Modified dynamic mixed subgrid-scale models for geophysical flows: Forced two-dimensional and beta-plane turbulence

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Abstract

Subgrid-scale (SGS) models for large-eddy simulations (LES) of geophysical turbulence typically need to balance dissipative regularization with backscatter, the upscale transfer of energy from unresolved to resolved scales. Dynamic mixed models (DMMs) combine functional eddy viscosity and structural closures through dynamically estimated coefficients that are least-squares optimal with respect to the Germano identity error (GIE). We show that this classical DMM least-squares estimation can be dominated by the structural component, thereby limiting the functional component's dissipative regularizing role. To address this limitation, we decompose the Gram matrix formed from the inner products of the functional and structural contributions to the GIE based on their self- and cross-interactions. We then develop a modified Gram-based framework to construct a novel parametric family of fully-coupled, sequential, and fully-decoupled DMMs with tunable structural-functional balance. Using an idealized forced two-dimensional mesoscale and beta-plane turbulence framework with the Leith model as the functional component and the fourth-order nonlinear gradient model as the structural component, we evaluate the resulting closures *a priori* and *a posteriori* across eddy ($\beta = 0$) and jet ($\beta > 0$) regimes at two LES resolutions. The *a priori* results show that structurally-dominated models achieve strong agreement with the *ideal* SGS forcing and accurately reproduce local SGS energy exchange, including backscatter. However, these instantaneous improvements do not directly translate into long-term LES accuracy: in *a posteriori* tests, structurally dominated models exhibit noise-like artifacts in the vorticity fields with high-wavenumber spectral deviations, indicating insufficient net dissipation. In contrast, the sequential DMM in which the functional component is determined first and then corrected by the structural component retains much of the *a priori* structural accuracy while improving the *a posteriori* vorticity fields, spectra, and domain-averaged diagnostics. Spectral SGS energy and enstrophy-transfer analyses show that this sequential DMM permits backscatter at scales larger than the forcing scale with enhanced dissipation at smaller scales, thereby improving the balance between instantaneous structural fidelity and long-term accuracy.

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