rain, which is a proxy for difficult flight conditions (convective weather). The question is: will rain avoidance lead to significantly different paths from the fastest ones, both in travel time and shape of the path?

Solving eq. (1), we obtain the rain-travel-time Pareto-optimal paths. In Fig. 2, we show four snapshots of such paths for three optimal travel times and accumulated rain, the first path (blue) being the strictly time-optimal path (ignoring rain). We find that, while the time-optimal path ignores the rain field and thus sums up a given amount of rain, other Pareto optimal paths can lower the exposure to rain, with moderate changes in travel time and shape of the path. The presence of non-Pareto-optimal portions on the hazard-time graph confirms

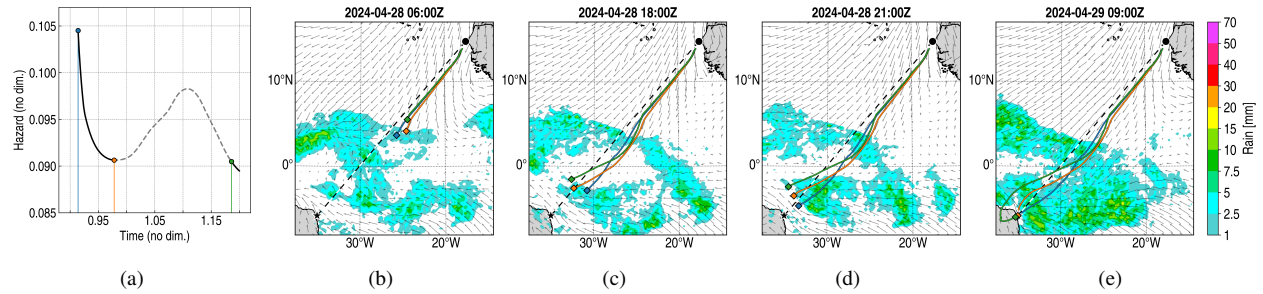


Fig. 2: Hazard-Time Reachability Analysis for an Air Drone Crossing the Atlantic. (a) Minimum total hazard for various travel times. Solid curves are Pareto optimal values. (b, c, d, e) Snapshots of hazard-time Pareto-optimal trajectories, overlaid on rain and wind fields.

that increasing the travel time is not a sufficient condition for lowering exposure to rain.

In the second application, an air drone (not shown) transports an ocean drone from Boston to a target location near the New England Seamounts (Fig. 3), in minimum time and avoiding atmospheric hazards. Once the ocean drone is dropped off, it completes its hazard-time optimal ocean mission in the fastest time while avoiding underwater hazards (Fig. 3).

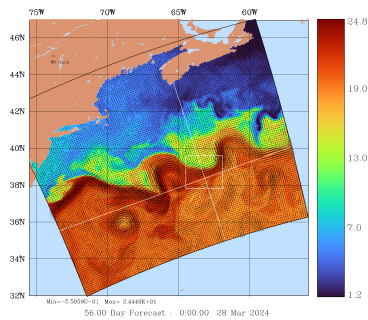


Fig. 3: Hazard-Time Collaborative Air-Sea Mission around the New England Seamounts: sea surface temperature overlaid with current vectors as hindcast by our multi-resolution MSEAS modeling system [16] including tides and atmospheric forcing.

All computations of the hazard-time reachable sets, Pareto fronts, and optimal paths are fast (seconds to minutes) compared to the mission durations. Multiple re-planning is possible as new weather, ocean, and hazard forecasts become available. Our results demonstrate that predicting hazard-time optimal paths for air and sea drones is feasible and useful for multiple applications, including for collaborative air-sea missions.

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