Towards Coupled Nonhydrostatic-Hydrostatic Hybridizable Discontinuous Galerkin Method

by

Aditya Karthik Saravanakumar

Submitted to the Center for Computational Science and Engineering in partial fulfillment of the requirements for the degree of Master of Science in Computational Science and Engineering at the

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Abstract

Numerical modelling of ocean physics is essential for multiple applications such as scientific inquiry and climate change but also renewable energy, transport, autonomy, fisheries, water, harvesting, tourism, communication, conservation, planning, and security. However, the wide range of scales and interactions involved in ocean dynamics make numerical modelling challenging and expensive. Many regional ocean models resort to a hydrostatic (HS) approximation that significantly reduces the computational burden. However, a challenge is to capture and study local ocean phenomena involving complex dynamics over a broader range of scales, from regional to small scales, and resolving nonlinear internal waves, subduction, and overturning. Such dynamics require multi-resolution non-hydrostatic (NHS) ocean models. It is known that the main computational cost for NHS models arises from solving a globally coupled elliptic PDE for the NHS pressure. Optimally reducing these costs such that the NHS dynamics are resolved where needed is the motivation for this work.

We propose a new multi-dynamics model to decompose a domain into NHS and HS dynamic regions and solve the corresponding models in their subdomains, reducing the cost associated with the NHS pressure solution step. We extend a high-order NHS solver developed using the hybridizable discontinuous Galerkin (HDG) finite element methodology by taking advantage of the local and global HDG solvers for combining HS with NHS solvers. The multi-dynamics is derived, and the first version is implemented in the HDG framework to quantify computational costs and evaluate accuracy using several analyses. We first showcase results on Rayleigh Taylor instability-driven striations to evaluate computational savings and accuracy compared to the standard NHS HDG and finite-volume solvers. We highlight and discuss sensitivities and performance. Finally, we explore parameters that can be used to identify domain regions exhibiting NHS behaviour, allowing the algorithm to dynamically evolve the NHS and HS subdomains.

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