Neural Closure Models for Chaotic Dynamical Systems

by

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Abstract

An important challenge in the problem of producing accurate forecasts of multiscale
dynamics, including but not limited to weather prediction and ocean modeling, is
that these dynamical systems are chaotic in nature. A hallmark of chaotic dynamical
systems is that they are highly sensitive to small perturbations in the initial condi-
tions and parameter values. As a result, even the best physics-based computational
models, often derived from first principles but limited by varied sources of errors,
have limited predictive capabilities for both shorter-term state forecasts and for im-
portant longer-term global characteristics of the true system. Observational data,
however, provide an avenue to increase predictive capabilities by learning the physics
missing from lower-fidelity computational models and reducing their various errors.
Recent advances in machine learning, and specifically data-driven knowledge-based
prediction, have made this a possibility but even state-of-the-art techniques in this
area have not been able to produce short-term forecasts beyond a small multiple
of the Lyapunov time of the system, even for simple chaotic systems such as the Lorenz
63 model. In this work, we develop a training framework to apply neural ordinary
differential equation-based (nODE) closure models to correct errors in the equations
of such dynamical systems. We first identify the key training parameters that have an
outsize effect on the learning ability of the neural closure models. We then develop a
novel learning algorithm, broadly consisting of adaptive tuning of these parameters,
designing dynamic multi-loss objective functions, and an error-targeting batching pro-
cess. We evaluate and showcase our methodology to the chaotic Balance Equations
in an array of increasingly difficult learning settings: first, only the coefficient of one
missing term in one perturbed equation; second, one entire missing term in one per-
turbed equation; third, two missing terms in two perturbed equations; and finally
the previous but with a perturbation being two orders of magnitude larger than the
state, thereby resulting in a completely different attractor. In each of these cases, our
new multi-faceted training approach drastically increases both state-of-the-art state
predictability (upto 15 Lyapunov times) and attractor-reproducibility. Finally, we
validate our results by comparing them with the predictability limit of the chaotic
BE system under different magnitudes of perturbations.

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