# Dynamically Orthogonal Equations for Stochastic Underwater Sound Propagation

by

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B.E., Mechanical Engineering (2016) American University of Beirut, Lebanon

Submitted to the Center for Computational Engineering in partial fulfillment of the requirements for the degree of

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#### Abstract

Grand challenges in ocean acoustic propagation are to accurately capture the dynamic environmental uncertainties and to predict the evolving probability density function of stochastic acoustic waves. This is due to the complex ocean physics and acoustics dynamics, nonlinearities, multiple scales, and large dimensions. There are several sources of uncertainty including: the initial and boundary conditions of the ocean physics and acoustic dynamics, the bathymetry and seabed fields; the models parameters; and, the models themselves. In the present work, we start addressing these challenges by deriving, implementing and verifying new optimally-reduced Dynamically Orthogonal (DO) differential equations that govern the propagation of stochastic acoustic waves for underwater sound propagation in an uncertain ocean environment. The developed methodology allows modeling environmental uncertainties in a rigorous probabilistic framework and predicting the uncertainties of acoustic fields, fully respecting the nonlinear governing equations and non-Gaussian statistics of the sound speed and acoustic state variables. The methodology is applied to the standard narrow-angle parabolic equation and is utilized to predict acoustic field uncertainties for three new stochastic idealized test cases: (1) an uncertain Pekeris waveguide with penetrable bottom, (2) an uncertain horizontal interface problem, and (3) an uncertain range-dependent sloping interface problem. For the first case, the solutions of the DO acoustic equations are validated against those obtained using standard Monte Carlo sampling. The second test case showcases results for predicting acoustic field probabilities due to uncertainties in the location of a sound speed channel. For the third test case, the advantages of the DO acoustic equations in predicting uncertainties in complex range-dependent environments are highlighted.

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