Neural Closure Model for Dynamic Mode Decomposition Forecasts

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Motivation

Dynamic Mode Decomposition (DMD) is a data-driven, equation-free dimensionality reduction algorithm [1, 6, 7, 8, 10] that constructs an approximately linear operator for a sequential data set.

$$\frac{d\mathbf{x}}{dt} = \mathbf{f}(\mathbf{x}, t; \mu) \longrightarrow \frac{d\mathbf{x}}{dt} = \mathscr{A}\mathbf{x}$$

DMD finds an analogous discrete best-fit $\mathbf{A}$ that aims to minimize the following

$$||\mathbf{X}' - \mathbf{A}\mathbf{X}||_F$$

where $\mathbf{X} = [\mathbf{x}_1 \ \ \mathbf{x}_2 \ \ \cdots \ \ \mathbf{x}_k]$ and $\mathbf{X}' = [\mathbf{x}_2 \ \ \mathbf{x}_3 \ \ \cdots \ \ \mathbf{x}_{k+1}]$.

DMD can be used as a computationally efficient forward model to provide forecasts of the ocean [4, 5, 11, 12]

Challenges

- Missing dynamics
- Unresolved sub-grid scale processes when DMD Is applied to low-fidelity simulations
- Static DMD may be irrelevant over time
- Truncated modes

Goal: Learn a neural closure model between low-fidelity DMD model and high-fidelity data

Theory

Consider a general, full, nonlinear dynamical model written as

$$\frac{du_k(t)}{dt} = R_k(u(t), t)$$

Upon using the Mori-Zwanzig (MZ) formulation and applying the P-projection, it could be rewritten as the following

$$\frac{\partial}{\partial t} u_k(\hat{u}_0, t) = \underbrace{PR_k(\hat{u}(\hat{u}_0, t))}_{\text{Low-fidelity}} + P\int_0^t \underbrace{K_k(\hat{u}(u_0, t-s)}_{\text{Memory}} ds$$

Hence for such systems, the closure model only considers the non-Markovian memory term, which requires time-lagged state information [2, 9].

The memory term can be represented using a neural delayed differential equations (nDDEs) for a hybrid closure model [5]

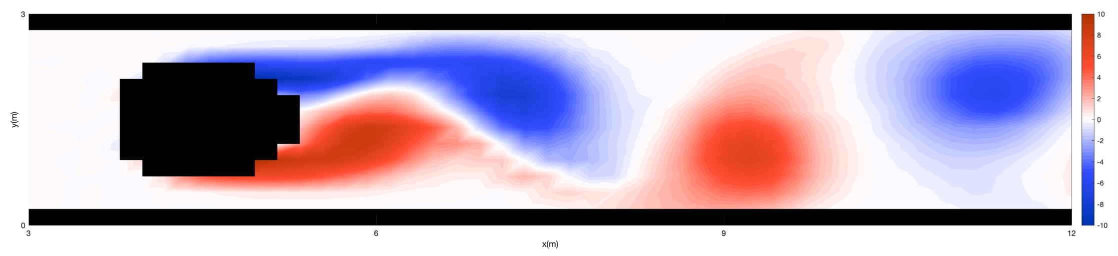
$$\frac{\partial \hat{u}(t)}{\partial t} = \underbrace{PR(\hat{u}(t))}_{\text{Low-fidelity model}} + f_{RNN}(\hat{u}(t), \underbrace{\hat{u}(t-\tau_1), \ldots \hat{u}(t-\tau_k)}_{\text{Neural Closure}}, t; \theta)$$

The amount of delay to be used also becomes a hyper parameter to tune in the nDDEs closure model

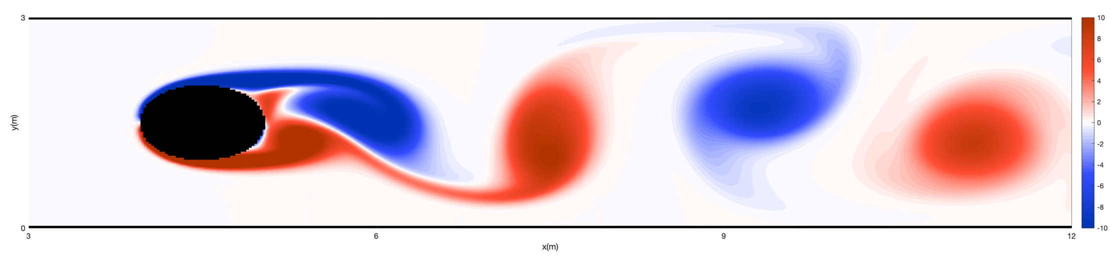
The linearity of DMD allows for simple calculation of the Jacobian, which is needed for efficient back propagation using the adjoint sensitivity method

Experiment: 2D Flow Behind a Cylinder

Low-resolution:
13 x 103



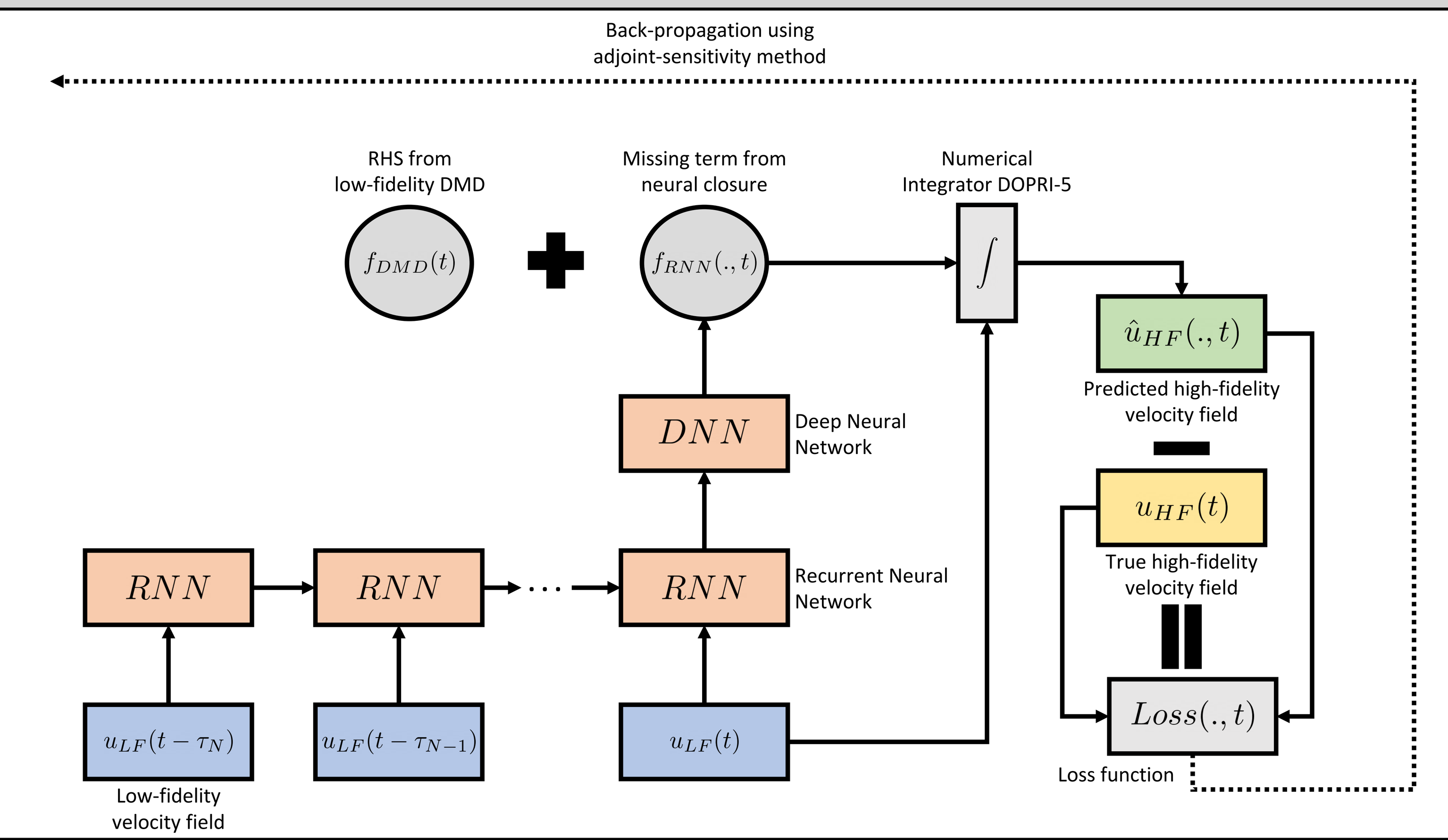
High-resolution:
105 x 855



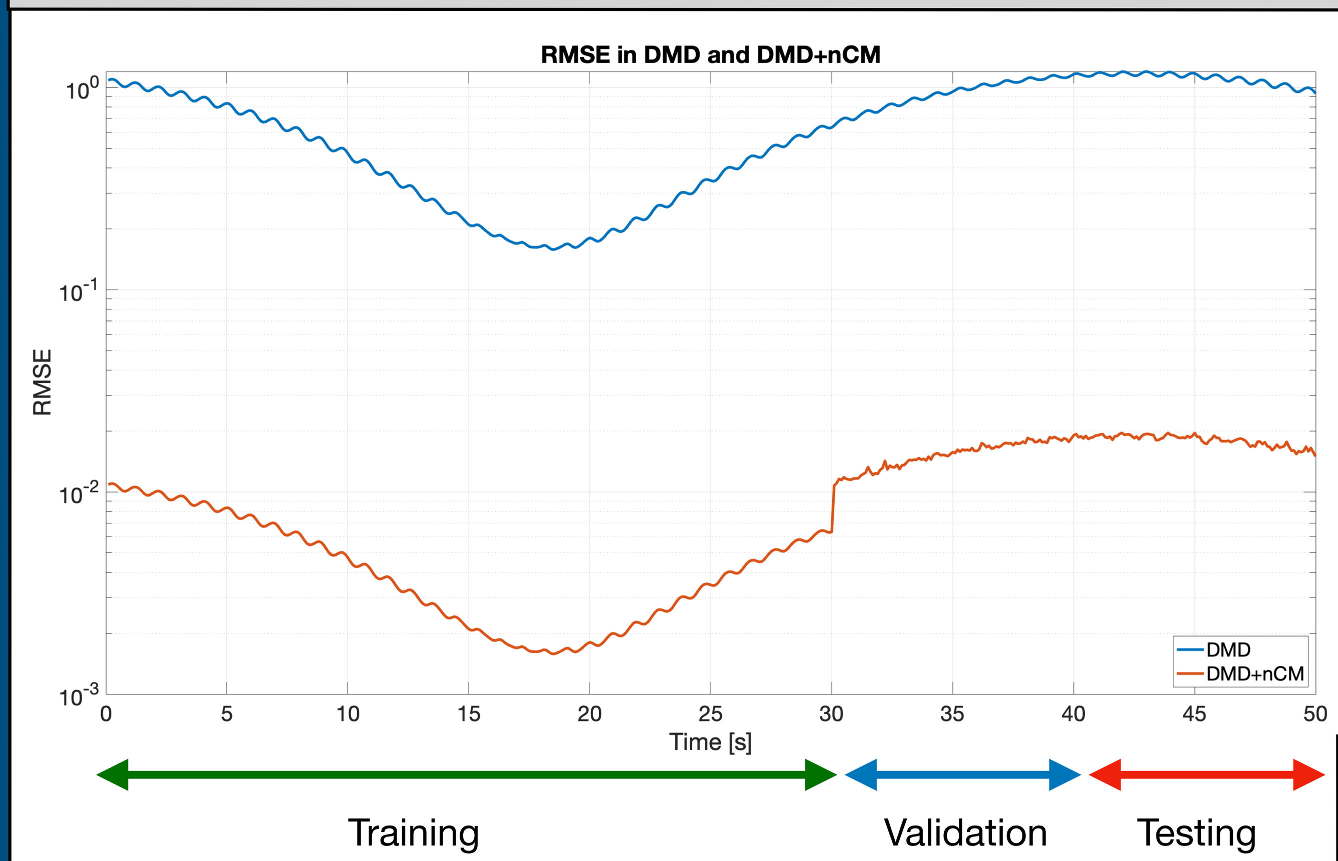
Remarks:

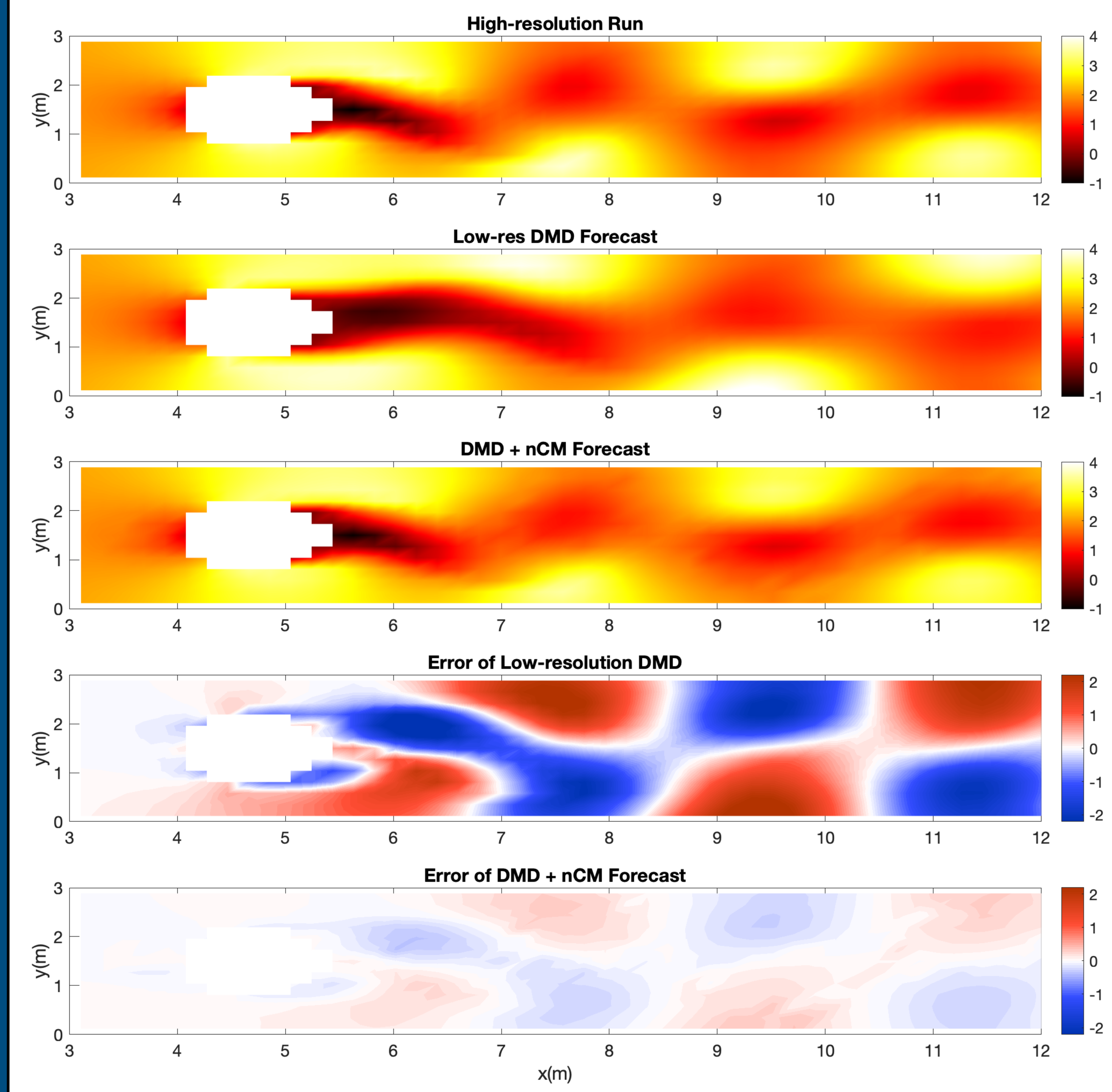
- Low-resolution simulation used to train the low-fidelity DMD model
- High-resolution simulation considered as high-fidelity data for the neural closure model
- Reynold's number (Re) = 200
- The low-resolution simulation sees a distorted square-shaped cylinder leading to:
  - I) longer recirculation region
  - II) different vortex shedding frequency when compared to the high-resolution simulation
- We only consider the **u-velocity** in this example here

Architecture



Results





Remarks:

- High-fidelity data was down-sampled to the low-resolution grid to maintain consistent dimensions when calculating errors and losses
- Error decreased by 2 orders of magnitudes
- DMD + nCM is able to correct for the difference in vortex shedding frequency

Follow the link or scan the QR code below for a video!

shorturl.at/dPST9

References

[1] Kevin K. Chen, Jonathan H. Tu, and Clarence W. Rowley. Variants of dynamic mode decomposition: Boundary condition, Koopman, and Fourier analyses. Journal of Nonlinear Science, 22:887-915, 2012.

[2] D. S. Glass, X. Jin, I. H. Riedel-Kruse. Nonlinear delay differential equations and their application to modeling biological network motifs. Nat. Commun. 12, 1-19. 2021.

[3] A. Gupta and P. F. J. Lermusiaux. Neural closure models for dynamical systems. Proc. R. Soc. A 477:20201004. August 2021.

[4] J. P. Heuss, P. J. Haley, Jr., C. Mirabito, E. Coelho, M. C. Schönau, K. Heaney, and P. F. J. Lermusiaux. Reduced order modeling for stochastic prediction onboard autonomous platforms at sea. In OCEANS 2020 IEEE/MTS, pages 1–10. IEEE, October 2020.

[5] Jacob P. Heuss. Reduced order modeling for stochastic prediction and data assimilation onboard autonomous platforms at sea. Master's thesis, Massachusetts Institute of Technology and Woods Hole Oceanographic Institute, Cambridge, Massachusetts, September 2021.

[6] J. N. Kutz, S. L. Brunton, B. W. Brunton, and J. J. Proctor. Dynamic Mode Decomposition: Data-Driven Modeling of Complex Systems. SIAM, Philadelphia, Pennsylvania, 2016.

[7] C. W. Rowley, S. B. Mezic, P. Schlatter, and D. S. Henningson. Spectral analysis of nonlinear flows. Journal of Fluid Mechanics 641:115-127, December 2009.

[8] P. J. Schmid. Dynamic mode decomposition of numerical and experimental data. Journal of Fluid Mechanics, 656:5-28, August 2010.

[9] P. Stinis. Renormalized Mori-Zwanzig reduced-order models for systems without scale separation. Proc. R. Soc. A. 471, 20140446. 2015.

[10] J. H. Tu, C. W. Rowley, D. M. Luchtenburg, S. L. Brunton, and J. N. Kutz. On Dynamic mode decomposition: Theory and applications. Journal of Computational Dynamics, 1(2):391-421, January 2014.

[11] T. Ryu, J. P. Heuss, P. J. Haley Jr., C. Mirabito, E. Coelho, P. Hursky, M. C. Schönau, K. Heaney, and P. F. J. Lermusiaux. Adaptive stochastic reduced order modeling for autonomous ocena platforms. In OCEANS 2021 IEEE/MTS, pages 1-9. IEEE, September 2021.

[12] T. Ryu, W. H. Ali, P. J. Haley Jr., C. Mirabito, A. Charous, P. F. J. Lermusiaux. Incremental low-rank dynamic mode decomposition model for efficient dynamic forecast dissemination and onboard forecasting. In OCEANS 2022 IEEE/MTS, October 2022. Sub-judice.

Conclusion and Future Work

Conclusion: Learned a neural closure model based on discrete delay differential equations using a low-fidelity DMD model

Future Works:

- Generalize further to 3D multivariate inputs and multi-field predictions
- Demonstrate on realistic ocean test cases

Acknowledgements

We are grateful to the Office of Naval Research for partial support under the grant no. N00014-20-1-2023 (MURI ML-SCOPE). We also thank members of our MSEAS group for their collaboration, especially Aman Jalan and Abhinav Gupta.