Time-Optimal Multi-Waypoint Mission Planning in Dynamic Environments

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Introduction. Autonomous underwater vehicles (AUVs) are currently fielded world-wide by commercial companies, militaries, and research institutions, and their use will only increase in the coming years. From a navigational standpoint for naval operations, a growing area of emphasis is “multi-waypoint missions”, i.e., missions in which a AUV travels to multiple target locations in the ocean for completing tasks [1]. For such missions, reducing operational costs by optimally utilizing ocean flow forecasts for navigation is crucial.

Recently, we developed partial differential equations (PDEs), efficient numerical schemes and computational systems to compute exact time-optimal paths [3], energy-optimal paths [4] in strong and dynamic deterministic currents, and stochastic time-optimal paths [6] in uncertain ocean currents. We also demonstrated our path planning in realistic ocean re-analysis, and in realtime with real AUVs [5].

In the present paper, we focus on demonstrating the use of our exact equations to predict time-optimal mission plans for a marine vehicle that visits multiple locations in a dynamic ocean flow field predicted by a data-assimilative ocean modeling system. These missions begin and end in the same location and visit a finite number of waypoints in the minimal time; this problem bears close resemblance to that of the classic “traveling salesman”, albeit with the added complexity of a continuously changing flow field.

Theory and Schemes. The exact time-optimal reachability front of a vehicle navigating in a strong and dynamic flow field is governed by a Hamilton Jacobi PDE [3]. The multi-waypoint mission planning proceeds in four steps. First, current forecasts for the planning horizon in the domain of interest is obtained, for which we utilize our data-driven 4-D primitive equation ocean modeling system (MSEAS [2]) forced by high-resolution tidal and real-time atmospheric forcing fields. Second, all tour permutations are enumerated. Third, the time-optimal PDEs are solved for each leg of the enumerated tour. The equations grow a reachability front from the starting location in all directions. Whenever the front reaches a waypoint, a new reachability front is immediately started from that location. This process continues until one set of reachability fronts has reached all goal waypoints and has returned to the original location. The process continues until one set of reachability fronts has reached all goal waypoints and has returned to the original location. The time-optimal path for the entire mission is then obtained by trajectory backtracking, going through the optimal set of reachability fields in reverse order. Due to the spatial and temporal dynamics, a varying start time results in different paths and durations for each leg and requires all permutations of travel to be calculated. Fourth, the permutation for which travel time is minimized is identified.

Even though the method is very efficient and the optimal path can be computed serially in real-time for common naval operations, for additional computational speed, a high-performance computing cluster was used to solve the level set calculations in parallel.

Applications. We illustrate our theory and distributed implementation by showcasing several idealized and realistic applications. The idealized examples utilize canonical flow fields such as a simulated river crossing and mission in a sinusoidally varying flow. For the realistic applications, we plan several simulated multi-waypoint missions with progressively increasing way-points in the Phillipines Archipelago. This region has complex geometry and strong dynamic currents. There are large-scale open ocean dynamics as well as small scale dynamics around islands, through narrow straits, and over steep shelfbreaks. Of the several that we completed, we present two large three-waypoint missions of durations assumed within the range of the U.S. Navy’s XLUUV (less than 30 days). We also show smaller five and six-waypoint missions within the U.S. Navy’s LDUUV capability (less than 5 days). Fig. 1 shows the results for one of the large three way-point missions. Here, a vehicle leaves the port of Ormoc and visits the shipwreck sites of USS Ommaney Bay lying West of Panay Island, USS Samuel B. Roberts lying East of the Semirara Islands, and USS Princeton lying east of Lamon Bay. As the crow flies, the targets are 200, 110, and 285 nautical miles from the starting location. Assuming a constant nominal relative vehicle-speed of 3 knots, the time optimal path, ABCDA, completes the mission in 15.80 days and the slowest path, ACBDA, would take 2.35 days (15%) longer.
<table>
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<th>Total Delta (Days)</th>
<th>% of Optimum</th>
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Figure 1: Three-Waypoint mission in the Philippines Archipelago. (a) Time-optimal paths for all tour permutations; (b) Mission duration of all tours and their deviation from the optimum tour; (c) The optimum tour computed by our approach. The paths are colored by the effective vehicle speed and the vectors represent the instantaneous currents encountered by the vehicle.

Fig. 1a shows that the complex geography of the islands forced tight constraints on the vehicle paths. The area where current can be seen to play the biggest role is the divergence of paths from C to D in the Western area. Critically, depending on the time which the vehicle would pass the small crescent island in the South-Western area (Homonhon Island) the vehicle either went to the East or West of the island. Fig. 1b shows the mission durations of all tour permutations and Fig. 1c shows the optimum tour. We also complete other complex missions with more way-points.

**Conclusion.** We demonstrate a novel application of exact time-optimal planning PDEs to complete multi-waypoint mission planning in strong and dynamic ocean flows. Since our approach calculates the global optimum, it serves two purposes. It can be either used in its present form to plan multi-waypoint missions offline in conjunction with a predictive ocean current modeling system, or it can be used as a litmus test for future algorithmic solutions to the traveling salesman problem in dynamic flow fields.

**References**


