Risk-Optimal Path Planning in Stochastic Dynamic Environments

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Abstract

We combine decision theory with fundamental stochastic time-optimal path planning to develop partial-differential-equations-based schemes for risk-optimal path planning in uncertain, strong and dynamic flows. The path planning proceeds in three steps: (i) predict the probability distribution of environmental flows, (ii) compute the distribution of exact time-optimal paths for the above flow distribution by solving stochastic dynamically orthogonal level set equations, and (iii) compute the risk of being suboptimal given the uncertain time-optimal path predictions and determine the plan that minimizes the risk. We showcase our theory and schemes by planning risk-optimal paths of unmanned and/or autonomous vehicles in illustrative idealized canonical flow scenarios commonly encountered in the coastal oceans and urban environments. The step-by-step procedure for computing the risk-optimal paths is presented and the key properties of the risk-optimal paths are analyzed.

Keywords:
Stochastic Path Planning, Level Set Equations, Dynamically Orthogonal, Ocean Modeling, AUV, Uncertainty Quantification

1. Introduction

In recent years, the use of autonomous platforms such as Autonomous Underwater Vehicles, gliders, floats, drones, and Unmanned Aerial Vehicles has rapidly increased, for both underwater and aerial applications. These vehicles operate in dynamic environments with strong and uncertain currents/winds that affect their motion. To pilot these vehicles efficiently between any two locations, it is important to utilize environmental predictions to plan paths that optimize travel time, energy consumption, data collection and/or safety. Such environmental predictions are, however, often uncertain and path planning has to rigorously account for these probabilistic predictions, if available. Recently, we developed governing stochastic partial differential equations and their efficient stochastic Dynamically Orthogonal (DO) counterparts for time-optimal path planning in uncertain, strong and dynamic flow fields (Subramani et al., 2018). With these equations, we can forecast the probability distribution of reachability fronts and time-optimal paths. Such probabilistic predictions raise important new questions: What is the optimal path choice under such uncertainty? Can/should concepts from rational decision making be utilized for informing this optimal choice? What is an appropriate risk measure and optimality criterion for such choices? Can such paths be computed efficiently? We answer these questions in the present paper.
The present work is based on fundamental differential equations that govern the evolution of the reachability and time-optimal paths in strong and dynamic currents (Lolla et al., 2014b), and is related to energy-optimal paths (Subramani et al., 2015; Subramani and Lermusiaux, 2016). These equations were used to compute optimal paths, both in realistic data-driven simulations (Lolla et al., 2014a; Subramani et al., 2017a) and with real vehicles (Subramani et al., 2017b). They were also employed to solve pursuit evasion problems (Sun et al., 2017a,b). The input probabilistic predictions of flow fields are obtained from the variance-optimal reduced-order stochastic dynamically orthogonal equations (Sapsis and Lermusiaux, 2009; Ueckermann et al., 2013; Feppon and Lermusiaux, 2018). The resulting stochastic PDEs (S-PDEs) for time-optimal path planning have some advantages (Subramani et al., 2018): (i) for a given stochastic environmental flow prediction, the stochastic time-optimal paths are exact; (ii) the computed paths naturally avoid stationary and dynamic obstacles; and (iii) the probability of a location being reachable (or non-reachable) are directly predicted.

From the probability distribution of time-optimal paths, we have to assess the risks and make a decision of risk-optimal paths. This subject of decision making under uncertainty has been well studied in the fields of economics and management. One widely used model is the expected utility theory and its several variants as reviewed in Schoemaker (1982). The key ingredients of this model are the evaluation of the utility cost of the outcome due to a decision and the probability of that outcome. The expected utility theory can be utilized in a prescriptive or normative framework to inform optimum decision making under complex decision scenarios and can be customized to fit the risk behavior of users (Schoemaker, 1982; Epstein, 1992; Von Neumann and Morgenstern, 2007). Specific utility functions are also available for different risk behaviors (e.g. Arrow, 1958; Fishburn, 1988; LiCalzi and Sorato, 2006).

Our goal here is to combine a principled risk optimality criterion grounded in decision theory with our stochastic dynamically orthogonal level-set equations to develop efficient computational schemes to predict risk-optimal paths from the distribution of stochastic time-optimal paths. We also seek to apply the new schemes to several stochastic flow scenarios, analyze the effects of different risk metrics and criteria, and discuss the properties of risk-optimal paths.

The remainder of the present paper is organized as follows. Next, we provide a brief review of prior path planning results. In Sec. 2, we develop the theory and schemes for risk-optimal path planning. In Sec. 3, we apply the new schemes to compute risk optimal paths for a variety of stochastic flow scenarios. In Sec. 4, we conclude and provide future research directions.

1.1. Previous Progress in Optimal Path Planning

Traditionally, path planning has focused on land-based robots in stationary environments (e.g., Hwang and Abuja, 1992; LaValle, 2006; Latombe, 2012); however, a major challenge for marine and aerial platforms is the effects of the uncertain, strong and dynamic currents/winds. Several authors have extended many of the algorithms for static environments to plan paths of autonomous vehicles in dynamic environments (for reviews, see e.g., Lolla et al. (2014b); Pereira et al. (2013)). These applications include graph based search methods such as the modified Dijkstra’s algorithm (Mannarini et al., 2016), A*-search (Garau et al., 2005), RRTs (Rao and Williams, 2009), kinematic tree-based navigation (Chakrabarty and Langeland, 2013), stochastic planners with uncertain edge weights (Wellman et al., 1995), and stochastic surface response methods (Kewlani et al., 2009). Other techniques such as nonlinear optimization methods (Kruger et al., 2007; Witt and Dunbabin, 2008), sequential quadratic programming (Beylkin, 2008), evolutionary algorithms (Alvarez et al., 2004; Aghababa, 2012), fast marching methods (Sethian, 1999; Petres et al., 2007), wave front expansion (Soulignac et al., 2009; Thompson