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Optimal Dynamic Formation and Coverage for Autonomous Platforms in Multiscale Ocean Flows

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Autonomous marine platforms are becoming essential tools for ocean science and operations because they can collect observations over large areas and long durations while reducing the cost, risk, and logistical burden of ship-based campaigns. They are now used for environmental monitoring, adaptive sampling, ocean forecasting, upper-ocean and air–sea observations, infrastructure inspection, and other persistent observing tasks [1–7].

However, there is increasing value in operating these vehicles as coordinated teams rather than as isolated platforms. Multi-vehicle systems can sample larger regions, observe evolving phenomena at multiple locations simultaneously, and adapt more effectively to dynamic environmental features [6, 8–12]. To realize these advantages, vehicles should remain sufficiently spread over the region of interest so that different parts of the environment are sampled and measurements do not cluster inefficiently in only a small portion of the domain. It can also be beneficial for the team to preserve an organized group structure, since maintaining prescribed relative positions, shapes, or coverage properties allows the vehicles to function as a coordinated sensing array for feature tracking and cooperative sampling [9, 11, 13–15].

In this work, we study the coordinated control of multiple autonomous marine vehicles in dynamic flow environments using the MIT-MSEAS general partial differential equations for reachability and optimal planning [7, 11, 16, 17]. We consider two related problems. The first is a formation-maintenance problem, in which a team of vehicles must travel through an unsteady flow while remaining in a specific configuration, so that the group retains a coherent geometry as it moves from an initial to a target region. The second is a stationary or dynamic coverage problem, in which vehicles must remain sufficiently distributed over a prescribed area. We partition this region of interest into subregions or grid cells, and seek to ensure that one vehicle remains within each cell in the presence of ambient advection. We investigate these questions first in canonical analytical flows, including rotational flows, the double-gyre, and flow past a cylinder, and then in realistic ocean flows, such as along the coast of California.

Our first problem is to compute minimum-time coordinated paths in a dynamic flow $V(\boldsymbol{x}, t)$ for vehicles with a maximum vehicle speed F. We use a leader–follower approach (or group-vehicles description), where we compute the time-optimal path for a leader and choose reachable intermediate positions for followers that best preserve the formation [11]. The leader's time-optimal trajectory $X_L(t)$ and travel time T_L are obtained from the level-set Hamilton–Jacobi equation,

$$\frac{\partial \phi}{\partial t} + F|\nabla \phi| + V(x,t) \cdot \nabla \phi = 0,$$

whose zero level set gives the reachability front and backtracking yields the optimal path. We then partition $[0, T^L]$ into times $0 = T^0 < \cdots < T^i < \cdots < T^N = T^L$. Over each interval, each follower k computes its short-time reachability front ∂R_k^{i+1}, and the next follower positions are selected by

$$\left(y_1^{i+1}, \ldots, y_{N_f}^{i+1}\right) = \arg \min_{z_k \in \partial R_k^{i+1}} \gamma\left(X_L(T^{i+1}), z_1, \ldots, z_{N_f}\right),$$

where γ penalizes deviations from a desired polygonal formation. The follower paths are then recovered by backtracking from the optimized points.

For our second problem of optimal coverage, we propose a novel solution method that is able to find controls so that each vehicle k remains inside an assigned grid cell or subdomain $D_k(t)$. We define a boundary function $g_k(x)$ such that $g_k(x) \leq 0$ if and only if $x \in D_k$, with $g_k(x) = 0$ on ∂D_k. We then define a new value function

$$W_k(x,t) = \inf_{\|u(\cdot)\| \leq F} \max_{\tau \in [t,T]} g_k(x(\tau)),$$

which measures the smallest worst-case boundary violation over the remaining time horizon. Thus, $W_k(x,t) \leq 0$ if and only if there exists a control that keeps the trajectory inside D_k for all $\tau \in [t, T]$, and the corresponding viability kernel is $\mathcal{K}_k(t) = \{x : W_k(x,t) \leq 0\}$. The new value function satisfies the Hamilton–Jacobi variational inequality

$$\min\left\{\frac{\partial W_k}{\partial t} + V(x,t) \cdot \nabla W_k - F|\nabla W_k|,\ g_k(x) - W_k(x,t)\right\} = 0,$$

with terminal condition $W_k(x,T) = g_k(x)$. Using insight from [18–22], stationary or dynamic coverage is then feasible precisely when each vehicle's initial condition lies in the viability kernel of its assigned grid, and the associated feedback control is obtained from the gradient of W_k, pushing the vehicle away from the boundary whenever the flow advects it toward escape.

We show an example of maintaining a square-formation for four vehicles along the coast of Southern California [23] in Figure 1, extending [11] beyond triangular shapes. In Figure 2, we show trajectories for vehicles maintaining coverage for each assigned cell in a square domain under an analytical double-gyre flow.

Our results demonstrate that level-set reachability provides a unified framework for computing formation preserving trajectories and coverage maintaining controls in dynamic ocean flows. They showcase opportunities for planning methods that explicitly couple navigation, formation maintenance, and coverage constraints. Our canonical and realistic flow test

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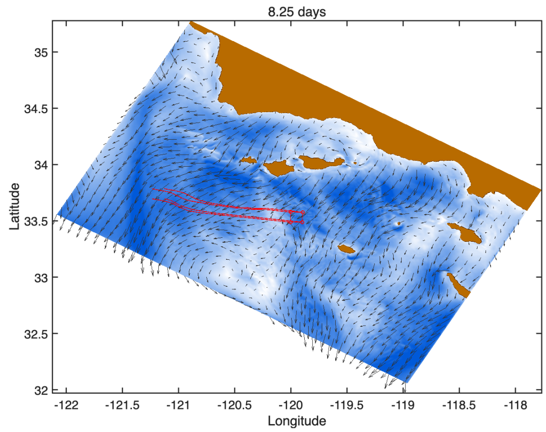

Fig. 1: Trajectories of surface vehicles in a square formation along the coast of Southern California. Currents estimated by the MSEAS ocean modeling system from June 2018 to August 2018 [23].

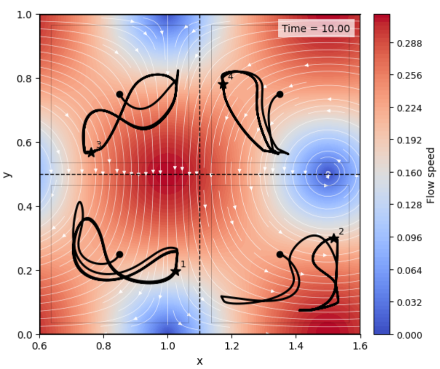

Fig. 2: Trajectories of Vehicles Remaining in Square Grids under a Double-Gyre flow. We show starting points as a circle and final positions as a star.

cases provide benchmarks for future development of robust multi-vehicle autonomy in complex ocean environments.

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